Large classical universes emerging from quantum cosmology

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It is generally believed that one cannot obtain a large Universe from quantum cosmological models without an inflationary phase in the classical expanding era because the typical size of the Universe after leaving the quantum regime should be around the Planck length, and the standard decelerated classical expansion after that is not sufficient to enlarge the Universe in the time available. For instance, in many quantum minisuperspace bouncing models studied in the literature, solutions where the Universe leave the quantum regime in the expanding phase with appropriate size have negligible probability amplitude with respect to solutions leaving this regime around the Planck length. In this paper, I present a general class of moving gaussian solutions of the Wheeler-DeWitt equation where the velocity of the wave in minisuperspace along the scale factor axis, which is the new large parameter introduced in order to circumvent the abovementioned problem, induces a large acceleration around the quantum bounce, forcing the Universe to leave the quantum regime sufficiently big to increase afterwards to the present size, without needing any classical inflationary phase in between, and with reasonable relative probability amplitudes with respect to models leaving the quantum regime around the Planck scale. Furthermore, linear perturbations around this background model are free of any transplanckian problem.

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I. INTRODUCTION

The existence of an initial singularity [1] is one of the major drawbacks of classical cosmology. In spite of the fact that the standard cosmological model, based in classical general relativity sourced by ordinary matter, has been successfully tested until the nucleosynthesis era, the extrapolation of this model to higher energies leads to a breakdown of the geometry in a finite cosmic time. It indicates the failure of this conventional approach at high energies, which should be complemented through the intervention of some new physics (presence of exotic matter, modifications of general relativity through non-minimal couplings, non linear curvature terms in the lagrangian, quantum effects of the gravitational field , etc), leading to a complete regular cosmological model.

In the framework of quantum cosmology in minisuperspace models, non singular bouncing models have been obtained[1], where the bounce occurs due to quantum effects in the background [6–8]. Some approaches have used an ontological interpretation of quantum mechanics, the Bohm-de Broglie [9] one, to interpret the results [7, 8] because, contrary to the standard Copenhagen interpre-

tation, this ontological interpretation does not need a classical domain outside the quantized system to generate the physical facts out of potentialities (the facts are there *ab initio*), and hence it can be applied to the Universe as a whole. Of course there are other alternative interpretations which can be used in quantum cosmology, like the many worlds interpretation of quantum mechanics [10], but I will not use them in this paper.

In the Bohm-de Broglie interpretation, quantum Bohmian trajectories, the quantum evolution of the scale factor $a_q(t)$, can be defined through the relation $\dot{a} \propto \partial S/\partial a$, where S is the phase of an exact wave solution $\Psi(a,t)$ of the Wheeler-DeWitt equation. It satisfies a modified Hamilton-Jacobi equation, augmented with a quantum potential term derived from $\Psi(a,t)$, and hence $a_q(t)$ is not the classical trajectory: in the regions where the quantum effects cannot be neglected, the quantum trajectory $a_q(t)$ performs a bounce which connect two asymptotic classical regions where the quantum effects are negligible. One then has in hands a definite function of time for the homogeneous and isotropic background part of the Universe, even at the quantum level, which realizes a soft transition from the contracting phase to the expanding one.

When studying the evolution of quantum cosmological perturbations on these backgrounds, which was done in the series of papers [12–15] for the case of one perfect fluid with equation of state $p = w\rho$, one arrives at the result that, in order to obtain wavelength spectra and amplitudes compatible with CMB data, one must have [15] $|w| << 1$ and $w^{1/4}L_0 \approx 10^2 l_{\rm pl}$, where L_0 is the curvature scale at the bounce and $l_{\rm pl}$ is the Planck length. Hence this analysis shows that the model is self consistent because observational constraints impose that the curvature scale at the bounce must be at least a few orders

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[1] There are many other frameworks where bounces connecting the present expanding phase with a preceding contracting one may occur [2, 3]. In this case, the Universe is eternal, there is no beginning of time, nor horizons. The new features of these models introduce a new picture, where the usual problems of initial conditions [4] (as, for instance, the almost homogeneous beginning of the expanding phase might be explained through the disipation of nonlinear inhomogeneities when the universe was very large and rarefied in the asymptotic far past of the contracting phase) and the evolution of cosmological perturbations [5] are viewed from a very different perspective.

of magnitude greater than the Planck length, a region where one can trust the Wheeler-DeWitt equation without been spoiled by high order quantum gravity effects. Of course this model should be extended to include radiation. In Ref. [4] it is shown that the requirement $|w| << 1$ is important only at the moment when the perturbation wavelength becomes greater than the curvature scale: the fluid which dominates at the bounce is irrelevant for the spectral index (but is important for the amplitude, as we will see in future publications).

However this scenario has a problem on the quantum background solution itself: in order for the model describe the big Universe we live in, the scale factor at the bounce a_0 must be somewhat large, and the probability one can obtain from the wave function of the model for the occurence of this value is incredibly small (in some cases $\exp(-10^{89})$, as we will see later on). Hence, either there is an inflationary phase after the bounce in order to enlarge the Universe from a small a_0 (which may lead to transplanckian problems [11] and non linear inhomogeneities at the bounce because of the growth of linear perturbations in the contracting phase if a_0 is small), or one should rely very strongly on some anthropic principle in a situation much worst than in the landscape scenario.

The aim of this paper is to overcome this difficulty by proposing some more general wave solutions of the Wheeler-DeWitt equation which lead to realistic bouncing scenarios with parameters with reasonable probabilities and without any transplanckian problem. In Ref. [15], the wave function at the bounce was chosen to be a static gaussian of the scale factor centered at $a = 0$. As we will see, the ratio between the scale factor at the bounce a_0 and the width of the gaussian must be very large in order to yield the big Universe we live in, yielding the very small probability of occurrence of these parameters I mentioned above. However, if one generalizes the wave function to be a moving gaussian on the a-axis with velocity u, there is a minimum value of this parameter from where one can obtain a large Universe, with reasonable probability of occurrence, and without any transplanckian problems. The parameter u induces a very large acceleration around the bounce, leading to a sufficiently large scale factor when the quantum regime is over.

This paper is organized as follows: in Section II I describe in detail the problem I want to solve. In Section III I present the generalized wave solutions from which this problem can be circumvented. I conclude in Section IV with a discussion of our results, their physical meanings, and prospects for future work.

II. QUANTUM BOUNCE SOLUTIONS FROM STATIC INITIAL GAUSSIANS AND THEIR PROBLEMS

The Hamiltonian constraint describing a cosmological model with flat homogeneous and isotropic closed space-like hypersurfaces with comoving volume $V = 1$, and a perfect fluid satisfying $p = \omega\epsilon$, where ϵ is the perfect fluid energy density, p is the pressure and ω is a constant, reads [12]

$$H_0 \equiv \frac{P_T}{a^{3\omega}} - \frac{P_a^2}{4a}, \qquad (1)$$

where a is the scale factor, P_a its canonical momentum, and the conserved quantity P_T, the momentum canonically conjugated to the degree of freedom of the fluid T [16], is associated with the constant appearing in the energy density of the fluid through the relation $\epsilon = P_T/a^{3(1+w)}$. All quantities are in Planck unities. One can verify that the Hamiltonian $H = NH_0$ generates the usual Friedmann equations of the model.

The wave function $\Psi(a,T)$ satisfies the Wheeler-DeWitt equation $H_0\Psi = 0$,

$$i\frac{\partial}{\partial T}\Psi(a,T) = \frac{a^{(3\omega-1)/2}}{4}\frac{\partial}{\partial a}\left[a^{(3\omega-1)/2}\frac{\partial}{\partial a}\right]\Psi(a,T), \quad (2)$$

where I have chosen the factor ordering in a in order to yield a covariant Schrödinger equation under field redefinitions. The fluid selects a prefered time variable.

I change variables to

$$\chi = \frac{2}{3}(1-\omega)^{-1}a^{3(1-\omega)/2},$$

obtaining the simple equation

$$i\frac{\partial\Psi(\chi,T)}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi(\chi,T)}{\partial\chi^2}. \qquad (3)$$

This is just the time reversed Schrödinger equation for a one dimensional free particle constrained to the positive axis. As a and χ are positive, solutions which have unitary evolution must satisfy the condition

$$\left(\Psi^\star\frac{\partial\Psi}{\partial\chi} - \Psi\frac{\partial\Psi^\star}{\partial\chi}\right)\bigg|_{\chi=0} = 0 \qquad (4)$$

(see Ref. [8] for details). I can choose the initial normalized wave function

$$\Psi^{(\text{init})}(\chi) = \left(\frac{8}{T_0\pi}\right)^{1/4}\exp\left(-\frac{\chi^2}{T_0}\right), \qquad (5)$$

where T_0 is an arbitrary constant. The gaussian $\Psi^{(\text{init})}$ satisfies condition (4), and it gives the probability density for the value of χ at $T = 0$ with minimum uncertainty.

Using the propagator procedure of Refs. [8], we obtain the wave solution for all times in terms of a:

$$\Psi(a,T) = \left[\frac{8T_0}{\pi\left(T^2 + T_0^2\right)}\right]^{1/4} \exp\left[\frac{-4T_0 a^{3(1-\omega)}}{9(T^2 + T_0^2)(1-\omega)^2}\right] \exp\left\{-i\left[\frac{4T a^{3(1-\omega)}}{9(T^2 + T_0^2)(1-\omega)^2} + \frac{1}{2}\arctan\left(\frac{T_0}{T}\right) - \frac{\pi}{4}\right]\right\}. \quad (6)$$

Due to the chosen factor ordering, the probability density $\rho(a,T)$ has a non trivial measure and it is given by $\rho(a,T) = a^{(1-3\omega)/2} R^2$, where $R^2 = |\Psi(a,T)|^2$. Its continuity equation, one of the equations coming from Eq. (2) after substitution of $\Psi = Re^{iS}$ in it, reads

$$\frac{\partial\rho}{\partial T} - \frac{\partial}{\partial a}\left[\frac{a^{(3\omega-1)}}{2}\frac{\partial S}{\partial a}\rho\right] = 0, \quad (7)$$

which implies, in the Bohm interpretation [9], the definition of a velocity field

$$\dot{a} = -\frac{a^{(3\omega-1)}}{2}\frac{\partial S}{\partial a}, \quad (8)$$

in accordance with the classical relations $\dot{a} = \{a, H\} = -a^{(3\omega-1)}P_a/2$ and $P_a = \partial S/\partial a$.

Note that S satisfies the other equation coming from (2),

$$\frac{\partial S}{\partial T} - \frac{a^{(3\omega-1)}}{4}\left(\frac{\partial S}{\partial a}\right)^2 + \frac{a^{(3\omega-1)/2}}{4R}\frac{\partial}{\partial a}\left[a^{(3\omega-1)/2}\frac{\partial R}{\partial a}\right] = 0, \quad (9)$$

which is a Hamilton-Jacobi-like equation with an extra quantum term, called the quantum potential, given by

$$Q \equiv -\frac{a^{(3\omega-1)/2}}{4R}\frac{\partial}{\partial a}\left[a^{(3\omega-1)/2}\frac{\partial R}{\partial a}\right]. \quad (10)$$

Hence, the trajectory (8) will not coincide with the classical trajectory whenever Q is comparable with the other terms present in Eq. (9) because S will be different from the classical Hamilton-Jacobi function.

Inserting the phase of (6) into Eq. (8), I obtain the Bohmian quantum trajectory for the scale factor:

$$a(T) = a_0\left[1 + \left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}, \quad (11)$$

or, in terms of $\chi(T)$,

$$\chi(T) = \chi_0\left[1 + \left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{2}}. \quad (12)$$

Note that χ_0 is the value of χ at $T = 0$, the moment of the bounce, $\chi_0 = \chi_{\text{bounce}}$, and the scale factor at the bounce a_0 is connected to χ_0 through

$$a_0 = \left[\frac{3}{2}(1-\omega)\chi_0\right]^{2/[3(1-\omega)]}. \quad (13)$$

Solution (11) has no singularities and tends to the classical solution when $T \to \pm\infty$. Remember that I am in the gauge $N = a^{3\omega}$, and T is related to conformal time through

$$N\text{d}T = a\text{d}\eta \implies \text{d}\eta = [a(T)]^{3\omega-1}\text{d}T. \quad (14)$$

The solution (11) can be obtained from other initial wave functions (see Ref. [8]).

However, the above solution suffers from the following drawback: the curvature scale at the bounce reads $L_{\text{bounce}} \equiv T_0 a_0^{3w}$, and the quantity P_T associated in the classical limit $|T| \to \infty$ with the constant appearing in the energy density of the fluid through the relation $\epsilon = P_T/a^{3(1+w)}$, can be obtained in the Bohmian approach from the wave function through the relation $P_T = \partial S/\partial T$. It reads

$$P_T = \frac{\partial S}{\partial T} = \frac{T_0}{2(T^2 + T_0^2)} - \frac{\chi(T)^2(T_0^2 - T^2)}{(T^2 + T_0^2)^2}. \quad (15)$$

Inserting the solution (12) in Eq. (15) and taking the classical limit $|T| \to \infty$, one obtains

$$P_T = \frac{\chi_0^2}{T_0^2}. \quad (16)$$

In the case of dust, P_T is the total dust mass of the Universe, yielding $P_T \geq 10^{60}$. If one takes the curvature scale at the bounce some few orders of magnitude larger than the Planck length, say 10^3, in order to not spoil the Wheeler-DeWitt approach used above due to strong quantum gravitational effects, one has to have $\chi_0 \geq 10^{33}$, with probability less than $\exp(-10^{63})$ to occur (see Eq. (5)).

The situation is similar with radiation, where now $P_T = \chi_0^2/T_0^2 \geq 10^{116}$. Note that in this case $\chi_0 = a_0$, $T = \eta$, and the curvature scale at the bounce reads $L_{\text{bounce}} \equiv T_0 a_0$. Combining the constraints $a_0/T_0 \geq 10^{58}$ and $a_0 T_0 \geq 10^3$, one arrives at the very low probability $\exp(-10^{89})$ for these parameters to occur.

The source of the problem is the fact that the constant χ_0 appearing in Eq. (16) is also the value of χ at $T = 0$, the χ at the bounce, and the fact that P_T must be large, induces a large χ^2/T_0 in the gaussian (5). One possibility to scape from this drawback is to find a different wave solution to Eq. (2) which either modifies Eq. (16) or yields Bohmian trajectories where χ_0 is not anymore the value of χ at $T = 0$, allowing the possibility of having a small initial χ, hence a small χ^2/T_0 in (5), and a huge χ_0, perhaps through the presence of an inflationary phase between the bounce and the standard decelerated

expansion. I will show in the next section that it is indeed possible to obtain a more general class of wave solutions of Eq. (2) where the above mentioned problem is circumvented.

the boundary condition (4), and now reds,

III. NEW BOUNCING SOLUTIONS

I will generalize the initial wave function by inserting a velocity term in Eq. (5), which of course must satisfy

$$\Psi^{(\text{init})}(\chi) = \left(\frac{2}{T_0\pi}\right)^{1/4} [1 \pm \exp(-u^2 T_0/8)]^{-1/2} [\exp(iu\chi/2) \pm \exp(-iu\chi/2)] \exp\left(-\frac{\chi^2}{T_0}\right). \tag{17}$$

This initial wave function represents two gaussians travelling from the origin in opposite directions (keeping in mind that only the tail of the gaussian traveling in the negative direction with suport on the positive a axis has physical meaning). The solution for all times read

$$
\begin{aligned}
\Psi(\chi, T) = & \left[\frac{2T_0}{\pi(T^2 + T_0^2)}\right]^{1/4} (1 \pm \exp(-u^2 T_0/8)^{-1/2} \\
& \left(\exp\left[-\frac{T_0(\chi - uT)^2}{(T^2 + T_0^2)}\right] \exp\left\{-i\left[\frac{T(\chi - uT)^2}{(T^2 + T_0^2)} + 2u(\chi - uT/2) + \frac{1}{2}\arctan\left(\frac{T_0}{T}\right) - \frac{\pi}{4}\right]\right\} \right. \\
& \left. \pm \exp\left[-\frac{T_0(\chi + uT)^2}{(T^2 + T_0^2)}\right] \exp\left\{-i\left[\frac{T(\chi + uT)^2}{(T^2 + T_0^2)} - 2u(\chi + uT/2) + \frac{1}{2}\arctan\left(\frac{T_0}{T}\right) - \frac{\pi}{4}\right]\right\}\right),
\end{aligned} \tag{18}
$$

which I write as,

$$\Psi = A(R_- e^{iS_-} \pm R_+ e^{iS_+}),$$

where

$$
\begin{aligned}
R_\pm &\equiv \exp\left[-\frac{T_0(\chi \pm uT)^2}{(T^2 + T_0^2)}\right], \\
S_\pm &\equiv \left[-\frac{T(\chi \pm uT)^2}{(T^2 + T_0^2)} \pm 2u(\chi \pm uT/2) - \frac{1}{2}\arctan\left(\frac{T_0}{T}\right) + \frac{\pi}{4}\right], \\
A &\equiv \left[\frac{2T_0}{\pi(T^2 + T_0^2)}\right]^{1/4} (1 \pm \exp(-u^2 T_0/8)^{-1/2}.
\end{aligned}
$$

From these equations one obtains the total amplitude and phase as

$$
\begin{aligned}
R &= A\,\sqrt{R_+^2 + R_-^2 \pm 2R_+R_-\cos(S_+ - S_-)} \\
S &= \arctan\left(\frac{R_+\sin(S_+) \pm R_-\sin(S_-)}{R_+\cos(S_+) \pm R_-\cos(S_-)}\right)
\end{aligned}
$$

The derivative of S with respect to some variable x reads

$$\frac{\partial S}{\partial x} = \frac{R_+^2 \frac{\partial S_+}{\partial x} + R_-^2 \frac{\partial S_-}{\partial x} \pm \left(\frac{\partial S_+}{\partial x} + \frac{\partial S_-}{\partial x}\right)R_+R_-\cos(S_+ - S_-) \pm \left(R_-\frac{\partial R_+}{\partial x} - R_+\frac{\partial R_-}{\partial x}\right)\sin(S_+ - S_-)}{R_+^2 + R_-^2 \pm 2R_+R_-\cos(S_+ - S_-)}$$

The guidance relation (8) leads to the exact differential equation

$$\dot{\chi}(T) = \frac{T\chi(T)}{T^2 + T_0^2} + \frac{uT_0^2}{T^2 + T_0^2}\left[\frac{\sinh\left(\theta\frac{T}{T_0}\right) \pm \frac{T}{T_0}\sin\theta}{\cosh\left(\theta\frac{T}{T_0}\right) \pm \cos\theta}\right], \tag{19}$$

where

$$\theta \equiv \frac{4uT_0^2\chi(T)}{T^2 + T_0^2}. \tag{20}$$

From the solution (18), the new P_T is now given by

$$P_T = \frac{\partial S}{\partial T} = \frac{T_0}{2(T^2 + T_0^2)} + \frac{[u^2T_0^2 - \chi^2(T)](T_0^2 - T^2)}{(T^2 + T_0^2)^2} + \frac{\left[4uT_0^2 T\chi(T)\sinh\left(\theta\frac{T}{T_0}\right) \pm 2T_0 u\chi(T)(T^2 - T_0^2)\sin\theta\right]}{(T^2 + T_0^2)^2\left[\cosh\left(\theta\frac{T}{T_0}\right) \pm \cos\theta\right]}. \tag{21}$$

From now on I will work with the plus sign solution given in Eq. (18). The minus sign solution yields the same qualitative results.

A.   Quantum solutions for small u

Taking $u << 1$, one has

$$\dot{\chi}(T) = \frac{T\chi(T)}{T^2 + T_0^2} + \frac{4\chi(T)u^2 T T_0^3}{(T^2 + T_0^2)^2}, \tag{22}$$

with solution

$$\begin{aligned}\chi(t) &= \frac{\chi_0}{T_0}\sqrt{T^2 + T_0^2}\exp\left[\frac{-2u^2 T_0^3}{T^2 + T_0^2}\right] \\ &= \chi_0\sqrt{x^2 + 1}\exp\left[\frac{-2u^2 T_0}{x^2 + 1}\right], \end{aligned} \tag{23}$$

where in the last step I wrote the solution in terms of $x = T/T_0$.

Solution (23) has very nice properties. First of all, one can see that the values of χ and the curvature scale at the bounce are now given by

$$\chi_{\text{bounce}} = \chi_0\exp(-2u^2 T_0), \tag{24}$$

and

$$L_{\text{bounce}} = \exp\left[-\frac{4wu^2 T_0}{(1-w)}\right]\frac{T_0 a_0^{3w}}{\sqrt{1 + 4u^2 T_0}}. \tag{25}$$

Inserting solution (23) into Eq. (21) in the limit $|T| \to \infty$, yields

$$P_T = \frac{\chi_0^2}{T_0^2}. \tag{26}$$

Note that now $\chi_0 \neq \chi_{\text{bounce}}$, the value of χ at $T = 0$. In fact, from Eq. (24), one may have $\chi_0 >> \chi_{\text{bounce}}$, depending on the value of T_0, because of the huge acceleration one may obtain near after the bounce as compared with the case where $u = 0$: it is a bounce followed by inflation. Hence, one may have reasonable probability amplitudes for the free parameters of the theory which are compatible with a huge P_T.

However, one must also check whether the Bohmian trajectory (23), with such apropriate choice of parameters, reachs classical evolution ($x >> 1$) before the nucleosynthesis epoch. Let us concentrate on the case of radiation ($w = 1/3$), as it is the most interesting physical situation (one expects the quantum effects and the bounce to occur in a very hot radiation dominated universe). Suppose the classical limit is already valid at a conformal time where the energy density of radiation is minimally greater than the energy density before nucleosynthesis, say, the energy density around the freeze-out of neutrons, $\rho_f \approx 10^{-88}$. Then, from Eqs. (23) and (26), and from $\rho_f = P_T/a_f^4$, with $a_f = a_0\eta_f/T_0$, one obtains $x_f \equiv \eta_f/T_0 \approx 10^{22}/(T_0 P_T^{1/4})$. Using that $P_T \geq 10^{116}$, one gets that $x_f \leq 10^{-7}/T_0$. Hence, $x_f >> 1$ if and only if $T_0 << 10^{-7}$. However, as $u << 1$, then $u^2 T_0 << 1$, and the exponential in (23) would be irrelevant, turning solution (23) very close to solution (11) for $w = 1/3$, taking us back to the previous problem. Concluding, the only way to obtain a huge P_T with parameters with reasonable probability amplitudes in this framework is through choices which will change the usual scale factor evolution during nuclosynthesis, spoiling its observed predictions.

B.   Quantum solutions for large u

If $u >> 1$, for $|T|$ not very small, and noting that the unique possible asymptotic behaviour of a solution $\chi(T)$ of Eq. (19) is $\chi(T) \propto T$, then the hyperbolic functions in (19) are very large and much greater than the terms with trigonometric functions, yielding

$$\dot{\chi}(T) = \frac{T\chi(T)}{T^2 + T_0^2} \pm \frac{uT_0^2}{T^2 + T_0^2}, \tag{27}$$

with solution

$$\chi(T) = \frac{\chi_0}{T_0}\sqrt{T^2 + T_0^2} \pm uT, \tag{28}$$

where the $\pm$ sign corresponds to positive and negative values of T, respectively. For $|T| \approx 0$, one has to rely on numerical calculations. However, as shown in figure 1 below, for large u this difference is quite unimportant. Hence, again, χ_0 is very close to the value of χ at the bounce.

FIG. 1: Figure showing $\chi(t)$ against t, comparing solution (28) (continuous line) for $u = 12, \chi_0 = 1, T_0 = 1$ with the numerical solution of equation (19) for $u = 12, \chi(10) = 130$ (dotted line). The almost parallel line corresponds to the bouncing solution for $u = 0, \chi_0 = 1.155$.

Inserting solution (28) into Eq. (21) in the limit $|T| \to \infty$, we obtain

$$P_T = \left(\frac{\chi_0}{T_0} + u \right)^2. \tag{29}$$

Hence, the huge values of P_T can be obtained from large values of u, without any imposition on the parameters χ_0 and T_0.

Let us calculate the constraints on the parameter space in order to obtain a sensible model. As discussed in section II, one should have $a_0^2/T_0 \leq 1$ in order to have a reasonable probability amplitude for a_0, and the curvature scale at the bounce should not be very close to the Planck length in order to avoid strong quantum gravitational effects. I will also impose that $a_0 > 1$ in order to avoid transplanckian problems (see below), which implies that $a_0/T_0 \leq 1$.

The curvature scale at the bounce reads

$$
\begin{aligned}
L_{\text{bounce}} &= \frac{a_0 T_0 \sqrt{a_0[1 + \cos(4ua_0)]}}{\sqrt{a_0[1 + \cos(4ua_0)] + uT_0[4ua_0 + \sin(4ua_0)]}} \\
&\approx \frac{a_0\sqrt{T_0}}{2u},
\end{aligned} \tag{30}
$$

where in the last approximation I used that $uT_0/a_0 >> 1$, which follows from $u >> 1$, $a_0/T_0 \leq 1$, and I assumed that $4ua_0 \neq (2n+1)\pi$, in order to avoid $L_{\text{bounce}} << 1$. Hence, as $a_0 \leq T_0^{1/2}$, then

$$L_{\text{bounce}} \leq \frac{T_0}{2u} \tag{31}$$

Demanding that $L_{\text{bounce}} \geq 10^3$, than

$$T_0 > u10^3 >> 1. \tag{32}$$

One must check again whether one recovers the classical radiation dominated evolution before nucleosynthesis. As before, I concentrate on the case $w = 1/3$, which implies that $\chi = a$ and $T = \eta$. I will do this in two steps: first I check whether solution (28) is valid before nucleosynthesis, and then whether quantum effects are negligible there.

The approximation leading to solution (28) requires that the argument of the hyperbolic functions in Eq. (19) be large, $\theta T/T_0 >> 1$, which implies that $x = \eta/T_0 >> (4u^2 T_0 - 1)^{-1/2} \approx 1/(2uT_0^{1/2})$. Hence, the values of the conformal time for which solution (28) is reliable are $\eta >> T_0^{1/2}/(2u)$.

Let us now verify whether solution (28) is valid around the freeze-out of neutrons, before nucleosynthesis. This will be true if $\eta_{\text{f}} >> T_0^{1/2}/(2u)$. However, $\eta_{\text{f}} \approx 10^{22}/P_T^{1/4} = 10^{22}/u^{1/2}$ which, when combined with $\eta_{\text{f}} >> T_0^{1/2}/(2u)$, implies that $10^{44} >> T_0/(4u) \geq L_{\text{bounce}}$, where I used Eq. (31). As the curvature scale around freeze-out of neutrons, L_{f}, satisfies $L_{\text{f}} \approx 10^{44}$, this condition is just the reasonable constraint that

$$L_{\text{bounce}} << L_{\text{f}} \approx 10^{44}. \tag{33}$$

Hence, if the curvature scale at the bounce is much smaller than the curvature scale around freeze-out of neutrons, than solution (28) must be valid at nucleosynthesis period.

Finally, one must check that in the regime where solution (28) is valid we are already in the classical limit, even though the above condition $x >> 1/(2uT_0^{1/2})$ may still contain a region were $x << 1$ because u and T_0 are large. To prove this, note first that at $\eta >> T_0^{1/2}/(2u)$, the term $u\eta >> T_0^{1/2}/2$ dominates over $a_0\sqrt{x^2+1}$ in Eq. (28), either for $x << 1$, because $a_0 \leq T_0^{1/2}$, as for $x >> 1$, because $a_0/T_0 << u$. Hence, the quantum potential given by

$$Q := -\frac{\partial^2 R}{4R\partial\chi^2} =: Q_1 + Q_2^2, \tag{34}$$

where

$$Q_1 = -T_0\{[4T_0(\chi^2 + u^2T^2) - (T^2 + T_0^2)]\cosh(\theta T/T_0) - 8uT_0T\chi\sinh(\theta T/T_0)$$
$$+ [4T_0(\chi^2 - u^2T_0^2) - (T^2 - T_0^2)]\cos\theta + 8T_0^2\chi u\sin\theta\}\{2(T^2 + T_0^2)^2[\cosh(\theta T/T_0) + \cos\theta]\}^{-1}, \quad (35)$$

and

$$Q_2 = T_0 \frac{X[\cosh(\theta T/T_0) + \cos\theta] - u[T\sinh(\theta T/T_0) - T_0\sin\theta]}{(T^2 + T_0^2)[\cosh(\theta T/T_0) + \cos\theta]}, \quad (36)$$

reads, around the Bohmian trajectory $a \approx u\eta$,

$$Q \approx \frac{1}{\cosh(2\theta T/T_0)} << 1, \quad (37)$$

while the kinetic term of the Hamilton-Jacobi-like equation (9) is given by,

$$\left(\frac{\partial S}{2\partial a}\right)^2 \approx u^2 >> 1. \quad (38)$$

Note that near the bounce at $\eta \approx 0$, the kinetic term is almost null, while the quantum potential is finite: quantum effects are dominant only very near the bounce. The transition from quantum to classical regime should be around $\theta T/T_0 \approx 1$, where $Q \approx 1$ and solution (28) is not reliable. A little bit later, when $x = \eta/T_0 > (4u^2T_0 - 1)^{-1/2} \approx 1/(2uT_0^{1/2})$, and knowing from Eq. (32) that $T_0 > 10^{61}$ because $u > 10^{58}$, we obtain that the scale factor at the beginning of the classical regime is $a > 10^{31}$ which is the minimum value required for the model to reach the size of the observed Universe without needing any classical inflationary phase afterwards. Note from figure 1 that the presence of the u term in Eq. (28) induces a much bigger acceleration at the bounce in comparison with the solution with the same a_0 but without the u term. It is this term which is the responsible for the big value $a > 10^{31}$ when the model enters the classical regime.

Concluding, solution (28) reaches the standard cosmological model before nucleosynthesis, and can indeed describe the observed Universe in the radiation dominated phase with parameters with reasonable relative probabilities.

In order to avoid the transplanckian problem for the scales of physical interest today, $10^{54} < \lambda_{\text{today}}^{\text{physical}} < 10^{60}$, one should have $\lambda_{\text{bounce}}^{\text{physical}}$ corresponding to these scales not smaller than, say, 10^3. As

$$\lambda_{\text{bounce}}^{\text{physical}} = \frac{a_0}{a_{\text{today}}}\lambda_{\text{today}}^{\text{physical}}, \quad (39)$$

this problem can be avoided if

$$\frac{a_0}{a_{\text{today}}} > 10^{-51}, \quad (40)$$

which implies that $a_0 > 10^9$.

Note that there is an upper limit for a_0 coming from

$$L_{\text{bounce}} \approx \frac{a_0 T_0^{1/2}}{2u} \geq 10^{64}\left(\frac{a_0}{a_{\text{today}}}\right)^2, \quad (41)$$

where I used that $a_0^2/T_0 \leq 1$, $P_T \approx u^2 \approx a_{\text{today}}^4 10^{-128}$. The constraint $L_{\text{bounce}} << 10^{44}$ (see Eq.(33)) then implies that

$$\frac{a_0}{a_{\text{today}}} << 10^{-10}. \quad (42)$$

Hence, there is a large domain of values of a_0 where the transplanckian problem can be avoided (see Eqs.(40,42)).

IV. CONCLUSION

I have shown in this paper how a sufficiently big universe can emerge from a quantum cosmological bounce, without needing any classical inflationary phase afterwards to make it grow to its present size. This is caused by a huge acceleration during the quantum bounce, which may be viewed as a quantum inflation. These results were obtained from a moving gaussian function of the scale factor, which is a solution of the Wheeler-DeWitt equation coming from the canonical quantization of general relativity sourced by relativistic particles. The solution is exact, there is no WKB approximation involved here. Its value at $T = 0$ yields reasonable relative probability amplitudes of having the scale factor at the bounce with the value a_0 required to avoid any transplanckian problem, and to allow that the curvature scale at the bounce be some few orders of magnitude greater than the Planck length, a region where one can rely on this simple quantization scheme. In fact, as the maximum value the curvature scale can have is at the bounce itself, one never reachs energy scales where more involved quantum gravity theories, like string theory and loop quantum gravity (see Ref. [17] about issues concerning this approach), must be invoked: the model is self-contained.

There are two internal parameters of the wave functon which must be big in order to obtain a large classical universe from a quantum bounce. The first one is the parameter T_0, the square root of the width of the gaussian at the moment of the bounce (see Eq. (17)), which must satisfy $T_0 > 10^{61}$ (in Planck unities). This value yields

the sufficient large value $a > 10^{31}$ for the scale factor in the beginning of the classical regime, and guarantees that the curvature scale at the bounce be some minimum orders of magnitude greater than the Planck length in order for the Wheeler-DeWitt equation I used be reliable. The other one is the velocity u of the gaussian along the scale factor axis which must satisfy $u > 10^{58}$ in order to yield the amount of radiation we observe in the Universe today, without appealing to some huge production of photons during the bounce.

From these considerations, one can see that the parameters emerging from the quantum era of the Universe are not necessarily Planckian: they depend also on the quantum state of the system, on the internal parameters of the wave function of the Universe. Hence, it is not surprising that one may have quantum gravity effects in large (when compared with the Planck length) Universes [18], which could be dramatically seen in a big-rip [19].

However, one may ask why the internal parameters of the wave function we obtained are so large. Note first that these are not coupling constants, but parameters in the quantum state of the Universe. Hence, to answer this question, one should rely on some deep understanding of quantum cosmology and/or new principles which are not available today. Note, however, that the big value of T_0 leads to a widely spread gaussian, and hence almost all scales at the bounce are equally probable. This is a reasonable assumption about the wave function of the Universe one can make: it should not intrinsically select any preferable scale at the bounce without any special reason. Concerning the u variable, its large value implies that the peak of the initial wave packet moves very fast towards large scale factors, which induces a large universe. Perhaps some version of the Anthropic Principle could justify the preference for large classical universes, and as consequence for a large u, but I think the important message here is the possibility of obtaining a large universe from a huge acceleration of the scale factor in the far past, whose origin differs fundamentally from those considered in usual inflationary scenarios.

In future publications, we will calculate the evolution of linear quantum perturbations and particle production on these quantum backgrounds, as in Ref. [15], and compare the results with observations.

As a final remark, I would like to repeat a comment we made elsewhere [4]: in contradistinction with models in which time begins, there is no point on asking what is the probability of appearance of some particular eternal model out of nothing. Contrary to usual perspectives, one can as well assume existence to be conceptually prior to non-existence, *i.e.* existence itself may not be deserving explanation. This is the idea underlying our category of models: the Universe always existed and its "appearance" is thus a non question.

Acknowledgements

[1] S. W. Hawking and G. F. R. Ellis, *The large scale structure of space-time*, Cambridge University Press (1973); R. M. Wald., *General Relativity*, Chicago University Press (1984); A. Borde and A. Vilenkin, Phys. Rev. D **56**, 717 (1997).

[2] M. Novello and S. E. Perez Bergliaffa, Phys. Rep. **463**, 127 (2008).

[3] R. C. Tolman, Phys. Rev. **38**, 1758 (1931); G. Murphy, Phys. Rev. **D8**, 4231 (1973); M. Novello and J. M. Salim, Phys. Rev. **D20**, 377 (1979); V. Melnikov and S. Orlov, Phys. Lett **A70**, 263 (1979); E. Elbaz, M. Novello, J. M. Salim and L. A. R. Oliveira, Int. J. of Mod. Phys. **D1**, 641 (1992); J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok, Phys. Rev. D **64**, 123522 (2001); V. A. De Lorenci, R. Klippert, M. Novello and J. M. Salim, Phys. Rev. D **65**, 063501 (2002); J. C. Fabris, R. G. Furtado, P. Peter and N. Pinto-Neto Phys. Rev. D **67**, 124003 (2003).

[4] P. Peter and N. Pinto-Neto, Phys. Rev. D **78**, 063506 (2008).

[5] R. Brandenberger and F. Finelli, J. High Energy Phys. 11 (2001) 056; P. Peter and N. Pinto-Neto, Phys. Rev. D **66**, 063509 (2002); F. Finelli, JCAP **0310**, 011(2003); V. Bozza and G. Veneziano, Phys. Lett. B **625**, 177 (2005); V. Bozza and G. Veneziano, JCAP **09**, 007 (2005); F. Finelli and R. Brandenberger, Phys. Rev. D **65** (2002) 103522; S. Tsujikawa, R. Brandenberger and F. Finelli, Phys. Rev. D **66**, 083513 (2002); P. Peter, N. Pinto-Neto and D. A. Gonzalez, JCAP **0312**, 003 (2003); J. Martin, P. Peter, Phys. Rev. Lett **92**, 061301 (2004); J Martin, P Peter, N Pinto-Neto and D. J. Schwarz, Phys. Rev. D **65**, 123513 (2002); J Martin, P Peter, N Pinto-Neto and D. J. Schwarz, Phys. Rev. D **67**, 028301 (2003); C. Cartier, R. Durrer and E. J. Copeland, Phys. Rev. D. **67**, 103517 (2003); L. E. Allen and D. Wands, Phys. Rev. D **70**, 063515 (2004); T. J. Battefeld, and G. Geshnizjani, Phys. Rev. D **73**, 064013 (2006); A Cardoso and D Wands, Phys. Rev. D **77**, 123538 (2008); Y Cai, T Qiu, R Brandenberger, Y Piao and X Zhang, J. Cosmol. Astropart. Phys. 03 (2008) 013.

[6] M. Bojowald, Phys. Rev. Lett. **86**, 5227 (2001); M. Bojowald, Phys. Rev. D **64**, 084018 (2001); A. Ashtekar, M. Bojowald and J. Lewandowski Adv. Theor. Math. Phys. **7**, 233 (2003).

[7] R. Colistete Jr., J. C. Fabris, N. Pinto-Neto, Phys. Rev. **D62**, 083507 (2000); N. Pinto-Neto, E. Sergio Santini and Felipe T. Falciano, Phys. Lett.**A 344**, 131-143(2005).

[8] F.G. Alvarenga, J.C. Fabris, N.A. Lemos and G.A. Mon-

erat, Gen.Rel.Grav. **34**, 651 (2002); J. Acacio de Barros, N. Pinto-Neto, and M. A. Sagioro-Leal, Phys. Lett. A **241**, 229 (1998).

[9] D. Bohm, Phys. Rev. **85**, 166 (1952); D. Bohm, B. J. Hiley and P. N. Kaloyerou, Phys. Rep. **144**, 349 (1987); P. R. Holland, *The Quantum Theory of Motion: An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanichs* (Cambridge University Press, Cambridge, 1993).

[10] *The Many-Worlds Interpretation of Quantum Mechanics*, ed. by B. S. DeWitt and N. Graham (Princeton University Press, Princeton, 1973).

[11] J. Martin and R. H. Brandenberger, Phys. Rev. D **63**, 123501 (2001); R. H. Brandenberger and J. Martin, Mod. Phys. Lett. A **16**, 999 (2001); J. C. Niemeyer, Phys. Rev. D **63**, 123502 (2001); M. Lemoine, M. Lubo, J. Martin, and J. P. Uzan, Phys. Rev. D **65**, 023510 (2001).

[12] P. Peter, E. Pinho, and N. Pinto-Neto, J. Cosmol. Astropart. Phys. 07 2005) 014.

[13] P. Peter, E. J. C. Pinho and N. Pinto-Neto, Phys. Rev. D **73** 104017 (2006).

[14] E. J. C. Pinho and N. Pinto-Neto, Phys. Rev. D **76** 023506 (2007).

[15] P. Peter, E. J. C. Pinho, and N. Pinto-Neto, Phys. Rev. D **75** 023516 (2007).

[16] V. G. Lapshinskii and V. A. Rubakov, Theor. Math. Phys. **33** 1076 (1977); F. J. Tipler, Phys. Rep. **137** 231 (1986).

[17] H. Nicolai, K. Peeters and M. Zamaklar, Class. Quant. Grav. **22** R193 (2005); T. Thiemann, Lect. Notes Phys. **721** 185 2007.

[18] A Linde, J. Cosmol. Astropart. Phys. 10 (2004) 004; J J. Halliwell and J B. Hartle, Phys. Rev. **D 41**, 1815 (1990); N. Pinto-Neto and E. S. Santini, Phys. Lett. A **315**, 36, (2003).

[19] E. M. Barboza Jr. and N. A. Lemos Gen. Rel. Grav. **38**, 1609 (2006); M. P. Dabrowski, C. Kiefer and B. Sandhofer Phys.Rev. D **74**, 044022 (2006).