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Modular localization and the holistic structure of causal quantum theory, a historical perspective

Dedicated to the memory of Jürgen Ehlers (1929-2008)

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Abstract

Recent insights into the conceptual structure of localization in QFT ("modular localization") led to clarifications of old unsolved problems. The oldest one is the Einstein-Jordan conundrum which led Jordan in 1925 to the discovery of quantum field theory. This comparison of fluctuations in subsystems of heat bath systems (Einstein) with those resulting from the restriction of the QFT vacuum state to an open subvolume (Jordan) leads to a perfect analogy; the globally pure vacuum state becomes upon local restriction a strongly impure KMS state. This phenomenon of localization-caused thermal behavior as well as the vacuum-polarization clouds at the causal boundary of the localization region places localization in QFT into a sharp contrast with quantum mechanics and justifies the attribute "holstic". In fact it positions the E-J Gedankenexperiment into the same conceptual category as the cosmological constant problem and the Unruh Gedankenexperiment. The holistic structure of QFT resulting from "modular localization" also leads to a revision of the conceptual origin of the crucial crossing property which entered particle theory at the time of the bootstrap S-matrix approach but suffered from incorrect use in the S-matrix settings of the dual model and string theory.

The new holistic point of view, which strengthens the autonomous aspect of QFT, also comes with new messages for gauge theory by exposing the clash between Hilbert space structure and localization and presenting alternative solutions based on the use of stringlocal fields in Hilbert space. Among other things this leads to a radical reformulation of the Englert-Higgs symmetry breaking mechanism.

Contents

Preface

The subject of this paper grew out of many discussions about Jordan's discovery of quantum field theory (QFT) which I had with the late Jürgen Ehlers. These conversations focussed in particular on events between the publication of Jordan's thesis on quantum aspects of statistical quantum mechanics in 1924 [1], and his discovery of QFT which was published in one section of the famous 1926 "Dreimännerarbeit" [2] together with Born and Heisenberg. This famous paper was in fact conceived as the second paper on quantum mechanics (QM). The resistance of Born and Heisenberg against Jordan's section has its natural explanation in that these two authors felt that Jordan was burdening the conceptual struggle to understand the new quantum mechanics with something which was even further out.

I met Jürgen Ehlers the first time around 1957 at the University of Hamburg when he was Jordan's assistant and played the leading role in Jordan's general relativity seminar. Our paths split, after I wrote my diploma thesis on a topic of particle theory at the time when particle physics moved away from the university to the newly constructed high energy laboratory at DESY away from the university institutes. Contacts with Ehlers and the relativity group became less frequent and ended when both of us took up research associate positions at different universities in the US.

Only 40 years later, when Ehlers moved to Potsdam/Golm in the 90s as the founding director of the new Albert Einstein Institute (AEI), we met a second time. At that time he was interested to understand some of Jordan's famous early work on quantum field theory about which we knew little at the time of Jordan's weekly relativity seminar[1]. Ehlers was in particular interested to understand some subtle points in a dispute between Jordan and Einstein concerning Einstein's use of statistical mechanics fluctuation arguments for black body radiation. The ensuing dispute around this purely theoretical argument in favor of the existence of photons has been more recently referred to as the *Einstein-Jordan conundrum [3].*

As the terminology reveals, the E-J conundrum was a poorly understood relation between fluctuations caused by restricting the vacuum state to the observables in a subvolume in Jordan's newly discovered field quantization and Einstein's use of statistical mechanics within the old Bohr-Sommerfield quantum setting. This led him to identify a particle-like component in the fluctuation spectrum of a black body radiation ensemble (which he termed "Nadelstrahlung") with his 1905 interpretation of the photo-electric effect as a manifestation of the corpuscular nature of light.

The E-J conundrum has sometimes been misunderstood as an illustration of the particle-wave dualism of quantum mechanics, but this was certainly not Ehler's view when he drew my attantion to this problem. Coming from general relativity and cosmology he thought about it as being analogous [4] to the problems which one encounters if one tries to explain the origin of the cosmological constant in terms of fluctuation properties of the quantum field theoretic vacuum. He hoped that with my experience of 40 years of QFT I could be of some help to solve this conundrum.

I learned recently through John Stachel that conjectures about possible connections

[1] After ww II Jordan interest was mainly focussed on general relativity and philosophical implications. At the time of the seminar Jürgen and I were quite ignorant about his important work on QFT.

between thermal aspects of the subvolume fluctuations in QFT with Hawking-Unruh type vacuum problems already existed in the 80s [5]. In fact it will become clear in the course of the present work that it indeed can and should be viewed this way.

For some time this problem remained out of my range of interest; I did not want to loose time on something which would draw me into opaque historical problems away from my research on new foundational insights into to QFT via "modular localization"[2] [6]. During a two year stay (2002/2003) in Brazil, a CNPq supported research project "The Modular Structure of Causal Quantum Physics" provided the chance to extend this research. Around 2007 I suddenly realized that the complete understanding of the E-J conundrum can be obtained with the help of precisely those newly gained insights. One just had to apply the *principle of modular localization,* which assigns a certain number of unexpected properties to localized subalgebras. Whereas the global vacuum state is pure, the restriction to a causally localized subalgebra renders it impure; in fact its impurity can be described as a thermodynamic KMS state [7] with respect to a "modular Hamiltonian". This is a general result of the application of the so-called Tomita-Takesaki modular theory of local operator algebras to the subalgebra which spacetime-localized observables localized in a causally complete spacetime region generate.

This reduced vacuum state is entangled in a much radical sense than the entanglement of particle states under a binary split of the system into an inside/outside subsystem in Schrödinger's quantum mechanic. The entanglement of quantum mechanical particle stated resulting from binary inside/outside splits of degrees of freedom resulting in an impurity from the reduction to the inside and the ensuing loss of information is a well-known phenomenon; it has been observed in quantum optical experiments and the results led to a Nobel prize. But the quantum mechanical "vacuum" (the mathematical reference state which one needs for the "second quantization" multiparticle description of QM) remains completely inert against entanglement. In fact *the singular vacuum entanglement caused by localization in QFT is characteristic for the enormous conceptual difference between the two quantum theories.* The terminology E-J "conundrum" refers to the fact that this aspect of the vacuum remained for a very long time outside theoretical attention; in fact theoretical physicists became for the first time aware of the KMS nature of the QFT vacuum state in connection with the Unruh's "Gedankenexperiment" in which the localization region is a spacetime wedge. This aspect of vacuum entanglement also points at the "fleeting" nature of this effect; it remains many orders of magnitude below the measured quantum optical entanglement of QM. But even if it will always remain a "Gedanken" concept [3], it is at the heart of QFT and follows directly from the *quantum adaptation of the* Faraday-Maxwell "action at the neighborhood" which Einstein converted into the Minkowski spacetime *causality principle.* As a result of its radically different and unexpected physical manifestations of its quantum formulation, its quantum counterpart will be referred to as *modular localization,* a terminology which relates to its mathematical formulation. In the present work it will be shown that its conceptual range is not limited

[2]Here modular localization stands for an intrinsic formulation of causal localization which is independent on what quantum field "coordinatization" one uses in order to describe the particular model. of QFT.

[3]The situation becomes less "fleeting" if the horizon of the localization region is an (Unruh observer-independent) black hole "event horizon".

to shed light into dark corners of QFTs history as the E-J conundrum, but it also plays an important role in an ongoing conceptual reformulation of QFT (which includes gauge theories and the recently much discussed "Higgs mechanism").

The two components in Einstein's statistical mechanics fluctuation properties are indeed, as Jordan claimed, also present in the physical vacuum state after restricting it to the ensemble of observables which are localized in a subvolume. It is important to not impose boundary restrictions (box quantization) but remain within the realm of "open systems". Here it is irrelevant whether Jordan's calculation treated this aspect correctly; many important observations in the history of quantum physics have been made within incorrect calculations.

When I was about to explain my findings [8][1][10] in 2008 to Ehlers, I learned that he passed away shortly before my return to Berlin.

The main aim of this paper, which I dedicate to the memory of Jürgen Ehlers, is to explain my findings and their relation to the ongoing research in QFT in more detail as I did in [8].

I remember that Ehlers, in his capacity as the founding director of the AEI in Potsdam, took an interest in string theory (ST). He was annoyed by the fact that he was unable to bridge the gaps between his understanding of spacetime properties of gravity and the (sometimes bizarre) claims of members of the ST group at the AEI; notwithstanding the fact of the enormous amount of mathematical sophistication and the reputation of some of the protagonists of ST.

The work on modular localization also led me to string-localized fields and their important improved short distance property, which promised a radical extension of renormalization theory to interaction between fields with higher spins. The reason why I mention this here is that this new concept of string-localized fields in Hilbert space also revealed that string theory (ST) and its derivatives (embeddings, dimensional reductions, properties of "branes") has no relation to causal localization in spacetime; it rather resulted of a fundamental misunderstanding on these issues. Hence Ehlers' problems with the ancient Einstein-Jordan conundrum and his problems with ST were interconnected in a curious way. His death in 2008 prevented me from conveying this insight.

It is the purpose of these notes to explain the constructive [8] as well as critical [11] power in a historical context.

Usually a historical paper revisits the past about already closed subjects; typical examples are research papers on the discovery and the conceptual development of QM. In contrast to such subjects, which are closed from a foundational point of view, the situation of the problems addressed in this paper are very different in that most of them, although present in QFT from its beginnings, were only solved recently; the context in which they appeared is still far from its closure.

The Einstein-Jordan conundrum was often misunderstood as a confirmation of the particle-wave duality which, since de Broglie's matter-wave idea and Schrödinger's wave equation, was an integral part of QM. But the E-J conundrum addresses a much deeper issue which before the appearance of modular localization concept in QFT was not really understood.

My posthumous thanks for introducing me to a fascinating topic from the genesis of QFT which, far from being a closed part of history exerts its conceptual spell over actual particle theory, naturally go to Jürgen Ehlers. The present exploration of the foundational principle of modular localization did not only change the view about hitherto incompletely understood problems at the dawn of QFT [8], but also promises to have an important say about its future [11].

1 Introduction

A dispute between Einstein and Jordan (referred to as the E-J conundrum [3]) led Jordan to propose the first quantum field theoretical model in order to show that there exists a quantum analog of Einstein's thermal subvolume fluctuations in open subvolumes (intervalls) of two-dimensional quantized Maxwell waves in a global vacuum state. For this pupose Jordan invented the simplest QFT which in modern terminology is the model generated by a conformal chiral current. A brief sketch of the pre-history which led to the E-J conundrum may be helpful:

- Einstein 1917 in [12]: calculation of mean square fluctuations in an open subvolume in statistical mechanics of the thermal black body radiation shows two components: wave- and particle-like ("Nadelstrahlung") fluctuation structure which Einstein interpreted as a theoretical evidence for photons (after his 1905 paper based on the observational support coming from the photoelectric effect).

- Jordan in his PhD thesis (1924, [13]) argued that the particle-like component $\sim \bar{E}_\nu h\nu$ is not needed for attaining equilibrium.

- Einstein's reaction [14] consisted in a publication in which Jordan's argument is shown to be mathematically correct but physically flawed (the absorption is incorrectly described). However he praised Jordan's statistical innovations ("Stosszahlansatz").

- Einstein's paper caused Jordan's radical change of mind; he fully accepted Einstein's view by demonstrating that he can obtain the same wave- and particle-like fluctuation components by restricting a "two-dimensional quantized Maxwell field" (modern terminology: d=1+1 chiral current model) to a subinterval. In this way he discovered field quantization probably without understanding *why* a vacuum in QFT behaves radically different from a quantum mechanical vacuum, in particular why the reduced vacuum shares the impurity with that of a KMS statistical mechanics state.

Shortly after this episode Jordan published his first field quantization in a separate section in the famous 1926 "Dreimännerarbeit" [2]. Gaps in Jordan's computation and his somewhat artistic treatments of infinities caused some ruffling of feathers with his coauthors Born and Heisenberg [3]. From a modern point of view the picture painted in some historical reviews, namely that this was a typical case of a young brainstorming innovator set against a scientific establishment (represented by Born), is not quite correct. Born and Heisenberg had valid reasons to consider Jordan's fluctuation calculations as incomplete, to put it mildly. Conceding this does however not lessen Jordan's merits as the protagonsist of QFT .

One reason why this discovery of QFT was not fully embraced at the time was that, although a free field on its own (staying with its linear properties) is a simple object, the problem of energy fluctuations in open subvolumes is anything but simple. To understand why subvolume fluctuations in the vacuum state of QFT are similar to Einstein's statistical mechanics thermal fluctuations is a deep conceptual problem which could not have been solved solely by calculations; especially because before the arrival of the concept of modular localization such calculations could only have been done in terms of conceptually uncontrolled approximations. But it can be satisfactory answered with the help of a new view of QFT which generically relates the restriction of the vacuum to the observables of a spacetime subvolume with thermal properties and vacuum polarization ("split inclusions" of modular localized algebras [7]); this is precisely what "modular localization" achieves. One may safely assume that Born and Heisenberg perceived that this new quantum field model of Jordan with infinitely many oscillator degrees of freedom did not quite fit into their quantum mechanical project which Heisenberg started a short time before; in particular Jordan's nonchalant way of handling infinities led to critical comments [3].

Nevertheless Heisenberg, who in comparison to Jordan understood very little about statistical mechanics at the time of the E-J conundrum, probably became aware of vacuum polarization (which is absent in QM) under the influence Jordan's fluctuation problem. A letter he wrote to Jordan before he published his famous vacuum polarization paper [3], mentions a logarithmic divergence $Lim_{\varepsilon\to\infty}\log\varepsilon$, with ε describing the "fuzziness" at the interval ends (next section). Indeed vacuum polarization and thermal manifestations of vacuum entanglement from causal localization are opposite sides of the same coin.

One note of caution. Since the terminology "particles" and "waves" played an important role in the Einstein-Jordan dispute, the reader may think that it refers to the quantum mechanical particle-wave dualismus (the two equivalent descriptions of QM); in this way its real significance, namely the thermal aspects of vacuum entanglement through causal localization of quantum matter is sometimes overlooked.

The important distinction between the global quantum mechanical nature of infinitely many oscillators and their holistic role in the implementation of causal localization in a quantum theory of local fields had to wait almost 5 decades before being understood on a foundational level. For some time QFT was suspected to be afflicted by internal inconsistencies which lead to ultraviolet divergencies (the "ultraviolet catastrophe"). Even after discovering the covariant renormalized perturbation theory for quantum electrodynamics and finding an impressively successful agreement of low order perturbation with experimental observations, some of these doubts lingered on. Renormalized perturbation theory remained for a long time a collection of recipes about how to extract finite time-ordered correlation functions from the quantization rules starting with classical Lagrangians.

The quantization parallelism to the classical field theory of Faraday and Maxwell as embodied in the Lagrangian or functional integral quantization prevented for a long time an awareness about some radical differences resulting from quantum causal localization as compared to its classical counterpart. One manifestation of such a difference was that quantum fields, in contrast to smooth causally propagating classical functions, were rather singular operator-valued Schwartz distributions. They require testfunction smearing in order to attain the status of (generally) unbounded operators with which one then can construct operator algebras which are causally localized in spacetime regions.

The other surprise was that these operator algebras have properties which were somewhat unexpected from the conceptual viewpoint of QM. Causal localization causes the global vacuum state to become impure upon restriction to a local operator subalgebra $\mathcal{A}(\mathcal{O})$ generated by covariant fields $A(x)$ smeared with $\mathcal{O}$-supported test functions. These impure "partial" states fulfill the so-called KMS property [7] with respect to a *modular Hamiltonian* which is intrinsically determined by the pair $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ of local algebra and vacuum state vector.

The mathematical theory of operator algebras which highlights such properties is the *Tomita-Takesaki modular operator theory* which is omnipresent in QFT thanks to its causal localization structure. The presentation of QFT in terms of a net of operator algebras and their properties was proposed by Rudolf Haag [15] shortly after Arthur Wightman published his characterization of covariant fields in terms of properties of their correlation functions [16]. Haag's textbook [7] on "local quantum physics" (LQP), based on an operator-algebraic approach to QFT, appeared only many decades after he gave a first account of this new formulation [15]. The terminology LQP in the present article is used whenever it is important to remind the reader that the arguments go beyond the view about QFT which he meets in most textbooks (which are usually restricted to a formulation of perturbation theory within the setting of Lagrangian quantization and its functional integral formulation).

The mathematical property which guaranties the applicability of the T-T modular operator theory, is the so-called *standardness* of the pair $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ i.e. the property that the operator algebra acts on Ω_{vac} (more generally on all finite-energy state vectors) in a cyclic ($\overline{\mathcal{A}(\mathcal{O})\Omega_{vac}} = H$) and separating ($\mathcal{A}(\mathcal{O})$ contains no annihilators of Ω_{vac}) manner. The cyclicity of the vacuum is closely related to the positivity of the energy of the representation of the Poincaré group, whereas the separating property results from spacelike commutativity of observables and is equivalent to the fact that the commutant, which contains the algebra of the causal complement $\mathcal{A}(\mathcal{O})' \supseteq \mathcal{A}(\mathcal{O}')$, acts also cyclic on Ω_{vac} as long as the spacelike complement $\mathcal{O}'$ is non-void. This physicists know under the name of the the "Reeh-Schlieder property" [7], whereas the operator algebraists call this the "standardness" of the pair $(\mathcal{A}(\mathcal{O}), \Omega)$. This property is not shared by QM and accounts for the significant differences between these two QT [17].

For a structural comparison it is convenient to rewrite (the Schrödinger form of) QM into the Fock space setting of "second quantization" which converts wave functions into fields. As mentioned before in this reformulation the newly introduced vacuum remains, as opposed to its active role in QFT, completely inert with respect to the action of the Schrödinger "quantum field" (no vacuum entanglement leading to vacuum polarization). Instead of the cyclic action the local algebra at a fixed time[4] corresponding to a spatial region $\mathcal{R} \subset \mathbb{R}^3$, one obtains a subspace and a tensor factorization of H

$$H(\mathcal{R}) = \overline{\mathcal{A}(\mathcal{R})\Omega_{QM}} \subset H = H(\mathcal{R}) \otimes H(\mathcal{R}^{\perp}) \tag{1}$$
$$\mathcal{A}(\mathcal{R}) = B(H(\mathcal{R})),\ \mathcal{A} \equiv \mathcal{B}(\mathcal{H}) = \mathcal{A}(\mathcal{R}) \otimes \mathcal{A}(\mathcal{R}^{\perp})$$

of with a factorizing vacuum Ω_{QM}. This inertness against entanglement of the quantum mechanical vacuum is very different from the "vacuum polarizability" of Ω_{vac} in QFT

[4]In LQP such an algebra at a fixed time $\mathcal{A}(\mathcal{R})$ is defined as the intersection of all spacetime algebras $\mathcal{A}(\mathcal{O})$ with $\mathcal{R} \subset \mathcal{O}$.

which is connected to the lack of tensor factorization (despite the commutation between $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O}'))$. In terms of structural properties of operator algebras these remarkable differences in the mathematical structure amount to the existence of two non-isomorphic factor algebras in QFT: the global $\mathcal{B}(H)$ algebra of all bounded operators on a Hilbert space (the unique type I_∞ factor) and the local *monad* algebras $\mathcal{A}(\mathcal{O})$ which are all isomorphic to the unique hyperfinite type III_1 factor algebra in the Murray-von Neumann-Connes classification of factor algebras [7].

The choice of terminology reveals the intention to see the new local quantum physical view of QFT in analogy to the way Leibnitz understood *reality in terms of relations between monads*. In this extreme relational view, a monad by itself is structureless, similar to a point in geometry. Indeed in the local quantum physical description of QFT all properties of quantum matter, including the Poincaré covariance of its localization in spacetime and its possible localization-preserving inner symmetries, can be shown to arise from the abstract (non-geometric) modular positioning of copies of the monad within a shared Hilbert space (section 3).

Together with the thermal KMS property of the locally restricted vacuum, there is the formation of a vacuum polarization cloud at the causal boundary of localization which accounts for a *localization entropy*. By replacing the boundary by a thin shell of size ε the localization entropy can be described in terms of a function of the dimensionless area $\alpha = area/\varepsilon^2$ which diverges in the limit $\varepsilon \to 0$. This relation between the increasing sharpness of localization and the increasing localization entropy is the *substitute of the lost quantum mechanical Heisenberg uncertainty relation*. The position operator $\mathbf{x}_{op}$ is, as all quantum mechanical observable of global nature; it does not belong to the observables obeying the causal localization principle of LQP but may be used in the (non-covariant) effective description of wave-function propagation. The divergence in the sharp localization limit $\varepsilon \to 0$ shows another aspect in which QFT differs from QM. The entanglement between the wedge-localized algebra and its opposite (that of the spacelike separated wedge) is always infinite in the sense that it is not possible to describe the associated state as density matrix (accounting for the singular nature of vacuum entanglement); indeed there are no pure states nor density matrix states on monad algebras; all states are impure in a very radical way. In quantum statistical mechanics such states appear as singular KMS states in the thermodynamic limit of density matrix Gibbs states. Local algebras $\mathcal{A}(\mathcal{O})$ in QFT have no density matrix states or pure states at all; every global state restricted to such an algebra will be rather singular (as a reminder: a state is a normalized linear positive functional on an algebra and only if this algebra consists of all bounded operators in a Hilbert space $B(H)$, states can be represented by vectors modulo phase factors).

The reduced vacuum state assign a *probability* to the ensemble of local observables contained in $\mathcal{A}(\mathcal{O})$; this is a consequence of the KMS (statistical mechanics-like) nature of the impure reduced vacuum state. Unlike the probability interpretation, which Born added to QM and which Einstein rejected ("God does not throw dice") the ensemble viewpoint of probability (as in statistical mechanics, which Einstein always accepted) is intrinsic to QFT. KMS states on the ensembles of $\mathcal{O}$-localized observables are like thermal states of statistical mechanics and not "Gedanken-ensembles" as in case of Born's individual mechanical systems of QM which they refer to the statistics of repeated measurements. Einstein had no problems with probability of real ensembles in statistical mechanics. Unfortunately the conceptual sophistication in the early days of QT (and many decades afterwards) led to probability.

There have been attempts to improve Jordan's approximations [3] since the subvolume fluctuation problem is not solvable in closed form. The characterization of the algebra of operators localized in a subvolume is a *holistic problem*; the enclosure of the subsystem in a quantization box is not the same as reducing the vacuum to the subvolume algebra. Dealing with open subsystems is an "holistic" challenge in which the knowledge of the global oscillators is of not much help. Standard QFT does not provide the means to characterize the ensemble of operators which is localized in a subvolume $\mathcal{O}$. On way of doing this would be to smear the quantum fields with $\mathcal{O}$-supported testfunctions and use the algebra which they generate. Even then one needs some knowledge about the "modular Hamiltonian" which is related to the kind of statistical mechanics associated with the KMS state corresponding to the restricted vacuum. In certain cases one can guess it in the form of a geometric transformation which leaves $\mathcal{O}$ invariant. For a noncompact wedge region in Minkowski spacetime e.g. $W_3 = \{x; x_3 > |x_0|\}$ this would be the wedge-preserving Lorentz subgroup $\Lambda_{W_3}(\chi)$, for Jordan's model (a chiral subalgebra on a lightlike interval, see section 4) it is the dilation subgroup of the Möbius group); but in the generic case on has to refer to modular theory. What is important in the historical review is not whether Jordan got this right, but rather that in his attempt to counter Einstein he invented QFT.

In order to avoid any misunderstandings, it should be emphasized that in saying that the concept of probability enters QFT in a more natural way than in QM, one is not implying that this is changing the epistemic aspects of the measurement theory in QT. All the conceptual aspects of entanglement (including Bell's inequality) remain valid in an appropriate modified form [18]. What QFT adds is a more radical realization of these phenomena on a much smaller scale; as already mentioned the scale of localization-caused vacuum entanglement is that of the Unruh effect and Hawking radiation. The reality of entanglement of particle states with respect to binary subdivisions in QM is experimentally accessible in terms of quantum optical arrangements, whereas the KMS impurity of the spacetime-restricted vacuum (e.g. the Unruh effect) will presumably always remain experimentally inaccessible (including even high energy nuclear experiments).

Part of the problem is that it is nearly impossible to describe precisely in terms of existing hardware how a perfect causal localization can be realized; even for noncompact spacetime regions as Unruh's Rindler wedges, the effect depends on the state of uniform acceleration of the observer; observer-independent manifestations appear only in the context of metric-induced event horizons of black holes. Fortunately foundational principles do not need to permit *direct* observational verification; they only have to be conceptually consistent, incorporate the reality which existed before their inception, and lead to new observable consequences. In this respect QFT, which only shares with QM the Hilbert space and $\hbar$ but not the causal locality principle, has been and promises to continue to be the most inclusive successful physical theory.

One can entertain wonderful dreams of what may have happened if important concepts would have appeared decades earlier. But in the real world big conceptual jumps against the prevalent ideas of the time (the Zeitgeist) are virtually impossible; even for getting from inertial systems in Minkowski spacetime to General Relativity it took Einstein many years and the same can be said about the development of QM out of the old semiclassical Bohr-Sommerfeld ideas. The problem for the case at hand is aggravated by the fact that, up to the middle of the 60s, there did not even exist a mathematical framework of operator algebras in which ideas about localization could have been adequately formulated.

It is interesting to note that modular operator theory and its physical counterpart of modular localization is the only theory to whose discovery and development mathematicians (Tomita, Takesaki, Connes) and physicists (Haag, Hugenholz and Winnink) contributed on par. They first realized this at a 1965 conference in Baton Rouge[5], with statistical mechanics of open systems and the role of the KMS property representing the physical side [7]. The study of the relation between modular operator theory and causal localization in LQP started a decade later [19], and its first application consisted in a more profound understanding [20] of the Unruh Gedankenexperiment [21]. The terminology "modular localization" is more recent and marks the beginning of a new constructive strategy in QFT based on the modular aspects of localization of states and algebras [44][6]. In mathematics the theory was decisive instrument which led to Connes closure of the Murray-von Neumann project of classifying von Neumann factor algebras.

The E-J conundrum represents in fact a precursor of the Unruh Gedankenexperiment and, as the latter, can be fully resolved in terms of the principle of modular localization. In fact in the special case of Jordan's chiral current model (the historically first and simplest model of a QFT), the solution of the E-J conundrum amounts to a unitary *isomorphism* between a system defined by the vacuum state restricted to the algebra $\mathcal{A}(I)$ localized in an interval I and an associated global statistical mechanics system at finite temperature. Such isomorphic relations are referred to as describing an "inverse Unruh effect", [24] and the Jordan model is the only known illustration. However in both cases the KMS temperature is not something which one can measure with a thermometer or use for "egg-boiling"[6] (and there is also no "boiling soup" of particle/anti-particle pairs) since the acceleration only affects the "Carnot-temperature" [48].

The attribute "holistic" will be used quite frequently in connection with modular localization. This terminology has been previously introduces by Hollands and Wald [25] in connection with their critique of calculations of the cosmological constant in terms of simply occupying global energy levels (with a cutoff at the Planck mass). In previous papers [26], it refers to the intrinsicness of localization which is connected with the cardinality of phase space degrees of freedom and their subtle local interplay. This distinguishes physical localization of quantum matter from mathematical/geometrical concepts. In fact it presents a strong barrier against attempts of geometrization of QFT and explains why the Atiyah-Witten attempt of the 70s to "geometrize" QFT did not lead to the physical insight which many people (including the author) hoped for.

[5]The mathematicians worked on the generalization of the modularity of Haar measures ("unimodular") in group representation theory whereas the physicists tried to understand quantum statistical mechanics directly in the thermodynamic infinite volume limit (open system statistical mechanics) by using the KMS identity instead of approaching this limit by tracial Gibbs states.

[6]These results remove certain bizarre alleged consequences of the Unruh effect (e.g. no generation of heat by whirling a thermometer through empty space) but maintain the impure KMS nature of the wedge-restricted vacuum.

The simplest illustration of the meaning of holistic consists in the refutation of the vernacular: "(free) quantum fields are nothing more than a collection of oscillators" which often students are told in courses of QM. Knowing continuous families of oscillators in the form of creation and annihilation operators $a^{\#}(\mathbf{p})$ does not reveal anything about free quantum fields and their associated local operator algebras. The free Schrödinger field and a free scalar covariant field share the same global oscillator creation/annihilation operators

$$a_{QM}(\mathbf{x},t) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int e^{i\mathbf{px}-\frac{\mathbf{p}^2}{2m}} a(\mathbf{p}) d^3p, \ \ [a(\mathbf{p}), a^*(\mathbf{p}')] = \delta^3(\mathbf{p}-\mathbf{p}') \tag{2}$$
$$A_{QFT}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(e^{-ipx}a(\mathbf{p}) + e^{ipx}a^*(\mathbf{p})\right) \frac{d^3p}{2\sqrt{\mathbf{p}^2+m^2}}, \ p = (\mathbf{p}, \sqrt{\mathbf{p}^2+m^2})$$

In both cases the global algebra is the irreducible algebra of all operators $B(H)$, generated by the shared creation/annihilation operators. But the local algebras[7] generated by test function smearing with finitely supported Schwartz functions $suppf(\mathbf{x}) \subset \mathcal{R}$ of the fields and its canonical conjugate at a fixed time in a spatial region $\mathcal{R}$ are very different in both cases. In the relativistic covariant case they are identical to the algebras $\mathcal{A}(\mathcal{O}_\mathcal{R})$, $\mathcal{O}_\mathcal{R} = \mathcal{R}''$ the causal spacetime completion of $\mathcal{R}$ (which is also generated by smearing with $\mathcal{O}_\mathcal{R}$-supported spacetime smearing functions). According to what was stated before, these algebras are of "monad" type and the $\mathcal{A}(\mathcal{O}_\mathcal{R})$-restricted vacuum state is a KMS state; in the case of the Schrödinger field the associated subalgebra $B(H(\mathcal{R}))$ is of the same type as the global algebra; the QM vacuum continues to be an inertial state in the "smaller" factor Hilbert space $H(\mathcal{R})$.

Whereas the global QM algebra is simply the tensor product of its factor algebras, the relation of the net of local algebras to its $\mathcal{A}(\mathcal{O})$ "pieces" is a more holistic relation; although together with its complement it generates the global algebra $\mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\mathcal{O})' = B(H)$, the global algebra $B(H)$ is not a tensor product of the two. The most surprising property which underlines the terminology "holistic" is the fact that the full net of local operator algebras which contains all physical informations can be obtained by "modular tuning" *of a finite number of copies of a monad* in a shared Hilbert space[8]; the reader who is interested in the precise formulation and its proof is referred to [27], see also [17]. The fact that the global oscillator variables are the same in both cases (2) does not reveal these fundamental holistic differences of spacetime organization of quantum matter which have very different physical consequences. The present quantization formalism (Lagrangian, functional integral) does not shed light on those properties of QFT which solve the Einstein-Jordan conundrum in a clear-cut way. If it comes to ensemble properties of localized observables, the global aspects of generating covariant fields (which have no definite localization region) on which covariant perturbation theory is founded are of lesser

[7]Technical points as the connection between fields and the algebras they generate are not important in the present context and therefore will be omitted.

[8]This number n is two for rhe simplest case of a chiral algebra, whereas for a net in four spacetime dimension the correct modular positioning can be achieved in terms of n=7 copies. The emergence of the spacetime symmetries in Minkowski spacetime as well as possible inner symmetries of quantum matter is a consequence of this holistic tuning.

importance than the local operator algebras $\mathcal{A}(\mathcal{O})$ which are generated by all smeared fields $A(f)$ with $\sup pf \subset \mathcal{O}$. The emphasis changes from covariance properties of fields to properties of relative localization of operator algebras and this change finds its appropriate mathematical form in the LQP ("local quantum physics") setting of QFT [7].

It is precisely this holistic aspect which renders any calculation of the subvolume fluctuation difficult; the simplicity of global oscillators is of no help here. A calculation in closed form is (even in the absence of interactions) not possible, and the imposition of covariance, which was the important step for obtaining the modern form of perturbation theory, also does not provide guidance. For renormalized perturbation theory one has clear recipes which were extracted from the imposition of covariance, but this is of not much help when one wants to find appropriate description of localized fluctuation in open subsystems. Saying that the global aspects can be described in terms of oscillators is almost as useless as trying to understand the holistic structure of a living body in terms of its chemical composition. Although modular localization theory asserts the existence of "modular Hamiltonians", in its present stage it does not provide a generic method to explicitly construct them. Jordan's chiral model is an exceptional case for which, similar to the Unruh Gedankenexperiment, an explicit identification of the modular Hamiltonian in terms of the spacetime symmetries of the model is possible. Actually one may view Jordan's fluctuation problem as a predecessor of the Unruh effect in other words: QFT was born with the "thermal"[9] localization aspects of the E-J conundrum which includes a completely intrinsic pre-Born notion of ensemble-probability; however the proximity of its date of birth to that of QM prevented an in-depth understanding of differences beyond the shared $\hbar$ and the Hilbert space.

This begs the question how, with the understanding of foundational properties of QFT still being that incomplete, it was possible to achieve the remarkable progress in renormalized perturbation theory. To phrase it in a more provocative historical context: how could one arrive at the Standard Model without having first solved the 1925 Einstein-Jordan conundrum? The answer is surprisingly simple: to get from the old Wenzel-Heitler formulation of perturbation theory, in which the vacuum polarization contributions were still missing, to the Tomonage-Feynman-Schwinger- Dyson perturbation theory for quantum electrodynamics (QED), one only needed to impose covariance and "exorcise" some ultraviolet divergences by finding plausible recipes. It was the internal consistency of the result and not its derivation from Lagrangian quantization which made renormalized perturbation theory successful.

Many years later there were also derivation of these renormalization rules by starting from invariant free field polynomials (without using Lagangian quantization[10]) and invoking spacelike commutativity in an inductive way (the causal perturbation setting of Epstein and Glaser [28]). But such conceptual refinements (of reducing prescriptions to to an underlying principle) had little impact on the Zeitgeist; in any case it would not have helped to obtain the foundational insight into modular localization which is required in order to solve the E-J conundrum.

This lucky situation of making progress by playfully pushing ahead and working once way through a yet conceptual incomplete formalism with the help of consistency checks

[9] The reason for the quotation marks will be explained in section 6.
[10] The free fields do not have to fulfill Euler-Lagrange equations.

did not extend much beyond Lagrangian quantization and renormalized perturbation theory. As will be shown in section 6, it is precisely this setting which determined the fate of QFT for more than half a century which is now being replaced by a more general setting based on modular localization. The latter has not only removed unnecessary restrictions from renormalization theory, but also led to a different view about on-shell constructions (section 5). When, in the aftermath of the Lehmann-Symanzik-Zimmermann (LSZ) scattering theory and the successfull adaptation of the Kramers-Kronig dispersion relations the first attempts of S-matrix based on-shell construction were formulated, the conceptual difficulties of analytic aspects of on-shell properties were underestimated. As one knows through more recent progress about modular localization, an important aspect of the S-matrix, namely its role as a relative modular invariant of wedge-localization was missing. As a result, the true nature of the particle crossing property was misunderstood by identifying it with Veneziano's dual model crossing which was than inherited by string theory (ST).

The correct formulation of the on-shell crossing property within a new S-matrix based construction project and the solution of the E-J conundrum are interconnected via the principle of modular localization. It is the aim of this paper to show the power of the latter by presenting the solution to these two problems. The first attempts to formulate particle physics and obtain an constructive access outside of quantization and perturbation theory was the S-matrix in Mandelstam's project [29]. As we know nowadays, and as it will be explained in detail in the present work, this failed as a result of the insufficient understood on-shell analytic properties, whose connection to the causality principles are much more subtle than those to the off-shell correlation functions. In retrospect it is clear that with the scant understanding of the central crossing property (and more generally the conceptual origin of on-shell analyticity properties), there was no chance in 70s for Mandelstam's S-matrix based particle theory project to succeed. In retrospect it is also clear why this happened precisely when Veneziano's mathematical construction of a crossing symmetric meromorphic function in two variables was accepted as a model realization of particle crossing for elastic scattering amplitudes. It is appropriate in an article, whose intention is to shed light on still ongoing misunderstandings, to explain this situation in its historical context.

The importance of the E-J conundrum in the development of QFT can be best highlighted by *following Galileio's example and imagine a dialog between Einstein and Jordan about subvolume fluctuations but placing it in the year 1927 after Max Born added his probability interpretation to Heisenberg's and Schrödinger's quantum mechanics.* Here it will be used as an artistic device to underline this importance.

Einstein: Dr. Jordan, I appreciate that you could finally accepted my invitation to come to Berlin and I am very interested to understand why, after first criticizing my fluctuation calculations in my statistical mechanics thermal blackbody radiation model, you now claim that you find the same fluctuation components in your new wave quantization at zero temperature.

Jordan: Thank you Professor Einstein for taking so much interest in my work. The appearance of such a fluctuation spectrum in my new setting of quantized waves in a vacuum state is indeed surprising because although my wave quantization of 2-dimensional Maxwell waves generalizes Heisenberg's quantization in some sense, the fluctuation properties obtained by restricting the vacuum to a subinterval are very different from those of his and Born's formulation of QM. It seems that my quantized Maxwell waves cannot be subsumed into a quantum mechanics of systems with an infinite number of oscillators.

Einstein: As you remember, I have some grave reservation against associating a probability to an individual measurement on a quantized mechanical system which I occasionally expressed in the formulation "the Dear Lord does not throw dice". But I never had any problem with probability in statistical mechanics, in fact my calculation of the Nadelstrahlung-component in the black body fluctuation spectrum, which led me to the particle nature of light on pure theoretical grounds, is based on the probability of statistical mechanics. Does the result of your subvolume fluctuation calculation in the pure ground state of your field quantization mean that this state appears impure if analyzed in the setting of an open subsysten?

Jordan: Professor Einstein, I am glad that you raised this question. I have been breaking my head over these unexpected consequences of my new quantized field theory and I would be dishonest with you, if I claim to understand these conceptual implications. But since the main difference to mechanics is the causal propagation, (which was already implicit in the Nahewirkungsprinzip of Faraday and Maxwell and which you then succeeded to generalize into your new relativity principle in a Minkowski spacetime), I am inclined to suspect that the ensemble aspect, which one needs in order to avoid the assignement of a probability to an individual mechanical system (as proposed by my adviser Prof. Max Born), has its origin in the quantum realization of causal localization. Somehow this principle creates a natural ensemble associated with its causal completion of a localization region, namely the ensemble of all local observables attached to that spacetime region. This is in contrast to QM which deals with individual mechanical systems for which the association to an ensemble is a useful mental construct for the interpretation of QM. I tried to convince Prof. Born and my colleague Werner Heisenberg, who despite their initial resistance finally agreed to permit me to present my idees in a separate section of a joint paper which was published two years ago. But I was not able to remove their doubts. It would be very helpful for me to obtain some support from your side.

Einstein: I need some time to think about this new situation. Your conjecture seems to suggest that your new theory of quantum fields, which is certainly much more fundamental than Heisenberg's and Schrödinger's quantized mechanics, comes with an intrinsic notion of localized ensembles of observables and an associated statistical mechanics type of probability. If one could better understand how the less fundamental global quantum mechanics can be related as a limiting case to your new fundamental quantum field theory in such a way that Born's postulated probability is a relict of your local ensemble probability, this may change my view and perhaps even influence my quantum physical Weltanschauung. Let us remain in contact and please keep me informed about future clarifications on the points raised in out conversation.//

In this imagined dialog, which could have radically changed the history of QFT, I avoided the use of advanced mathematical concepts. of modular localization (there was no mathematical support in the 20s). The E-J conundrum is best understood as a progenitor of an Unruh-like Gedankenexperiment.

The organization of this paper is as follows. In the next section the vacuum polarization on the boundary of causal localization is derived for the "partial charge", which is a modern formulation of Heisenberg's original observation. Section 3 sketches the issue of modular localization and its KMS property with special emphasis on the difference between a KMS temperature and that measured by a thermometer. In section 4 the KMS property is used for the explicit construction of an isomorphism between the thermal subvolume (interval in Jordan's chiral model) fluctuations in Jordan's model with a corresponding statistical mechanics model representing Einstein's side. Section 5 explains modular localization and its relation with the Tomita-Takesaki modular operator theory. The ongoing impact of modular localization on on-shell constructions of QFT, with particular emphasis on the connection of particle crossing with the KMS identity, is addressed in section 7.

The most important consequence of modular localization for the ongoing research in particle theory is the generalization of renormalized perturbation to interactions involving arbitrarily high spin through the use of string-localized fields in section 6. In the case of spin s=1 it leads to a much deeper understanding of why gauge theory requires the indefinite metric Krein space setting and how modular localization allows a formulation which remains throughout in Hilbert space.

The same ideas which lead to unexpected progress also permit to expose the misunderstandings which led to the dual model and ST as presented in section 7. In contrast to the stringlocal fields in higher spin QFT the "string" in ST has no relation to spacetime. Section 8 addresses some old and in the maelstrom of time lost insights about the connection between the cardinality of phase space degrees of freedom and causal localization which includes problems concerning dimensional changes which came from ST but which can also be formulated in the setting of QFT. The critique of the Maldacena conjecture, concerning the nature of the AdS-CFT correspondence, addresses one of those problems. The concluding remarks attempt to position the present situation in particle theory within the historical context and expectations about its future.

2 Vacuum polarization, area law

In 1934 Heisenberg [30] finally published his findings about vacuum polarizations (v. p.) in the context of conserved currents which are quadratic expressions in free fields. Whereas in QM they lead to well-defined partial charges associated with a volume V,

$$\partial^\mu j_\mu = 0,\ Q_V^{clas}(t) = \int_V d^3x j_0^{clas}(t,\mathbf{x}) \tag{3}$$

$$Q_V^{QM}(t) = \int_V d^3x j_0^{QM}(t,\mathbf{x}),\ Q_V^{QM}(t)\Omega^{QM} = 0$$

there are no such sharp defined "partial charges" Q_V in QFT, rather one finds (with g_T a finite support smooth interpolation of the delta function) [31]

$$Q(f_{R,\Delta R}, g_T) = \int j_0(\mathbf{x},t) f_{R,\Delta R}(\mathbf{x}) g_T(t) d\mathbf{x}dt,\ f_{R,\Delta R} = \begin{pmatrix} 1, & \|x\| \leq R \\ 0, & \|x\| \geq R + \Delta R \end{pmatrix} \tag{4}$$

$$lim_{R\to\infty} Q(f_{R,\Delta R}, g_T) = Q,\ \ \|Q(f_{R,\Delta R}, g_T)\Omega\| = \begin{cases} F_2(R,\Delta R) \overset{\Delta R\to 0}{\sim} C_2 ln(\frac{R}{\Delta R}),\ n=2 \\ F_n(R,\Delta R) \overset{\Delta R\to 0}{\sim} C_n(\frac{R}{\Delta R})^{n-2},\ n>2 \end{cases}$$

The *dimensionless* partial charge $Q(f_{R,\Delta R}, g_T)$ depends on the "thickness" (fuzziness, roughness) $\Delta R = \varepsilon$ of the boundary and becomes the f and g-independent (and hence t-independent i.e. conserved) global charge operator in the large volume limit. The deviation from the case of QM are caused by v. p.. Whereas the latter fade out in the $R \to \infty$ limit, they grow with the dimensionless area $\frac{R}{\Delta R}$ for $\Delta R \to 0$. The simplest calculation is in terms of the two-point function of conserved current of a zero mass scalar free field. In the massive case the leading term in the limit $\Delta R \to 0$ remains unchanged. We leave the elementary calculations (not elementary at the time of Heisenberg) to the reader.

The presence of v. p. causes relativistic quantum fields to be more singular than Schrödinger fields and requires the formulation in terms of Schwartz distribution theory as used in the above smearing of the current with smooth finitely supported test function. The LQP setting on the other hand avoids the direct use of such singular objects in favor of local operator algebras. In such a description the singular nature of vacuum polarization is not directly perceived in the individual operators, but rather shows up in ensemble properties of operator algebras. It turns out that under rather general conditions there exists between two monad algebras a distinguished (by modular theory) intermediate type I_∞ algebra N [7]

$$\begin{aligned}
&\mathcal{A}(\mathcal{O}_{\mathcal{R}+\Delta R}) \supset N \supset \mathcal{A}(\mathcal{O}_{\mathcal{R}}),\ H \xrightarrow{V} H(N) \otimes H(N'),\ \eta \equiv V(\Omega \otimes \Omega) \\
&VAB'\Omega = A\Omega \otimes B\Omega,\ A \in \mathcal{A}(\mathcal{O}_{\mathcal{R}}), B' \in \mathcal{A}(\mathcal{O}_{\mathcal{R}+\Delta R}),\ VNV^* = B(H) \otimes \mathbf{1}
\end{aligned} \tag{5}$$

i.e. there exists a unitary operator V which permits to write the full Hilbert in terms of a tensor product such that $\mathcal{A}(\mathcal{O}_{\mathcal{R}}) \subset N$, $\mathcal{A}(\mathcal{O}_{\mathcal{R}+\Delta R})' \subset N'$ where the "split vacuum" η is a state in the original Hilbert space which corresponds to the tensor product of vacua.

In QM the unitary V would be simply the identity operator expressing the fact that the vacuum is a auxiliary mathematical state which remains physically inert under splitting, i.e. the QM vacuum is not entangled under spatial subdivisions. In QFT it is a an state which on $N \otimes N'$ is nontrivially entangled in the sense of quantum information theory. However in the sharp localization limit $\Delta R \to 0$ the "quantum mechanical" type I_∞ converge towards the monads $A(\mathcal{O}_R), A(\mathcal{O}'_R)$ which commute but do not tensor-factorize. The limiting entanglement is of a very singular kind which has no counterpart in quantum information theory and is characteristic for monad algebras which do not admit density matrix states. The situation is analogous to that encountered in finite temperature statistical mechanics in the thermodynamic infinite volume limit when the tracial nature (the Gibbs formula) of the state is lost and only the KMS property remains[11].

The above described nontrivial behavior under splitting leads to a nontrivial ΔR dependent *localization entropy* which is consistent with the KMS impurity of the restricted vacuum. In fact since the vacuum polarization happens in a layer of size ΔR (the "fuzzy" boundary) the entropy is a function of the dimensionless area

$$\begin{aligned}
a = \frac{area}{R^2},\ En(a) = split\ localization\ entropy \\
En(a)|_{\Delta R \to 0} \simeq ca,\ a = \frac{area}{\Delta R^{d-2}},\ for\ d > 2
\end{aligned} \tag{6}$$

[11]Whereas the thermodynamic limit monad is approximated from the inside, the split property approximates the local monad from the outside.

where the second line is the leading order in the sharp localization limit which one expects if the "polarization clouds", which determine the singular behavior of smeared fields as Heisenberg's partial charges (4), are the same as those which appear in the above entropy argument.

The logarithmic behavior for d=2 split entropy can actually be derived [47] and is well-known to condensed matter physicists. For Jordan's chiral current model used in the E-J conundrum, the entropy can be directly obtained from the isometry with a chiral statistical mechanics model (section 4). This situation is very special and has been termed "the inverse Unruh effect [24]. For d=1+3 't Hooft has obtaind the area behavior in terms of the "brickwall picture" **[32],** but a rigorous derivation, solely based in the split property of modular localization, is not yet available. Bekenstein's area law results if one relates ΔR with the Plank length.

There exists a conjecture that even in the general case there could be a weak form of the "inverse Unruh effect" [24] in which the spatial volume factor is replaced by the "volume factor" if a box with two spacelike and one lightlike direction. In that case the two spacelike extensions would account for the dimensionless area factor and the lightlike contribution would be (as in the chiral Jordan model) logarithmic [47] so that the net result is a logarithmically modified area law.

This behavior of localization-entropy shows that although there are genuine infinities in QFT, they are limited to sharp localization of fields (smearing with non-smooth spacetime test functions) or the entropy content of sharply localized algebras. Unlike the ultraviolet divergencies in the old formulation of perturbation theory, they have no relation to the "ultraviolet catastrophe" i.e. they threaten in no way the consistency of QFT; to the contrary, they are a consequence of its most foundational modular localization property. In a certain sense the divergence of thermodynamic infinite volume limit correspond to the infinity obtained in the sharp boundary limit (vanishing "fuzzyness" or roughness of the boundary) $\varepsilon \rightarrow 0$.

With the notion of "localization temperature" and energy one has to be much more careful than with the dimensionless localization entropy. When one naively interprets the Unruh temperature as that measured by a thermometer, one enters a conceptual mine field. The equality of the thermometer temperature (related to the zeroth thermodynamic law) with the "Carnot temperature" of the second fundamental law of an KMS equilibrium state is only correct in an inertial system, but the Unruh temperature refers to an accelerated observer. In fact the thermometer *temperature in a vacuum state remains zero*; it is a "local temperature" which does not depend on the Unruh trajectory [48]. The same holds for other situations described by modular theory (next section); although there is always a dimensionless modular Hamiltonian and a dimensionless temperature $\beta = 2\pi$ associated with modular KMS states, the *standard form of thermodynamics holds only in inertial systems.* The still ongoing hot topic about "firewalls" [50] is dangerously close to the Unruh "cooking temperature" and more investigations about possible differenced between causal horizons (Unruh) and event horizons are necessary.

A useful conceptual step in passing from classical fields to quantum fields is to avoid to attribute a direct physical meaning to fields, but rather to view them in a similar role as that which coordinates play in the description of geometry. This is facilitated by the fact that quantum fields are not directly measured (no experimentalist has measured a nuclear field); rather the notion of a quantum field serves as a *device to describe particles* which are related to a particular subset of quantum field. But the same particles can be associated to many different fields. It has turned out that to view fields in their role as coordinatizing or generating local algebras is the most useful way of keeping track of the differences of description.dependent fields from intrinsic particles. In this way particles do not correspond to individual fields but rather to local field classes which carry the same superselection charges. All structural properties of LQP and the resulting general theorems can be expressed in terms of local nets of operator algebras, but the present formulation of renormalized perturbation theory still needs fields.

Note that the well known entropy conjecture by Bekenstein, based on equating a certain area behavior in classical General Relativity (which parallels that of entropy) with quantum entropy, results formally from the above area law by equating ΔR with the Planck length. Quantum Gravity is often thought of that still elusive theory which explains why and how the quanta of gravity can escape the consequences of modular localization for sharp localization an evade causal localization. If Bekenstein's conjecture really describes quantum aspects of gravity black (and not just quantum matter in curved spacetime) then modular localization cannot be extended to Quantum Gravity.

As mentioned before the relation between ΔR and the entropy is reminiscent of Heisenberg's quantum mechanical uncertainty relation in which the uncertainty in the position is replaced by the split distance ΔR within which the vacuum polarizations can attenuate, so that outside the vacuum returns to play its usual role (if tested with local observables in the causal complement of $\mathcal{O}_{\mathcal{R}+\Delta R}$).

It should be stressed again that the probability interpretation, which Born had to add to Heisenberg's and Schrödinger's formulation of QM, is completely intrinsic to LQP. It is a consequence of the "thermal" KMS property of ensembles of operators contained in a localized algebra $\mathcal{A}(\mathcal{O})$ in the $\mathcal{O}$-restricted vacuum. As such it is not different from the statistical mechanic probability, which Einstein used in his fluctuation arguments in terms of which he challenged the physical content of Jordan's thesis. It is only with the modern concept of modular localization and the hindsight of more than eight decades of QFT that one realizes how close the E-J conundrum came to the intrinsic probability coming from the quantum formulation of the Faraday-Maxwell-Einstein causal locality principle in Minkowski spacetime. Einstein's problem was the assignement of a probability to an individual mechanical system (which requires to *imagine* it as a member of an ensemble for which the probabilistic nature is seen in repeated measurements).

The fact that probability is intrinsic to QFT affects in no way the discussion around quantum entanglement and Bell's inequality. The effects of the (more radical form of) entanglement of the vacuum through localization are however orders of magnitudes below the quantum mechanical entanglement of particle state which can never be measured by quantum optical methods; in fact such effects which are characteristic for QFT (they sharply separate the latter from QM) may never be directly measurable.

A particular radical illustration of the conceptual differences between QFT and QM is the reconstruction of a net of operator algebras from the relative modular position of a finite number of copies of the monad [17]. For chiral theories on the lightray one needs two monads in a shared Hilbert space in the position of a *modular inclusion*, for d=1+2 this "modular GPS" construction needs three and in d=1+3 seven modular positioned monads are sufficient [27] to create the full reality of a quantum matter world, including its Poincaré symmetry (and hence Minkowski spacetime) from the abstract modular groups. This possibility of obtaining concrete models by modular positioning of a finite number of copies of an abstract monad (stuff with no inner structure) is the strongest "holistic outing" of QFT and the reader is encouraged to look at this application of modular theory [27]. For d=1+1 chiral models the modular positioning leads to a partial classification of chiral theories as well as to their explicit construction (section 5).

Apart from d=1+1 factorizing (integrable) models, where modular properties in the form of *nuclear modularity* were used for existence proofs of models [38], QFT has not yet reached the state of maturity where such holistic properties can be applied for classifications and existence proofs of families of models and their mathematically controlled approximation. An extension to curved spacetime would be very interesting; the simplest question in this direction is the modular construction of the local diffeomorphism group on the circle in the setting of chiral theories.

3 Modular localization and its thermal manifestation

The aim of this section is to present the concept of *modular localization* which is the backbone of LQP and exposes the intrinsic formulation of causal quantum localization. Since, as mentioned before, subalgebras $\mathcal{A}(\mathcal{O})$ localized in spacetime regions $\mathcal{O}$ with $\mathcal{O}'' \subsetneq R^4$ are known to act cyclic and separating on the vacuum (the Reeh-Schlieder property [7]), the "standardness" condition for the validity of the Tomita-Takesaki modular theory is always fulfilled for local subalgebras. This leads to a uniquely defined Tomita operator $S_{\mathcal{A}(\mathcal{O})}$ whose properties will be the main subject of this section.

It has been known for a long time that the algebraic structure underlying free fields allows a functorial interpretation in which operator subalgebras of the global algebra $B(H)$ are the functorial images of *subspaces of the Wigner wave function spaces* ("second quantization"[12]).

Before presenting some mathematical details, it is useful to recall some conceptual/philosophical points. LQP avoids the parallelism to classical field theory which charaterizes the Langrangian quantization approach of QFT and the closely related functional integral representation, by starting from principle which charaterize causal quantum matter. If one accepts that QFT is more fundamental than classical field theory then the content of QFT should reveal itself in terms of its own principles without the detour of a "quantization parallelism" to classical field theory.

In contrast to QM, the LQP setting of QFT de-emphasizes individual operators in QFT in favour of *ensembles of operators* which share the same spacetime localization region. This intends to follow more closely the situation in the laboratory where the experimentalist measures coincidences between events in spacetime; all the measured particle properties, including the nature of spin and internal quantum numbers, were obtained by repetitions and refinements of observations based on counters which are placed in compact spatial region and remain "switched on" for a limited time. Their detailed

[12]Not to be confused with quantization; to quote a famous saying by Ed Nelson: "quantization is an art, but second quantization is a functor".

internal structure is generally not known, what matters is their localization in spacetime and the sensitivity of their response. As previously mentioned the causal localization property of quantum matter contains from the start the notion of ensembles of localized observables and the probabilistic aspects resulting from the local restriction of the vacuum whereas for an individual quantum mechanics system one has to (follow Born and) add it for interpretative reasons. Without a precise mathematical backup which matches these physical concepts, LQP would however have remained in the philosophical realm. Such a presentation will be the aim of this section

The role of covariant quantum fields in LQP is that of *generators* of a net of local operator algebras $\{\mathcal{A}(\mathcal{O})\}_{O\in R^4}$ which act in a fixed Hilbert space. In the Wightman setting a field is a covariant operator-valued distribution $A(x)$ which is globally defined for all $x \in R^4$. From its global definition one passes to (unbounded) $\mathcal{O}$-localized operators by testfunction smearing, formally written as $A(f) = \int A(x)f(x)d^4x$, $supp f \subset \mathcal{O}$. According to Wightman's axioms, fields define a system of polynomial (generally unbounded) operator *-algebras $\mathcal{P}(\mathcal{O})$. Formally these unbounded operators can be associated with an aforementioned net of (mathematically easier manageable) bounded operators forming von Neumann algebras which define Haag's LQP setting. The advantage is that one obtains access to the well-developed mathematical theory of operator algebras (from now on omitting "bounded"). Certain causality aspects allow a more natural definition and more profound understanding in the LQP setting. The mathematical details, which allow to pass between Wightman's description to the algebraic local nets of observables in the LQP setting and vice versa, are tedious and still technically incomplete [7], but this had little effect on progress.

Whereas both settings are different formulations of closely related physical concepts, there is a significant distinction between these settings and constructions based on Lagrangian or (closely related) functional integral based quantization methods. Quantization is not a physical principle; whereas it is conceivable that certain successful classical descriptions of nature can be pictured as limiting cases of quantum theories, there is no general correspondence in the opposite direction. The fact that the less fundamental QM (it lacks causal localization and its holistic consequences which make QFT "fundamental") is capable to maintain an almost (up to ordering prescriptions of operators) unique connection to classical mechanics does not imply that such a close relation must continue to hold in QFT. The strong link between classical mechanics and its quantum counterpart finds its best known expression in the fact that Lagrangian quantization (canonical quantization) and functional quantization (path integrals) enjoy solid mathematical support from measure theory.

All this breaks down in interacting QFT with realistic short distance behavior[13]. Apart from d=1+1 integrable models (section 5), for which rigorous methods of LQP led to existence proofs [38][55], there is of course renormalized perturbation theory; but since perturbative expansions in the coupling strengths (which are consistent on the level of polynomial relations) inevitably lead to divergent series, they are not the right objects for an intrinsic formulation of QFT. In fact there exists not even a mathematical argument that they define an asymptotic small coupling approximation in the limit of vanishing

[13]Free field short distance behavior of polynomialyl coupled scalar fields is still in the reach of measure-theoretical functional methodes [33].

coupling to an existing model of QFT, although the use of low order perturbative results led in certain cases to quite spectacular agreements with observations. Whereas the setting of QM has reached its closure a long time ago, the conceptual/mathematical flanks remained open.

The *causal perturbation* setting of Epstein and Glaser [28] avoids the ultraviolet divergencies of the Lagrangian or functional setting by implementing causal locality in terms of time-ordered products in an inductive way. A specific model is defined in terms of its free field content, and the starting point is a first order interaction density in form of a Lorentz-invariant (scalar) Wick-polynomial. The scaling degree of the interaction density is determined in terms of the scaling degrees of the participating fields and their derivatives. If the scaling degree of the interaction does not surpass $d_{s.d.}^{int} = 4$ one obtains a renormalizable model in which the short distance dimensions of quantum fields remain bounded independent of the iterative steps (order of perturbation). Although this kind of *causal* renormalized *perturbation theory* is independent of Lagrangian quantization (the problem of whether a free field associated to a Wigner representation is "Euler-Lagrange" is irrelevant), it cannot prevent the divergence of the perturbative series.

The problem with this setting is its limitation with respect the spin of pointlike free fields in a Hilbert space setting. The short distance dimension of pointlike free fields in Hilbert space increases with spin as $d_{s.d.} = s + 1$. Hence a $m > 0,\ s = 1$ Proca potentials with $d_{s.d.} = 2$ does not admit any renormalizable interaction (below the power-counting limit) in Hilbert space. Wigner's 1939 classification of particles in terms of positive energy representations led to a clear statement about the field content of covariant $(m = 0, s \geq 1)$ representations: there are covariant field pointlike field strengths[14] but no covariant pointlike potentials. This is the famous *clash between Hilbert space positivity and pointlike localization*. The conventional way out is that of keeping the pointlike structure and allow indefinite metric (so-called Krein-) spaces instead of Hilbert spaces.

This problem is not present in classical Maxwell theory; in that case the use of vectorpotentials contains a redundancy which affects the connection of Cauchy data and their causal propagation and is conveniently taken care of in terms of the concept of gauge transformations and gauge invariance (the return to field strengths and currents). Lagrangian quantization and functional integral presriptions for gauge theories lead out of the Hilbert space, in fact pointlike interaction-free massless vector potentials are well known to require a Krein space formulation (the Gupta-Bleuler formalism). Since the Hilbert space setting is the foundational pillar of QT, the setting of *quantum gauge theory* in the presence of interactions of massive or massless vectormesons is an undesired but inevitable consequence of quantization of classical gauge theory. In particular physical matter fields remain outside the reach of the quantum gauge which only secures the return of Hilbert space for gauge invariant local observables.

This makes it desirable to turn to another description which the previously mentioned alternative suggests: *abandon pointlike localization and keep the Hilbert space.* Since this is inconsistent with the quantization of classical gauge theory, it is not surprising that such an alternative requires a radical change of the Epstein-Glaser causal perturbative setting

[14]Massive pointlike potentials and their associated field strengths have the same $d_{s.d.} = s + 1$, but whereas the zero mass limit of field strengths exists, that of potentials does not.

[28]. Although the latter does not depend on quantization of a classical field structure[15], it uses pointlike generating fields in an essential way. The safest procedure is to try to extract an information from the foundational localization principles of LQP by asking the following structural questions: what is the tightest localization which can be derived solely from the mass gap property? The type of models for which such a question could be relevant are interacting massive vectormesons. As mentioned before pointlike interactions of such fields are nonrenormalizable, and since the concept of renormalizability is intimately related to the short distance aspects of localization, it is natural to think about such models in which renormalizabilty was obtained at the prize of sacrificing the Hilbert space.

The answer is part of a theorem by Buchholz-Fredenhagen [7]: all LQP with a mass-gap (which are known to admit scattering theory) can be generated by spacelike semi-infinite stringlocal fields[16] whose localization is stringlike. Covariant generating stringlocal fields $\Psi(x,e)$, $e^2 = -1$ are localized on $x + \mathbb{R}_+ e$ and commute for spacelike separated strings (appropriately modified for Fermions). In section 6 the string-extended E-G perturbation theory will be exemplified in massive gauge theories. Whereas the local observables (field strengths, currents) remain pointlocal and the interacting physical matter fields are stringlocal, the S-matrix turns out to be e-independent. Massive vectormesons also permit a coupling to *neutral matter* (scalar Hermitian fields).

These couplings reveal what was known to some researchers for a long time: the Higgs mechanism about a mass-creating symmetry breaking is not supported by QFT. What is looming behind is nothing else than the renormalizable coupling of massive vectormesons to Hermitian (charge-less) fields, a theory which has no classical counterpart which is related to the vanishing of the interaction in the massless limit. The intrinsic property of all couplings of massive vectormesons to matter, independent of whether the latter is charged or neutral, is the "Schwinger-Higgs screening" of the Maxwell charge. Although this is consistent with the BRST gauge setting, the new Hilbert space setting using renormalizable couplings of stringlocal massive vectormesons lead to these results without having to rely on unphysical Krein space methods (section 6).

The fundamental idea which is behind the ongoing radical changes is a much deeper understanding of *quantum* causal locality in the algebraic operator setting of modular localization. Individual quantum fields never played a similar distinguished physical role as classical fields. They are hardly ever directly measured (measuring a hadronic field ?) and the particles which are identified with counter events are always associated with an infinite class of (composite) fields which carry the same superselected charge. Whereas in QM it makes sense to think in terms of a hierarchy of particles namely the ones in terms of whose dynamical variables one defines the model and their bound states, such a division is rather meaningless in QFT since the omnipresence of vacuum fluctuations only respects the superselected charges and couples all states which have the same such charge. The fields within one superselected class are distinguished by their short distance scale

[15] In particular it does not depend on whether the quantum fields are solutions of Euler-Lagrange equations.

[16] Since LQP avoids generating fields in favor of localized alsgebras, the localization regions in the theorem is "arbitray narrow spacelike cones" (whose cores are strings). Pointlke localization is a special case. .

dimensions (the renormalizable Lagrangian couplings highlight fields with low $d_{s.d.}$), but the particle field relation is based on infinite timelike separations (time-dependent scattering theory) for which low $d_{s.d.}$ values are irrelevant. But it is precisely this "Murphy's Law" behavior of QFT in the presence of interactions: *everything which can be coupled will be coupled* (there is always a process in which this coupling is activated) which is the *prize to be paid for a fundamental theory.* Modular localization theory brings all these foundational properties (which still remain somewhat hidden in the perturbation theory in terms of individual fields) into the forefront.

The central issue in LQP refers to two physically motivated requirements on the local net of operator algebras

$$\begin{aligned} [\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)] = 0,\ \mathcal{O}_1 >< \mathcal{O}_2,\ &\textit{Einstein causality} \\ \mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}''),\ &\textit{causal completeness} \\ \mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})',\ &\textit{Haag duality} \end{aligned} \tag{7}$$

The first line is a condensed notation for the commutativity of operators from spacelike separated regions; it is only required for observable fields. The commutation property for *non-observable* operators, as those coming from spinor fields or fields carrying superselected charges, are determined by the local representation properties of the observables (the superselection theory to their associated observable subalgebras [7]).

The *causal completeness property* (7) is a local adaptation of the old time-slice property [36]. In classical relativistic field theory the field values in the relativistic "causal shadow" (causal completion) V'' are uniquely determined in terms of the (properly defined) initial values of fields in a finite volume V at fixed time. Its quantum adaptation in the LQP setting is the algebraic causal completeness property. Often particle theoreticians only consider the simpler Einstein causality property and ignore causal completeness. But there are situations which are consistent with Einstein causality but violate causal completness[17]. In fact in [36] the simplest model, a so-called generalized free field with a suitable continuous mass distribution was used as an illustrative example for a physically unacceptable Einstein-causal field. Whereas in the Lagrangian quantization setting causal completeness is a formal relic of the quantization of a classical hyperbolically propagating theory field theory, this property needs to be checked outside of quantization. Unfortunately the old knowledge about these important property has been lost within the string-theory community, otherwise Maldacena would not have been able to convince a world wide community that the mathematically consistent $AdS_{n+1}-CFT_n$ isomorphism is also physically acceptable. Only holographic projections onto n-1 null-surfaces lead to a right "thinning out" of degrees of freedom (loss of information). As a consequence one cannot return to the original theory without some additional information.

There exist however situations in certain quantum field theories, which contain massless $s \geq 1$ in which for multiply connected spacetime regions the Haag duality is violated in a specific way; the prototype is the quantum Aharonov-Bohm effect for the net of algebras generated by the quantum electromagnetic field strength [37]. Moreover in chiral QFTs such topological violations of Haag's duality happens for disconnected intervals.

[17] In quantum physical terms a completeness violating situation exhibits a "poltergeist" behavior: new degrees of freedom (which were not present in $\mathcal{A}(\mathcal{O})$) enter $\mathcal{A}(\mathcal{O}'')$ from "nowhere".

According to my best knowledge these are the only physically acceptable (no degrees of freedom problems) violations. In the case of zero mass field strengths for $s \geq 1$ these cases are related to the clash between pointlike localization of potentials and the positivity of Hilbert space and its resolution in terms of stringlocal potentials.

Mathematically it is very easy to construct Einstein-causal theories which violate causal completeness and as a consequence (apart from the aforementioned (topological exceptions) lead to pathological physical properties with respect to their "degrees of freedom" behavior[18]. Well known cases in addition to the mentioned Maldacena conjecture arise from embedding lower dimensional quantum field theories and its reverse: Kaluza-Klein dimensional reductions and "branes".

As a result of a subtle relation between the cardinality of phase-space degrees of freedom with localization (split property, causal completeness,..), the nuclearity property (introduced by Buchholz and Wichmann [7]) became in conjunction with modular theory ("modular nuclearity") an important concept for the classification and nonperturbative construction of models of QFT [38] [26].

After having presented some of the physical requirements of the LQP formulation, we now pass to a brief description of its main mathematical support: the Tomita-Takesaki modular operator theory. This theory has its origin in the operator-algebraic aspects of group representation algebras from which Tomita took the terminology "modular" (originally referring to properties of Haar measures). A conference in the US (Baton Rouge, 1967), which was attended by mathematicians (Tomita, Takesaki, Kadison,..) and mathematical physicists (Haag, Hugenholz, Winnink, Borchers,..), is marks the beginning of the Tomita-Takesaki modular operator theory as a joint project [39]. The participating physicists had already obtained important partial results of that theory through their project of formulating quantum statistical mechanics directly in the thermodynamic limit (statistical mechanics of *open systems*) [7]. In their new way of thinking, the Kubo-Martin-Schwinger property (originally an analytic shortcut for computing Gibbs traces) assumed a conceptual role in the new formulation of thermal equilibrium states for *open quantum systems.* Although these ideas originated independently, the mentioned conference united them; there is hardly any area in which the contribution of mathematicians and physicists have been that much on par as in modular operator theory/modular localization.

One reason for this perfect match was that the area of physical application of modular theory widened the scope of statistical mechanics and, combined with *causal localization,* became the most important mathematical/conceptual tool of LQP. The basic fact which led to this new connection was the Reeh-Schlieder theorem [7] which secures the validity of the "standardness" requirement for the applicability of the Tomita-Takesaki theory. Standardness of a pair $(\mathcal{A},\Omega)$ (algebra and state) means that the action of the operator algebra $\mathcal{A}$ on the state vector Ω generates the Hilbert space (cyclicity of Ω) and that there are no annihilators of Ω in $\mathcal{A}$ (Ω is separating)

$$cycl.:\ \overline{\mathcal{A}\Omega}=H,\quad sep.:\ A\Omega=0\curvearrowright A=0,\ A\in\mathcal{A}$$

The Reeh-Schlieder theorem guaranties the validity of this property for any pair $(\mathcal{A}(\mathcal{O}),\Omega),\ \mathcal{O}''\subset\mathbb{R}^4$; in fact this even holds if the vacuum is replaced by any finite energy state. The im-

[18]The breakdown of causal completeness leads to a "poltergeist" effect where degrees of freedom apparently enter from "nowhere"; one finds them in $\mathcal{O}''$ but they were not in $\mathcal{O}$.

portance of the relation between localization and the T-T theory was noted a decade after the Baton Rouge conference by Bisognano and Wichmann [7]; these authors found that in the context of localization in a wedge region $\mathcal{O} = W$ the Tomita-Takesaki theory makes contact with known geometrical/physical objects.

The general T-T theory is based on the existence of an unbounded antilinear closable involution S with a dense domain $domS$ in H which contains all states of the form $\mathcal{A}\Omega,$ in case of a standard pair [41][18]. Whereas the cyclicity secures the existence of its dense domain, the absence of annihilators of Ω in $\mathcal{A}$ guaranties its uniqueness.

$$
\begin{aligned}
&S_{\mathcal{O}}A\Omega = A^*\Omega,\ A \in \mathcal{A} \subset B(H),\ S = J\Delta^{\frac{1}{2}} = \Delta^{-\frac{1}{2}}J \qquad (8)\\
&J\ antiunit.,\ \Delta^{it}\ mod.\ unitary,\ \sigma_t(A) = Ad\Delta^{it}A
\end{aligned}
$$

The existence of a polar decomposition in terms of a antiunitary J and a positive generally unbounded operator Δ follows from the closability of S (in the following S stands for the closure). The modular unitary gives rise to a *modular automorphism* group of the localized algebra $\mathcal{A}$.

The physical interpretation is only known only for $\mathcal{O} = W =$ wedge regions, which are Poincaré transforms of the standard t-z wedge $W_0 = \{z > |t|\,; \mathbf{x} \in \mathbb{R}^2\}$. In that case the modular objects are the unitary transformation representing the W-preserving Lorentz ("boost") subgroup $\Delta_W^{it} = U(\Lambda_W(-\pi t))$ and the reflection on the edge of the wedge J which is, up to a π-rotation within the edge, equal to the TCP operator. Since in a theory with a complete particle interpretation (to which the considerations of this paper are restricted, unless stated otherwise) the interacting TCP operator and its incoming (free) counterpart are known to be related by the scattering operator S_{scat} [42], we obtain for all J independent of the position of W [6]

$$J_W = S_{scat} J_{W,in} \quad for\ all\ W$$

This expresses a property of S_{scat} which turns out to be indispensable for the constructive use of modular localization in QFT: *S_{scat} is a relative modular invariant between the interacting and the associated free (particle) wedge algebra.* This property was recently used in a more physical proof [40] of the Bisognano-Wichmann theorem which reduces the interacting case in theories with mass gaps and a complete particle interpretation to that of free fields (see below).

The relative modular invariance of S_{scat} is the crucial property which accounts for the analyticity of on-shell objects as S_{scat} and the related formfactors. These on-shell analytic properties find their important manifestation in the *particle crossing property*. It is also the starting point of the algebraic construction of integrable QFT [6]. The connection between algebraic and analytic properties is much more subtle for on-shell objects as the S-matrix and formfactors than for off-shell correlation function. Since most of these properties were not understood in the 60s, it is not surprising that Mandelstam's project of formulating particle physics as a quantization-free on-shell project failed on the lack of understanding of on-shell analytic properties.

The misunderstandings about the particle crossing property in the construction of the *dual model*, which later entered string theory, have their origin in confusions about the meaning of localization in QFT as opposed to QM. In section 7 these misunderstandings will be analyzed in the light of recent progress.

Since it is not possible to present a self-consistent complete account of the mathematical aspects of modular localization and its physical consequences in a history-motivated setting as the present one, the aim in the rest of this section will be to raise awareness about their existence and their physical content.

It has been known for a long time that the algebraic structure associated to free fields allows a functorial interpretation in which operator subalgebras of the global algebra $B(H)$ are the *functorial images of certain real subspaces* of the Wigner space of one-particle wave functions (the famous so-called "second quantization"[19]), in particular the spacetime localized algebras are the images of localized real subspaces. This means that the issue of localization to some extend can be studied in the simpler form of localized subspaces of the Wigner particle representation space (unitary positive energy representations of the $\mathcal{P}$-group).

These localized subspaces can be defined in a intrinsic way [44] i.e. without quantization, only using operators from the positive energy representation U of the proper Poincaré group $\mathcal{P}_+$ ($det = +1$) on the direct sum of two copies of the Wigner representation u of the connected component (proper orthochronous $\mathcal{P}_+^\uparrow$) on the one-particle space H_1. For simplicity of notation the transformation formulas are limited to the case of a spinless charged particle:

$$H_1 : (\varphi_1, \varphi_2) = \int \bar{\varphi}_1(p)\varphi_2(p)\frac{d^3p}{2p_0}, \quad \hat{\varphi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int e^{ipx}\varphi(p)\frac{d^3p}{2p_0} \tag{9}$$

$$U(g)(\varphi_1 \oplus \varphi_2) = u(g)\varphi_1 \oplus u(g)\varphi_2, \;\; u(a,\Lambda)\varphi(p) = e^{ipa}u(\Lambda^{-1}p)$$

$$\Theta \equiv TCP, \;\; \Theta(\varphi_1 \oplus \varphi_2) = C\varphi_2 \oplus C\varphi_1, \;\; C\varphi(p) = \overline{\varphi(p)} \tag{10}$$

Any $\mathcal{P}_+$ transformation can be generated from $U(g)$ and Θ. For representations with $s > 0$ the Lorentz group acts through Wigner rotations (Wigner's "little group") on the little Hilbert space which in the massive case is the 2s+1 component representation space of rotations. The massless case leads to a 2-dimensional Euclidean "little space" whose degenerate representation (with trivially represented "little translations") form a two-component little helicity space, whereas faithful representation acts in an infinite dimensional Hilbert space ("infinite spin") [43]. The Lorentz transformations as well as Θ act also (through representations of the little group) on the little Hilbert space.

It is precisely through the appearance of this little Hilbert space that the problem of causal localization of states (wave functions) cannot be simply solved by Fourier transformation and adding positive frequency contributions of particles with those of negative frequency from antiparticles. Whereas in the case of the two classes of finite little spaces (the massive and zero mass finite helicity class) of positive energy Wigner representation, their "covariantization" was easily achieved in terms of group theoretic methods [45] and led to local pointlike generating wave functions and fields, this third infinite spin class posed a series obstacle. Attempt to convert its members into covariant pointlike wave

[19] Not to be confused with quantization; to quote a famous saying by Ed Nelson: "quantization is an art, but second quantization is a functor".

functions and corresponding fields remained unsuccessful and there was no understanding of the origin of this failure[20]. Weinberg dismissed this large positive energy representation class by stating that nature does not make use out of it [45]. Since all important physical properties are connected to aspects of localization which are precisely those properties which at that time remained poorly understood, such a dismissal could be premature, in particular in times of dark matter.

The localization problems of the infinite spin class were finally solved [44] with the help of modular localization which for different problems was already used in [6]. In fact the main theorem in that paper states [44] that all positive energy wave functions are localizable in noncompact spacelike cones and only the first two classes permit the sharper localization in double cones (the causal shadow of a 3-dim. sphere). Since the (topological) core of arbitrarily small double cones is a point and that of arbitrary narrow spacelike cones is a semiinfinite spacelike string, the remaining problem consisted in actually constructing the generating fields of these representation; this was achieved in [43]. The result can be described in terms of operator-valued distributions $\Psi(x,e)$ which depend in addition to the start x of the semiinfinite string also on the the spacelike string direction $e,\ e^2=-1$. They are covariant under simultaneous transformations of x and e and fulfill Einstein causality

$$[\Psi_1(x_1,e_1),\Psi_2(x_2,e_2)]_{gr}=0,\quad x_1+\mathbb{R}_+e >< x_2+\mathbb{R}_+e_2 \tag{11}$$

where gr stands for graded (fermionic strings anticommute).

The modular localization of states uses the following construction. With a wedge $W=(x\mid x_3>|x_0|)$ there comes a wedge preserving one-parametric group of Lorentz-transformation $\Lambda_W(\chi=-2\pi\tau)$ where χ is the hyperbolic boost parameter and Θ_W denotes the x_0-x_3 reflection. The latter differs from the total reflection Θ by a π rotation r_W around the x_3 axis (in the x_1-x_2 plane) and therefore acts on the wave functions as $J_W=U(r_W)\Theta$. Both transformations Λ_W and J_W commute. Since the generators of one-parametric strongly continuous unitary groups are selfadjoint operators, there exists an "analytic continuation" in terms of positive unbounded operators with dense domains which decrease with the increase of distance from the real axis. This forces the W-localized wave functions to have certain analyticity properties in the momentum space rapidity θ $(p_0,p_3)=\sqrt{m^2+p_\perp^2}(ch\theta,sh\theta)$ which relate the analytic continuation of particle wave function to the complex conjugate of the antiparticle wave function[21] Using the notation $\Delta_W^{i\tau}\equiv U(\Lambda_W(-2\pi\tau))$, the commutation with the antiunitary J_W leads to

$$\begin{aligned}
&S_W=J_W\mathfrak{d}_W^{\frac{1}{2}}=\mathfrak{d}_W^{-\frac{1}{2}}J_W,\ S_W^2\subset 1,\ \textit{acts on } H_1\oplus H_1 \\
&S_W\psi=\bar{\psi},\ K_W\equiv\{\varphi\in domS_W;S_W\varphi=+\varphi\},\ S_Wi\varphi=-i\varphi \\
&K_W\ \textit{"is standard"}: K_W\cap iK_W=0,\ K_W+iK_W\ \textit{dense in } H_1\oplus H_1
\end{aligned} \tag{12}$$

where $\bar{\psi}$ denotes the localization-independent S-conjugate wave function (the complex

[20]Reference [46] is an exception in that certain aspects of the localization problem were already noted.

[21]If there exists an operator creating a partice, the negative frequency part associated with the antiparticle annihilation must be related to the positive frequency part of the antiparticle creation in its hermitian adjoint.

conjugate for the case at hand)[22]. The properties are straightforward consequences of the commutation between the boost and the associated reflection [44]. The important point here is that S relates wave functions to their conjugates in a way which involves analytic continuation where the analyticity came from spacetime localization.

The properties in (12) result simply from the commutativity of $\Lambda_W(\chi)$ with the reflection J on the edge of the wedge; since J is anti-unitary it commutes with the unitary boost, there will be a change of sign in its action on the analytic continuation of u. Hence it has all the properties of a modular Tomita operator. The K-spaces $K(\mathcal{O})$ for causally closed sub-wedge regions $\mathcal{O}$ can be obtained by intersections i.e. $\cap_{W\supset\mathcal{O}}K(W)$; this intersection may however turn out to be trivial (see below) if the region is "too small".

The surprise resides in the fact that the transformation of wave functions to their S-conjugate (12, second line) does not only encode the information about two geometric objects: a one-parametric modular group leaving a wedge invariant and a reflection on that wedge into its opposite but (and at this point the positive energy property of the Wigner representation becomes relevant [44]) it also contains the information about the spacetime localization of the wave function. This is certainly something which is totally incomprehensible in QM; it points to an incomplete understanding of the foundations of QFT which becomes fully revealed in the relation between localized subalgebras and modular operator theory in the presence of interactions.

The connection with causal localization is of course a property which only appears in the physical context. The general setting of modular real subspaces is a Hilbert space which contains a real subspace $K\subset H$ which is standard in the sense of (12). The abstract S-operator is then defined in terms of K and iK.

The above application to the Wigner representation theory of positive energy representations[23] also includes the *infinite spin representations* which lead to semiinfinite string-localized wave functions i.e. there are no pointlike covariant wave function-valued distributions which generate these representations; they are genuinely string-localized (which the superstring representation of the Poincaré group is not; so beware of terminology!). The application of the above mentioned second quantized functor converts the modular localized subspaces into a net of $\mathcal{O}$-indexed interaction-free subalgebras $\mathcal{A}(\mathcal{O})$. Interacting field theories can clearly not be obtained in this way. The relation between particles and fields becomes much more subtle in the presence of interactions and this applies even to models which have a complete particle interpretation i.e. in which the particles related to fields via the LSZ large time behavior of fields (the LSZ scattering formalism) lead to the identification of the Hilbert space as a WignerFock particle space (section 7).

The algebraic setting in terms of modular localization also gives rise to a physically extremely informative type of inclusion of two algebras which share the vacuum state, the so-called *modular inclusions* $(\mathcal{A}\subset\mathcal{B},\Omega_{vac})$ where modular means that the modular group of the bigger $\Delta_{\mathcal{B}}^{it}$ compresses (or extends) the smaller algebra [27]. A modular inclusion automatically forces the two algebras to be of the monad type. The above

[22] Although the action of S_W is diagonal, the definition of the J_W needs the antiparticle doubling of the Wigner space.

[23] The positive energy condition is absolutely crucial for obtaining the prerequisites (12) of modular localization.

mentioned "GPS construction of a QFT" from a finite number of monads positioned in a common Hilbert space uses this concept in an essential way. It is perhaps the most forceful illustration of the holistic nature of QFT.

There are two properties which always accompany modular localization and which are interesting in their own right. Both are related to the statistical mechanics nature of impure $\mathcal{A}(\mathcal{O})$-restricted vacuum

- *KMS property.* By ignoring the world outside $\mathcal{O}$ one gains infinitely many KMS modified commutation properties with modular Hamiltonians $\hat{K}$ associated to the $\widehat{\mathcal{O}}$ restricted vacuum.

$$\langle AB \rangle = \left\langle Be^{-K}A \right\rangle, \quad \Delta = e^{-K}, \ A, B \in \mathcal{A}(\mathcal{O}), infinitly\ many\ \widehat{K} \quad for\ \widehat{\mathcal{O}} \supset \mathcal{O}$$

In contrast to the inert factorizing vacuum of QM in the Fock space ("2nd quantization") description, the spatially restricted QFT vacuum fulfills infinitely many KMS relations associated with modular Hamiltonians of larger spacetime regions.

- Area law for localization-entropy, see (6)

$$Entr = f(\frac{area}{\varepsilon^2}), \ \varepsilon = split\ size$$

As mentioned in the previous section, one needs to invoke the so-called split property in order to approximate the singular KMS state by a sequence of density matrix states; this is similar to the construction of the thermodynamic limit state in statistical mechanics. In contrast to the approximation of the latter in terms of box-quantized finite volume Gibbs states, the split formalism for open subsystems is a part of the (presently computational rather inaccessible) modular localization theory. It is in particular not clear whether the density matrix from the split property leads to a plain dimensionless area law $f \simeq area/\varepsilon^2$ [24] as in (6) or to a logarithmically modified area law [47]. For chiral conformal theories on the lightray there is a rigorous derivation of the localization entropy for an interval with vacuum attenuation length ε (surface fuzziness) from the well-known linear length $l \to \infty$ behavior (the "one-dimensional volume factor" l). They are related as $ln\varepsilon^{-1} \sim l \times kT$. This *inverse Unruh effect* plays an important role in the full understanding of the E-J conundrum and will be presented in the next section.

Great care needs to be taken in identifying the modular localization "temperature" with that measured with a thermometer. This is because the notion of thermometer temperature is based on the zeroth thermodynamic law (the *local temperature* in [48]), whereas the KMS temperature refers to the second law according to which it is impossible to gain energy from equilibrium states by running a Carnot cycle (the absolute temperature). In inertial systems those two definitions coalesce (after proper normalization), whereas in a accelerated systems (used e.g. in the Unruh Gedankenexperiment to achieve the Rindler-wedge localization) this is not the case.

[24]This is suggested by the vacuum polarization clouds of smeared fields in the limit of a aharply cut-off smearing function (see previous section).

A closer examination shows [48] that the conclusion about "egg-boiling" and particle radiation claimed to be observed by an accelerated observer are incorrect, a fact which has been consistently ignored in the literature on the Unruh effect. The correct local temperature, different from the Carnot temperature, does not depend on the acceleration and since it vanishes at spacelike infinity, it vanishes everywhere. Although the black hole situation is different, the application of Einstein's equivalence principle suggests caution about the relation of a rescaled modular temperature with that measured by a thermometer. This includes also the presently very popular ideas about *firewalls* which are allegedly created by restricting generically locally normal states to a causal/event hotizon.

Using again the more protected form of a Sagredo-Simplicio dialog for a critical look at some topics presented in this section and also to recall some important pre-electronic results which were lost in the maelstrom of time and as the result of sociological changes, one may imagine the following conversation:

Sagredo: Dear friend Simplicio, I noticed that you have some critical opinions about the topic of extra dimensions and dimensional reductions. Can you explain your arguments against these extremely popular ideas?

Simplicio: Such ideas originated a long time ago when Kaluza and Klein realized that in classical field theories and quasiclassical approximation of quantum theories one may relate models in different spacetime dimensions by appropriately reinterpreting the field content, for example identifying vectorpotentials with certain components of the metric tensor in higher dimensions. However the foundational understanding of the issue of causal localization in its mathematically precise form of *modular localization of quantum matter* is inconsistent with such ideas.

Causally localizable quantum matter is inexorably linked to spacetime and as a result the connection between the cardinality of degrees of freedom does not allow a physically meaningful "transplantation" of quantum matter. Contrary to classical causality, as formulated by Einstein in Minkowski spacetime, quantum causality does not allow such a separation; whereas the use of quantum fields as "coordinates" of spacetime-localized LQP nets may incorrectly suggest such a picture (since it is consistent with classical fields), the use of the more intrinsic LQP description of quantum matter in spacetime disproves this idea. Already Wigner's theory of positive energy representations of the Poincaré group shows the subtle relation between spacetime and matter; whereas in QM the spacetime interpretation is in the hands of the computing physicist (he may want to view an oscillator chain in 1 or n dimension), every spacetime dimension leads to different particle representations (depending on Wigner's little group). None of the defenders of such ideas has any mathematical controlled argument; their reasoning for dimensional reductions ("the curling up of small dimensions") are either based on quasiclassical approximations or on "massaging" Lagrangians. Arguments directly based on correlation functions are conspicuously absent during the more than 80 years which have passed since the time of Kaluza and Klein. Using modern concepts of modular localization we now know why any such attempt has to fail.

Sagedo: But there *are* relations between theories in different spacetime dimensions, as the AdS-CFT correspondence.

Simplicio: Yes, but at what physical prize! Nobody with a profound conceptual understanding of particle physics would attribute to that overpopulated "stuff" obtained on the CFT side the status of physical matter. Degrees of freedom streaming into the region which is protected by the causal shadow property (QFT "poltergeists") are not acceptable. It is of course not forbidden to use mathematical facts in technical tricks as e.g. doing computations about CFT on the AdS side (in case these computation are simpler) before returning to the physics on the CFT side. The Maldecena conjecture which alleges that the AdS-CFT correspondence connects two physical theories has not only problems with mathematical facts, but also has its problems with a more naive picture of quantum degrees of freedom.

Of course incorrect ideas have always accompanied research on foundational frontiers, and even if they did not lead to a new concept (as Dirac's antiparticles from the incorrect "hole theory") their refutation played a catalyzing role. This was not different at the time of Heisenberg, Dirac, Jordan, Pauli or Feynman. But in those times there was an ongoing process of cleansing; no incorrect idea had a chance to become the accepted viewpoint of a globalized community; in fact such communities simply did not yet exist. The phenomenon of a loss of important insights from pre-electronic times about the connection between causal propagation and cardinality of degrees of freedom is new; it needed the intellectual arrogance within globalized monocultures and hegemony of Big Science in order to turn the vernacular "many people cannot err" into its converse.

4 The E-J conundrum, Jordan's model

With the *locally restricted vacuum* representing a highly impure state with respect to *all* modular Hamiltonians $H_{mod}(\check{\mathcal{O}})$, $\check{\mathcal{O}} \supseteq \mathcal{O}$ on local observables $A \in \mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'')$, a fundamental conceptual difference between QFT and QM has been identified. QM (type I_∞ factors) is the conceptual home of *quantum information theory*[25], whereas in case of localized subalgebras of QFT a direct assignment of entropy and information content to a monad, if possible at all, can only be done in a limiting sense. The present work shows that QFT started with this conceptual antagonism in the E-J conundrum, but its foundational understanding only began more than half a century later and is still far from its closure.

For this reason it is more than a historical retrospection to re-analyze the E-J conundrum from a contemporary viewpoint. In a modern setting Jordan's two-dimensional photon[26] model is a chiral current model. As a two-dimensional zero mass field which solves the wave equation it can be decomposed into its two u,v lightray components

[25] Another subject which would have taken different turn with a better appreciation of the problems in transfering notions of quantum information theory to QFT is the decades lasting conflict about the problem of "black hole information loss".

[26] This terminology was quite common in the early days of field quantization before it was understood that that in contrast to QM the physical properties depend in an essential way on the spacetime dimension. Jordan's photons and his later neutrinos (in his "neutrino theory of light" [9]) do not have properties which permits to interprete the real 4-dimensional objects as higher dimensional versions in the same sense that a chain of oscillators is independent embedding space..

$$\partial_\mu \partial^\mu \Phi(t,x) = 0,\ \Phi(t,x) = V(u) + V(v),\ u = t + x,\ v = t - x \tag{13}$$
$$j(u) = \partial_u V(u),\ j(v) = \partial_v V(v),\ \langle j(u), j(u')\rangle \sim \frac{1}{(u - u' + i\varepsilon)^2}$$
$$T(u) =: j^2(u) :,\ T(v) =: j^2(v) :, \quad [j(u), j(v)] = 0$$

The scale dimension of the chiral current is $d(j) = 1$, whereas the energy-momentum tensor (the Wick-square of j) has $d(T) = 2$; the u and v world are completely independent and it suffices to consider the fluctuation problem for one chiral component. The logarithmic infrared divergence problems of zero dimensional chiral $d(V) = 0$ fields arise from the fact that the zero mass field V, different from what happens in higher dimensions[27], are really stringlike instead of pointlike localized. In fact the V is best pictured as a semiinfinite line integral (a string) over the current [9]; this underlines that the connection between infrared behavior and string-localized quantum matter also holds for chiral models on the lightray. It contrasts with QM where the infrared aspects are not related to the infinite extension of quantum matter but rather with the *range of forces* between particles. Exponentials of string-localized quantum fields involving integration over zero mass string localized d=1+3 vectorpotentials share with the exponentials of integrals over d=1+1 currents $expi\alpha V$ the property that their infrared behavior requires a representation which is inequivalent to the vacuum representation of the field strength or currents; the emergence of superselection rules ("Maxwell charges") is one of the more radical consequences of string-localization.

The E-J fluctuation problem can be formulated in terms of j (charge fluctuations) or T (energy fluctuations). It is useful to recall that vacuum expectations of chiral operators are invariant under the fractionally acting 3-parametric acting Möbius group (x stands for u,v)

$$U(a)j(x)U(a)^* = j(x+a),\ U(\lambda)j(x)U(\lambda)^* = \lambda j(\lambda x) \quad dilation \tag{14}$$
$$U(\alpha)j(x)U(\alpha)^* = \frac{1}{(-sin\pi\alpha + cos\pi\alpha)^2} j(\frac{cos\pi\alpha x + sin\alpha}{-sin\pi\alpha x + cos\pi\alpha})\ rotation$$

The next step consists in identifying the KMS property of the locally restricted vacuum with that of a global system in a thermodynamic limit state. For evident reasons it is referred to as the *inverse Unruh effect,* i.e. finding a localization-caused thermal system which corresponds (after adjusting parameters) to a heat bath thermal system. In the strong form of an isomorphism this is only possible under special circumstances which are met in the Einstein-Jordan conundrum, but not in the actual Unruh Gedankenexperiment for which the localization region is the Rindler wedge.

Theorem 1 *([24]) The global chiral operator algebra $\mathcal{A}(\mathbb{R})$ associated with the heat bath representation at temperature $\beta = 2\pi$ is isomorphic to the vacuum representation re-*

[27] The V are semiinfinite integrals over the pointlike $j's$, just as the stringlike vectorpotentials in QED are semi-infinite integrals over pointlike field strength [37].

stricted to the half-line chiral algebra such that

$$(\mathcal{A}(\mathbb{R}),\Omega_{2\pi}) \cong (\mathcal{A}(\mathbb{R}_+),\Omega_{vac}) \qquad (15)$$
$$(\mathcal{A}(\mathbb{R})',\Omega_{2\pi}) \cong (\mathcal{A}(\mathbb{R}_-),\Omega_{vac})$$

The isomorphism intertwines the translations of $\mathbb{R}$ *with the dilations of* $\mathbb{R}_+$*, such that the isomorphism extends to the local algebras:*

$$(\mathcal{A}((a,b)),\Omega_{2\pi}) \cong (\mathcal{A}((e^a,e^b)),\Omega_{vac}) \qquad (16)$$

This can be shown by modular theory. The proof extends prior work by Borchers and Yngvason [51]. . Let $\mathcal{A}$ denote the C^* algebra associated to the chiral current j[28]. Consider a thermal state ω at the (for convenience) Hawking temperature 2π associated with the translation on the line. Let $\mathcal{M}$ be the operator algebra obtained by the GNS representation and $\Omega_{2\pi}$ the state vector associated to ω. We denote by N the half-space algebra of $\mathcal{M}$ and by $N'\cap\mathcal{M}$ the relative commutant of $\mathcal{N}$ *in* $\mathcal{M}$. The main point is now that one can show that the modular groups $\mathcal{M},\ \mathcal{N}$ and $\mathcal{N}'\cap\mathcal{M}$ } generate a "hidden" positive energy representation of the Möbius group $SL(2,R)/Z_2$ where hidden means that the actions have no geometric interpretation on the thermal net. The positive energy representation acts on a hidden vacuum representation for which the thermal state is now the vacuum state Ω.The relation of the previous 3 thermal algebras to their vacuum counterpart is as follows:

$$\mathcal{N}=\mathcal{A}(1,\infty),\ \mathcal{N}'\cap\mathcal{M}=\mathcal{A}(0,1),\ \mathcal{M}=\mathcal{A}(0,\infty) \qquad (17)$$
$$\mathcal{M}'=\mathcal{A}(-\infty,0),\ \mathcal{A}(-\infty,\infty)=\mathcal{M}\vee\mathcal{M}'$$
$$\mathcal{M}(a,b)=\mathcal{A}(e^{2\pi a},e^{2\pi b}) \qquad (18)$$

Here $\mathcal{M}'$ is the "thermal shadow world" which is hidden in the standard Gibbs state formalism but makes its explicit appearance in the so called *thermo-field* setting i.e. the result of the GNS description in which Gibbs states described by density matrices or the KMS stated resulting from their thermodynamic limits are described in a vector formalism. The last line expresses that the interval algebras are exponentially related.

In the theorem we used the more explicit notation

$$\mathcal{M}(a,b)=(\mathcal{A}(a,b),\Omega_{th})=(\mathcal{A}(e^{2\pi a},e^{2\pi b}),\Omega_{vac})$$

Moreover we see, that there is a natural space-time structure also on the shadow world i.e. on the thermal commutant to the quasilocal algebra on which this hidden symmetry naturally acts. Expressing this observation a more vernacular way: the thermal shadow world is converted into virgin living space[29]. In conclusion, we have encountered a rich hidden symmetry lying behind the tip of an iceberg, of which the tip was first seen by Borchers and Yngvason.

[28] One can either obtain the bounded operator algebras from the spectral decomposition of the smeared free fields $j(f)$ or from a Weyl algebra construction.

[29] In [7] it is shown how to extract the shadow world description from the density matrix (Gibbs states) formalism with the help of the canonical GNS construction.

Although we have assumed the temperature to have the Hawking value $\beta = 2\pi$, the reader convinces himself that the derivation may easily be generalized to arbitrary positive β as in the Borchers-Yngvason work. A more detailed exposition of these arguments is contained in a paper *Looking beyond the Thermal Horizon: Hidden Symmetries in Chiral Models* [24].

In this way an interval of length L (one-dimensional box) passes to the size of the split distance ε which plays the role of Heisenberg's vacuum polarization cloud $\varepsilon \sim e^{-l}$. Equating the thermodynamic $l \to \infty$ with the the limit of a fuzzy localization converging against a sharp localization on the vacuum side in $(e^{-2\pi l},\ e^{2\pi l})$ for $l \to \infty$ with the fuzziness $e^{-2\pi l} \equiv \varepsilon \to 0,$ the thermodynamic limit of the thermal entropy passes to that of the localization entropy in the limit of vanishing ε

$$entr\ |_{kT=2\pi} \simeq -\ln \varepsilon \tag{19}$$

where the left hand side is proportional to the (dimensionless) heat bath entropy and the right hand side is proportional to the localization entropy.

Although it is unlikely that a localization-caused thermal system is isomorphic to a heat bath thermal situation in higher dimensions (the *strong inverse Unruh effect*), there may exist a "weak" inverse Unruh situation in which the volume factor corresponds to a logarithmically modified dimensionless area law i.e. $(\frac{R}{\Delta R})^{n-2} ln(\frac{R}{\Delta R})$ where R is the radius of a double cone , $\frac{\Delta R}{R}$ its dimensionless fuzzy surface and the box with two transverse- and one lightlike- directions is the counterpart of the spatial box so that the volume factor V corresponds to a box where one direction is lightlike. This would be different by a logarithmic factor from the area law which is suggested by the analogy to the behavior of vacuum polarization of a partial charge in the sharp localization limit (see previous section) and which also appears in the Bekenstein's work and in 't Hooft's proposal to make the derivation of the Hawking radiation consistent with Bekenstein's area law with the help of a brickwall picture [32]. The present state of computational control of the split property is not able to decide between these two possibilities for n>2.

The above isomorphism shows that Jordan's situation of quantum fluctuations, i.e. fluctuations in a small subinterval of a chiral QFT restricted to a halfline, is isomorphic to Einstein's Gedankenexperiment of thermal fluctuations in a heat bath thermodynamic limit state on a line restricted to an interval. Such a tight relation, also referred to a an *inverse Unruh effect* [24], can not be expected in higher spacetime dimension. Although the thermal aspect of a restricted vacuum in QFT is a structural consequence of causal localization, the general identification of the dimensionless modular temperature with an actual temperature of a heat bath system, or, which is equivalent, the modular "time" with the physical time is not correct; the modular Hamiltonian is does not describe the inertial time for which the local temperature defined in terms of the zeroth thermodynamic law agrees with the "Carnot temperature" of the second law [48].

The mean square energy fluctuation in a subinterval requires to compute the fluctuations of integrals over the energy density $T(u)$ and compare them to the calculation in a thermal heat bath calculation (the Einstein side). This would go beyond our modest aim of showing that both systems are structurally (independently of the chiral model) identical.

Properties of states in QFT depend on the nature of the algebra: a monad does not have pure states nor density matrices, but only admits rather singular impure states as singular (non Gibbs) KMS states. The identification of states with vectors in a Hilbert space up to phase factors becomes highly ambiguous and physically impractical outside of QM. The state in form of a linear expectation functional on an algebra and the unique vector (always modulo a phase factor) obtained by the intrinsic GNS construction [7] leads to a vector representation, but this depends on the particular state used for the GNS construction. In QM the algebras are always of the B(H) type where this distinction between vector states and state vectors is not necessary.

5 Particle crossing, on-shell constructions from modular setting

An important insight into "particles & fields" comes from a derivation of the *crossing property* of particle physics from the modular properties of wedge-localization. The formfactor crossing states that the n-particle to vacuum matrixelement of a local operator B is analytically related to to the connected part of the formfactors of B between k incoming and n-k outgoing particles in terms of the following identity

$$\begin{aligned} &\langle 0\,|B|\,p_1,..p_n\rangle^{in} = {}^{out}\langle -\bar{p}_{k+1}..,-\bar{p}_n\,|B|\,p_1,..p_k\rangle^{in}_{con} \\ &B\in\mathcal{A}(\mathcal{O}),\ \mathcal{O}\subseteq W,\ \bar{p} = antiparticle\ of\ p \end{aligned} \tag{20}$$

Here the momenta $-\bar{p}$ on the backward mass-shell refer to the anti-particles of the n-k crossed particles of the original n-particle state where the transition to the negative momenta involves an analytic continuation within the complex mass-shell. The analyticity following from principle of modular wedge-localization is however not in the Mandelstam invariants associated to the momenta, but rather in the rapidity θ variables. It turns out that the better known crossing property of the S-matrix do not have to be considered separately, they can be related to those of formfactors. Although the crossing appears trivial from a formal Feynman graph point of view, the nontrivial aspect is the mass-shell restriction of the analytic continuation. The LSZ reduction formalism relates the crossing property of the S-matrix (crossing of pairs of particle) to that of formfactors.

The physical content of formfactor crossing is that the different k to n-k formfactors are related to one master formfactor which may be taken to be the n-particle to vacuum formfactor. The only known non-perturbative general derivation of formfactor crossing uses modular theory[30], to be more precise the modular theory of a wedge-localized subalgebra [26]. Before a derivation will be sketched, some remarks about its conceptual relation to other better known consequences of modular localization theory may be helpful. Its conceptual proximity to the Unruh [21] effect through the shared wedge localization is somewhat unexpected. Whereas the latter together with the Einstein-Jordan subvolume fluctuations will probably remain a "Gedankenexperiment" about consequences of vacuum entanglement, the particle crossing is observational accessible [22] and constitutes an

[30]For a special case (elastic scattering) Bros, Epstein and Glaser [23] derived crossing of the S-matrix within the rather involved setting of functions of several analytic variables.

important concept of high energy particle physics. This changes the conceptual setting of crossing from that attributed to it in the dual model and ST, a topic which will be taken up in section 7.

The modern conceptual understanding came from the recognition that models of QFT with mass-gaps, which are known to relate quantum fields with the scattering theory of particles, also possess wedge-localized subalgebras with special properties [6][26]. In such theories the S-matrix turns out to be a *relative invariant* between the interacting wedge algebra $\mathcal{A}(W)$ and it interaction-free incoming counterpart $\mathcal{A}_{in}(W)$. Namely the two modular reflections are related through [6]

$$J = J_{in}S_{scat} \tag{21}$$

a relation which can be derived from Jost's formula [42] for the action of the TCP operator on the incoming free fields. Another idea from modular wedge-localization which is used in the derivation of formfactor crossing is *emulation* of interacting wedge-localized states (state vectors obtained by applying smeared fields $B(f)$ with $suppf \subset W$ to the vacuum Ω) in terms of free wedge-localized states obtained by applying operators $A_{in}(f)$ to the vacuum [26] [10]. Emulation involves different algebras acting in the same Hilbert space and sharing the same $\mathcal{P}$-representation.

To get some technicalities out of the way, let us first formulate the *free field KMS* relation in the way we need it for later purpose. With B a W-smeared composite of a free field, and the modular KMS relation for wedge-localized free fields reads

$$\begin{aligned}
&\left\langle BA^{(1)}A^{(2)}\right\rangle = \left\langle A^{(2)}\Delta BA^{(1)}\right\rangle,\ \Delta^{it} = U(\Lambda(-2\pi t)) \\
&A^{(1)} =: A(f_1)..A(f_k):,\quad A^{(2)} =: A(f_{k+1})..A(f_n): \\
&\Delta A^{(2)*}\left|0\right\rangle = \Delta SA^{(2)}\left|0\right\rangle = \Delta^{1/2}JA^{(2)}\left|0\right\rangle
\end{aligned} \tag{22}$$

A smeared free field can be written in terms of creation/annihilation operators integrated with wavefunctions which are the mass-shell restriction of the Fourier transforms of W-supported test functions (for economy of notation f will also be used for the Fourier transform)

$$\begin{aligned}
A(f) &= \int (f(p)a^*(p) + \bar{f}_a(p)b(p))\frac{d^3p}{2p_0},\ p \in H_m \\
A(f)^* &= \int (f_a(p)b^*(p) + \bar{f}(p)a(p))\frac{d^3p}{2p_0}
\end{aligned} \tag{23}$$

where f_a is the wavefunction of the b-antiparticle. We take the wedge W in the 0-3 directions, so that it is left invariant by $\Lambda_{0\text{-}3}$ Lorentz boosts, and parametrize the mass-shell momenta in terms of W-affiliated rapidities. It is well-known that the Fourier transforms of W-supported testfunctions lead to wavefunctions $f(p)$ which are boundary values of functions holomorphic functions $f(p(z))$ holomorphic in the rapidity strip in such a way that the analytic continuation of the particle wave function to the other side of the strip is equal to the complex conjugate of the antiparticle wavefunction.

$$\begin{aligned}
p(z) &= (mshz, mchz; p_\perp),\quad 0 < \operatorname{Im} z < \pi \\
f(p(\theta + i\pi)) &= f_a(p(\theta))
\end{aligned}$$

Rewriting the KMS relation (22) in terms of particle states we obtain

$$\int .. \int \langle B|p_1, ..p_n\rangle \frac{d^3p_1}{2p_{0,1}}..\frac{d^3p_n}{2p_{0,n}} + contr. = \tag{24}$$
$$\int .. \int (\Delta^{1/2} J\,|\bar{p}_{k+1}..\bar{p}_n\rangle\,, B\,|p_1, ..p_k\rangle) f(p_1)..f(p_n)\frac{d^3p_1}{2p_{0,1}}..\frac{d^3p_n}{2p_{0,n}} + contr.$$
$$\Delta\ (A^{(2)})^*\,|0\rangle = \Delta S A^{(2)}\,|0\rangle = \Delta^{1/2} J A^{(2)}\,|0\rangle \tag{25}$$

where round bracket denotes the scalar product between the bra and ket vectors and *contr.* stands for the contraction terms between two Wick-products. They contain a lower number of particles and hence do not contribute to the n-particle terms and therefore can be omitted. The third line in (22) is used inside the inner product in order to rewrite the right hand side of the KMS relation as a matrix element of B between particle states.

To pass to the crossing relation (20) we must show that one can omit the integration with the dense set of strip-analytic wavefunction. Since matrix element between momentum space eigenstates are generally distributions, this is not possible without knowing that the formfactors are locally square integrable; in this case the relation on a dense set of wave functions implies its validity on all locally L^2 integrable functions and hence (20) follows. For formfactors of composite of free fields this is trivial.

In the presence of interactions the extraction of the particle crossing from the KMS relation is more demanding. Particles are related to (incoming/outgoing) free fields whereas the fields in the KMS relation are interacting. The crossing relation (20) which we want to derive contains in and outgoing particles which are associated with in/out free fields. We need to know a relation between incoming and interacting wedge localized states. Using the notation: Recalling that both algebras $\mathcal{A}(W)$ and $\mathcal{A}_{in}(W)$ share the same representation of the Poincaré group, one obtains from the equality of the W-preserving Lorentz boosts the equality of the domains of their Tomita operators $domS_{\mathcal{A}(W)} = domS_{\mathcal{A}_{in}(W)}$. This means that for a vector state created by applying a wedge-local operator from $\mathcal{A}_{in}(W)$ to the vacuum there will be a corresponding uniquely defined operator in $\mathcal{A}(W)$ operator which, applied to the vacuum creates the same vector. Existence and uniqueness is secured by modular theory applied to the wedge region [53]. We refer to this bijection between wedge local operators as: *emulation of wedge localized free fields within the interacting wedge algebra [10][26]* and denote the emulated image by a subscript $\mathcal{A}(W)$

$$:A_{in}(f_1)...A_{in}(f_k): \longrightarrow (:A_{in}(f_1)...A_{in}(f_k):)_{\mathcal{A}(W)},\ suppf \subset W,\ A(f_i) \in \mathcal{A}_{in}(W) \tag{26}$$
$$:A_{in}(f_1)..A_{in}(f_k):|0\rangle = (:A_{in}(f_1)..A_{in}(f_k):)_{\mathcal{A}(W)}\,|0\rangle = |f_1, ..f_k\rangle_{in}$$

where, as before, the f inside the bracket state vectors are the wave functions associated with the W-supported testfunctions.

The KMS relation for interacting fields, from which the particle crossing is to be derived, reads now [54]

$$\left\langle B(A_{in}^{(1)})_{\mathcal{A}(W)}(A_{in}^{(2)})_{\mathcal{A}(W)}\right\rangle = \left\langle (A_{in}^2)_{\mathcal{A}(W)}\Delta B(A_{in}^1)_{\mathcal{A}(W)}\right\rangle \tag{27}$$
$$\Delta(A_{in}^{(2)})^*_{\mathcal{A}(W)}\,|0\rangle = \Delta^{\frac{1}{2}} J A_{out}^{(2)}\,|0\rangle\,,\ J = S_{scat}J_{in}$$

The identification of the right hand side with a (analytically continued) particle formfactors is similar to the free case; the difference is the presence of the scattering matrix which converts an incoming bra-state into an outgoing state

$$\left\langle B|A_{in}^{(1)}(p_1,..p_k)_{\mathcal{A}(W)}|p_{k+1},..p_n\right\rangle^{in} \simeq {}^{out}\left\langle -\bar{p}_{k+1},..-\bar{p}_n\left|B\right|p_1,..p_k\right\rangle^{in} \tag{28}$$

The equivalence sign expresses the fact that the equality according to (27) only holds after integration with wavefunctions from a dense set of W-localized wave functions, and the Ψ stands for a state obtained by applying an emulated k-particle operator to an n-k incoming state. It depends on n on-shell particle momenta but is not an incoming n-particle state (+ contributions from contractions)[31]; the product of emulations of free field states is not the emulation of the product of the latter. In order to relate the action of an k emulat on a n-k particle state one needs an additional idea.

There exists a concept which achieves this: *the analytic on-shell order change.* It arose in the setting of integrable models [56] and consists in an analytic interchange of particle momenta within formfactors which, in the presence of interactions, is different from the kinematical interchange in terms of statistics. For simplicity of notation we restrict to d=1+1 in which case on-shell formfactors are fully described by rapidities θ. We define a new object (denoted by a superscript an) in a special configuration

$$\langle B|\theta_1..\theta_n\rangle^{an} \equiv \langle B|\theta_1,..,\theta_n\rangle_{in} \;\; for\ \theta_1 > .... > \theta_n \tag{29}$$

Using (bosonic) particle statistics, formfactors can always be written in this naturally ordered form. An analytic ordering change along a certain path leads from the natural order to a different formfactor function which depends not only on the new order but also on the path of the analytic continuation. The resulting object is still on-shell, but one generally does not know its representation in terms of particle states.

Fortunately for the derivation of the momentum space crossing one does not have to know the particle content after the analytic changes. If the formfactors are locally square integrable one can, by using wave functions with ordered θ-supports, always "filter out" the natural order. This is achieved by passing from wedge-local wave functions which are spread (27) over all θ to wave functions supported in naturally ordered θ-intervals. In other words the on-shell analytic ordering property permits to reduce the derivation of the crossing property in the presence of interactions to that of the interaction-free case; the presence of interactions would only show up in the contributions from different orders. Before we attempt to algebraize the analytic ordering idea it is helpful to take a look at the simpler case of integrable models.

Integrable models permit an explicit illustration of the previous arguments, including an operator-encoding of analytic ordering changes into a representation of the permutation group (with the analytic transpositions being defined in terms of the 2-particle elastic scattering matrix). In fact the emulated free fields[32] turn out to be identical to the Fourier

[31] A outgoing free creation operator applied on a n-1 incoming state is not an n-particle state. Similarly the action of emulated incoming fields on an incoming state is an infinite superposition of incoming particle states even though the emulated momenta are on-shell.

[32] In earlier publications the special case of an emulated incoming field was referred to as a vacuum polarization free generators (PFG) [53].

transforms of the Zamolodchikov operators which obey the Zamolodchikov-Faddeev algebra.

This simplicity has its mathematical origin in restrictive domain properties of emulats which characterize integrability [53]. Emulats in general QFT only inherit the invariance property of their domains under the wedge-preserving subgroup. The requirement that the domain is also invariant under translations turns out to be extremely restrictive [53]. In $d > 1+1$ the definition of integrability in terms of domain properties of PFG's forces the S-matrix to be trivial $S_{scat} = 1$, whereas in $d = 1+1$ it allows nontrivial S-matrices which are suitable combinatorial products of elastic 2-particle S-matrices which fulfill the bootstrap properties (matrix-valued scattering functions)[33]. In other words the *connected* higher particle contributions vanish, which is the standard definition of integrability in terms of S-matrices (the infinite number of conservation laws is a consequence). The elastic S-matrices are given in terms of (possibly matrix-valued) scattering functions which have to obey certain analytic properties in order to come from a field theory; these scattering functions permit a classification.

Using these scattering functions as structure functions in a Zamolodchikov-Faddeev algebra [57] one obtains the creation/annihilation components of wedge-localized temperate PFGs. At this point one realizes that the above abstract definition in terms of domain properties of PFGs coalesces with the standard definition of d=1+1 integrability. Such models are susceptible to solutions in closed form and are therefore called "integrable". Compared with the classical integrability which requires to find a complete set of "conservation laws in involution" (and where integrable systems exist in every dimension), integrability in QFT is limited to d=1=1 and appears simpler. Integrable models possess explicitly computable formfactors

The so-called bootstrap-formfactor construction program relates the scattering functions to explicitly computed formfactors [56]. The last step consists in showing that these formfactors really belong to an existing model of LQP. In order to achieve this on has to show the nontriviality of double cone localized intersections of wedge-local algebras. This is a very nontrivial step which has been accomplished with the use of modular nuclearity in the work of Lechner [38]. The same author also showed how (in the absence of boundstates) one can construct the wedge-algebra generating PFG's in terms of deformations of free fields [55].

This simplicity of integrable S- matrices (the absence of connected parts for n>2) keep integrable models in the proximity of interaction-free models. Hence it is not so surprising that their wedge-generators (the Zamolodchikov-Faddeev algebra generators) can be obtained by *deformations* of free fields instead of the more complicated emulation.

For the convenience of the reader and for later use we add some details on the algebraic structure of emulated free fields for integrable models.

[33] In d=1+1 the cluster factorization does not distinguish a nontrivial elestic scattering amplitude from $S_{scat} = 1$.

$$(A_{in}(f))_{\mathcal{A}(W)} = \int_C f(\theta)Z^*(\theta)d\theta, \ C = \partial strip, \ p = m(ch\theta, sh\theta) \tag{30}$$
$$strip = \{z \mid 0 < Imz < \pi\}, \quad Z(\theta) \equiv Z^*(\theta + i\pi)$$
$$Z^*(z_1)Z^*(z_2) = S(z_1 - z_2)Z^*(z_2)Z^*(z_1), \ z \in C$$

Since integrable models preserve the particle number in scattering processes, the n-fold application of the creation parts $Z^*(\theta)$ to the vacuum are n-particle states. Identifying the velocity-ordered particle state with the incoming states

$$Z^*(\theta_1)Z^*(\theta_2)..Z^*(\theta_n)\left|0\right\rangle = \left|\theta_1, \theta_2, ., \theta_n\right\rangle_{in}, \ \theta_1 > \theta_2 > .. > \theta_n \tag{31}$$
$$anal.\ transpos.\ \left\langle 0\left|B\right|\theta_2, \theta_1, ..\theta_n\right\rangle_{in} = S(\theta_1 - \theta_2)\left\langle 0\left|B\right|\theta_2, \theta_1, ..\theta_n\right\rangle_{in}$$

the old degenerate representation related to (bosonic) statistics has been "dumped" into the incoming configuration which frees the left hand side for another nontrivial representation in of the permutation group in which the transposition of two neighboring $\theta's$ involves the scattering function. This nontrivial representation takes care of the *analytic exchange* of $\theta's$ inside a formfactor (second line in (31)).

It follows from repeated application of (31) that the analytic change of a θ through a k-cluster of θ on its right hand side will be a *product* of of scattering functions which *in terms of the full k+1 S-matrix* corresponds to a *grazing shot S-matrix* defined as [58]

$$S_{g.s.}(\theta; \theta_1, ..\theta_k) = S^k(\theta_1, ..\theta_k)^{-1} S^{k+1}(\theta, \theta_1, ..\theta_k) \tag{32}$$

This grazing shot concept has been used to generalize the properties of integrable emulations of free fields to the generic situation [10][26] by converting the idea of analytic changes of ordering into an algebraic structure; in this sense it tries to generalize the structure of the Zamolodchikov-Faddeev algebra. This is according to my best knowledge the first *attempt* to find a model-independent constructive on-shell access into nonperturbative QFT; the importance of such a step outweighs the risk of failure on such a subtle project.

The first attempt of an on-shell construction of particle theory after the failure of the S-matrix bootstrap was that by Mandelstam. It ignored the subtlety of analytic on-shell properties by trying to guess their structure instead of understanding them as a result of the causal locality principles of QFT. It was derailed by the incorrect idea of identifying the meromorphic function of the dual model with that particle crossing in scattering amplitudes (more in section 7).

The idea is to relate the on-shell analytic order changes to the action of emulats. For two relatively naturally ordered clusters, the analytic ordering idea for the left hand side in (28) reads

$$\left\langle B|A_{in}^{(1)}(\theta_1, ..\theta_k)_{\mathcal{A}(W)}|\theta_{k+1}, ..\theta_n\right\rangle^{in} = \left\langle B|\theta_1, \theta_2, ..\theta_n\right\rangle^{in} + contr. \tag{33}$$
$$(\theta_1, ..\theta_k) > (\theta_{k+1}, ..\theta_n), \ order\ within\ each\ cluster\ is\ irrelevant \tag{34}$$

were the contractions result from the incoming Wick product $A_{in}^{(1)}(\theta_1, ..\theta_k)$ acting on the n-k particle state; they do not contribute if all θ are different. For other orderings the on-shell formfactor will contain an infinite number of particles. For the simplest case

$$_{in}\left\langle \chi_1, .\chi_m \right| Z^*(\theta)_{\mathcal{A}(W)}) \left|\theta_1; \theta_2.\theta_n\right\rangle_{in} = S_{gs}^{(m,n)}(\chi, \theta_1; \theta) \tag{35}$$

For more details about the possible constructions of PFG's outside integrability see .

That the ordering prescription is crucial for the derivation of the standard property (in which the interaction does not explicitly appear) is corroborated by the derivation of the time-dependent LSZ reduction formula from the foundational properties of QFT [58]. In that derivation overlapping wave functions have to be avoided because the overlap causes the change of threshold singularities. The previous idea to view the algebraic action of emulats of particle states as particular analytic ordering changes along particular (”minimal”) paths has not yet been tested, but fortunately the ”kinematical” form of the ordered crossing property does not depend on this conjecture.

The ideas about PFGs and of wedge-localized particle states in terms of emulated fields can (and in my opinion should) be viewed as an extension of Wigner's representation-theoretical approach for noninteracting particles and its functorial relation (”second” quantization) with quantum fields in the presence of interactions. The conceptual distance between the functorial particle-free field relation and emulation in the presence of interactions is immense. Modular localization, as a mathematical precise formulation of the causal locality principle of LQP, is intrinsic; it has no relation to Born's localization in QM obtained by calling a certain Hermitian operator a ”position operator” and using its spectral representation to define localized wave function and identify their absolute square as a spatial probability density of localization. The particle-field relation in QFT has also no connection to the particle-wave dualism of QM[34]. Most of the misunderstandings, including those resulting from certain Lagrangian manipulations (see next section) and those which led to ST, result from incomplete understanding of the intrinsic localization principle which separates LQP from QM.

6 Impact of modular localization on gauge theories

It is well-known that the *Hilbert space formulation* for renormalizable couplings of *pointlike* fields is limited to spin $s < 1$. For $s = 1$ vectorpotentials one is forced to use a Krein space formulation either in the form of Gupta-Bleuler or, in the massive case, in terms of the ghost fields of the well-known Becchi-Rouet-Stora-Tyutin (BRST) setting of operator gauge theory. In the following the BRST gauge setting will be sketched in a slightly different form from what the reader may be familiar with; in this way formal similarities and conceptual differences with the new stringlocal field setting (SLF) in Hilbert space will be clearer visible.

The description in the *massive* case starts from the observation that by adding an indefinite metric scalar Stückelberg ϕ^K free field (two-point function with the opposite sign) to the $d_{sd} = 2$ transverse Proca field A_μ^P with $\partial^\mu A_\mu^P = 0$, one compensates the

[34] This applies in particular to the E-J conundrum.

leading short distance singularity of the Proca field with the derivative of a lower short distance dimensional "Krein vectorpotential" $d_{sd} = 1$

$$A_\mu^K(x) \simeq A_\mu^P(x) + \partial_\mu \phi^K, \quad \curvearrowright \partial^\mu A_\mu^K(x) + m^2\phi^K \simeq 0 \tag{36}$$

The equivalence sign is meant to indicate that relations between the Krein vectorpotential and its physical Proca counterpart cannot be written in terms of equalities in Krein- or Hilbert- space; in order to turn this relation into an equality in Krein space and afterwards descend to Hilbert space one needs additional ghost variables and to a nilpotent s-operation which has similar cohomological properties as a differential acting on differential forms. The second relation in (36) passes to the Lorentz gauge relation in the $m = 0$ Gupta-Bleuler formalism and it is well-known that this is not yet an operator relation but becomes one on a suitably define Hilbert space whose vectors consist of equivalence classes. Whereas the second relation survives in this way, the first relation breaks down since there is simply no massless Proca field. The choice of the ϕ^K and hence A_μ^K depends on the "gauge description" which one wants to use; taking for the Stückelberg field the pointlike massive scalar field with the opposite sign of the two-point function, one obtains the a gauge which for $m \to 0$ becomes the Lorentz gauge.

The BRST formalism adds additional "ghost" operators which permit to replace equivalences by equalities and plays a pivotal role in relating Krein spaces to Hilbert spaces and in the construction of local observables; This formalism is synonymous with operator gauge theory, in the massless limit it is identical to what one obtains by quantizing classical gauge theory. Its weaknesses show up in the construction of interacting physical matter fields which couple to the massive vectormesons; this problem extends to self-interacting massive gluon which have no linear related field physical pointlike strengths. The well-known nonrenormalizability of pointlike massive gauge in *Hilbert space* indicates that such pointlike fields have no polynomial bounds and hence are more singular than tempered distributions (Wightman fields). The problem of physical (gauge invariant) matter fields is acerbated in the massless limit; in this case one knows from nonperturbative structural investigations that physical electron operators cannot be described by pointlike Hilbert space fields at all [73]. Another manifestations of the same problem is the absence of Maxwell charges in the pointlike description of QED (necessarily in Krein space, i.e. gauge dependent matter fields) [74].

Before passing to a new formalism which is based on the use of string-localized fields in Hilbert space it is helpful for later comparisons to present the BRST operator gauge formalism in some detail. The mentioned enlargement adds additional anticommuting scalar ghost and anti-ghost fields $u, \tilde{u}$ in terms of which one defines a nilpotent s-operation which is implemented by a "ghost charge" operator Q

$$\begin{aligned} sA_\mu &= \partial_\mu u, \; s\phi = u, \; s\tilde{u} = -(\partial A + m^2\phi) \\ sB &= [Q, B]_{\text{grad}}, \; Q \textit{ ghost charge}, \; Q^2 = 0 \end{aligned} \tag{37}$$

where the graded commutator is an anti-commutator if B contains an odd number of ghost fields $u, \tilde{u}$. The result is the aforementioned BRST gauge setting, in which the physical observables and the Hilbert space are defined as kernel modulo image of s. As shown[35] in [65] [66] [67] [68] this leads to *renormalizable gauge theories for massive*[36] *vectormesons* coupled to charge-carrying as well as to neutral matter fields.

Whereas the charged couplings follow to a large degree (apart from the presence of the ghost degrees of freedom) the rules of the classical gauge group formalism, the coupling to neutral matter has no classical counterpart and comes and shows some unexpected features. There are many more "gauge-induced" second order terms than in the charged case[37], which (in view of the charge neutrality of the coupled Hermitian field H) requires the presence of odd powers in H and therefore is not surprising. What may however be unexpected by those who accepted the Higgs mechanism (mass creation by symmetry breaking) is that these *BRST-induced terms have the form of a Mexican hat potential* (subsection 3). The numerical coefficients of the various even and odd powers are fixed in terms of the vectormeson coupling g and mass ratios of the two fields (the vectormeson and the H-field). This is not surprising in view of the fact that the neutral H interaction with an abelian massive vectormeson is a theory with three parameters. The broken symmetry interaction, which results from the symmetric quartic potential by shifting the field value by a constant, is also described in terms of three parameters: the vectormeson coupling g and the two parameters of a Mexican hat; since the model is massless there are no additional parameters.

But there is a world of difference on the physical side; the broken symmetry picture, for which the input is the Mexican hat potential, is a formal manipulation with Lagrangians whose consistency with principles of QFT is questionable[38]. On the other hand the Mexican hat potential as a direct result of counterterm "induction" from the second order BRST gauge "principle"; this is a certainly natural presentation of a coupling of a vectormeson to a neutral field. The word "principle" appears in quotation mark since the BRST formalism is strictly speaking not a quantum field theoretic principle but rather a prescription about how to extract from an unphysical Krein space setting.

It is the main point of this section to replace the gauge description by formulation in a Hilbert space which replaces pointlocal vectormesons by their stringlike counterpart; in this way the pointlike vectormeson, which is the culprit for the appearance of the Krein space setting and the "gauge principle" (the return to Hilbert space) is replaced by a localization property. The latter is not imposed on a model but rather results from leaving it up to the foundation causal localization principle of QFT to decide which is the tightest possible localization for a given interacting model which is consistent with a Hilbert space description. The pointlike localization is only the correct answer for $s < 1$ interactions.

Even without the knowledge of these mathematical facts it is difficult to understand why the strange philosophical consequences of the Higgs mechanism concerning the *ex-*

[35]There are of course many references to the BRST operator gauge theory, but this one is best suited for a comparision with SLF,

[36]The free field transformation rules (37) refer to the incoming free fields of scattering theory. In massless gauge theories as QED the ghost charges depend on the coupling [69].

[37]With induced interactions we denote counterterms with fixed numerical values (no free coupling parameters).

[38]The difficulties to maintain consistency with classical aspects of gauge theory was a frequent point of contention.

istence of a distinguished particle which creates the mass of vectormesons as well as its own mass (the God particle) found widespread acceptance, even though it is well-known that QFT that the only hierarchy consistent with the localization principles of QFT is that of the superselected charges which are carried by the particles (nuclear democracy within one superselection sector). Masses of interacting massive particles can of course be written in terms of other Lagrangian parameters but this does not mean that there is a physical interpretation behind such rewriting. The only known formulation of renormalized perturbation theory is based on particle masses of a given field zero order content and dimensionless coupling strengths; masses of particles related to composites of the defining field content ("bound states") are expected to be uniquely determined, but there is no mathematically controlled way to decide which composites have bound states.

A mass creation in form of a Higgs mechanism is not supported by QFT. The "massaging" of Lagrangians is simply not part of physics. In addition of offering no computational help in the perturbative calculation, the Higgs mechanism is misleading because it tries to explain a simpler situation of a QFT with mass gaps in terms of a much more subtle zero mass description in which the basic physical matter fields are stringlike. QFT is not a theory in which one can compute masses; claims to the contrary always disclose themselves as reparametrizations in which parameters which have a direct physical meaning have been traded by unphysical ones. The description of a coupling of a massive vectormeson to a massive Hermitian scalar field (a selfconjugate particle) in terms of a coupling strength g and the two parameters of a Mexican hat potential is a good illustration; it hides the fact that this potential is rather the result of implementing the BRST gauge principle in a pointlike setting in Krein space (or better: the consequence of implementing locality in a Hilbert space) for a mass m vectormeson to a mass m_H scalar Hermitian field. The confusion arises because the "induction of interactions" from gauge restrictions (or better from locality in Hilbert space) is a phenomenon which has no counterpart for pointlike interactions between fields of spin $s < 1$.

This critique is also supported by looking at the situation from the viewpoint of spontaneous symmetry breaking. To apply Goldstone's physical idea of a spontaneous symmetry breaking (a conserved current whose charge diverges as a result of its coupling to a long range Goldstone boson) to a pointlike *gauge dependent matter field* is certainly not supported by QFT since the prerequisites of the proof (namely the existence of a conserved current with diverging charge in a Hilbert space setting) are violated.

On the other hand it is conceivable that the implementation of the BRST gauge requirement for interacting massive vectormesons (or that of the locality principle in a Hilbert space setting) requires additional $s = 0$ degrees of freedom in addition to its "minimal field content". This is certainly *not the case for abelian interactions*, where it is well-known that the imposition of the BRST formalism does not require an extension beyond the minimal field content. But the author is not aware of a theorem which insures that this remains valid in case of massive Y-M interactions. The problem of consistency of massive Y-M interactions with a minimal field content can presently only be decided by explicit second order S-matrix calculation. In subsection 4 this question will be posed in the context of our new SLF Hilbert space setting. If selfinteracting $s \geq 1$ stringlocal potentials require the presence of lower spin (possibly pointlike) companions, this would certainly enrich our knowledge about QFT of higher spin objects. Presently the only known characteristic property of massive abelian vectormeson coupled to charged or neutral (Hermitian) matter is that the charge associated to the Maxwell current (the divergence of the field strength) vanishes; this is the *famous Schwinger-Swieca screening* to which we will return in subsection 3.

Despite the many new insights into higher spin interactions presented in this section it is worthwhile to emphasize again that these new results in no respect affect the principle of nuclear democracy among particles. The only hierarchy consistent with the causal localization principle of QFT is that of superselected charges: the basic charges are those which cannot be obtained by fusion of others, all others a fused charges. Within one superselected charge sector all states are coupled to each other, at least if they do not consist of localized pieces separated by a large spacelike distance. Nuclear democracy holds between particles which carry the same superselection charges. In particular there are no "God particles" which "fatten others" and receive their own mass by a Higgs mechanism.

Hermitian fields coupled to massive vectormeson have no counterpart in classical field theory which may explain why there presence has not been directly seen but rather entered QFT as that what remains behind the metaphoric haze of the Higgs mechanism. They constitute the chargeless version of massive scalar QED; as a result of absence of bilinear currents and the general absence of the even/odd selection rule of charged (complex) fields they have a richer structure of interaction terms in the form of a Mexican hat. But in contrast to the Higgs model which starts from a massless model with a Mexican hat, these terms are not put in but are rather the result of either implementing the BRST gauge formalism in Krein space or of the adaptation of locality in the presence of stringlocal vectormeson fields in a Hilbert space setting.

The BRST renormalization formalism has some obvious shortcomings. In order to maintain the renormalizability for pointlike fields one has to introduce ghosts whose structure is incompatible with that of a Hilbert space and work in an indefinite metric Krein space; in this way one avoids the structural clash between pointlike $m = 0,\ s \geq 1$ potentials and the Hilbert space. The easy part of the BRST formalism is the perturbative calculation of pointlike correlations, but this comes at the very high prize since the physical correlations, with the exception of the gauge invariant observables, remain outside computational reach. Everybody has already seen the formal representation of stringlike matter fields in terms of pointlike gauge dependent fields:

$$\varphi(x,e) = \varphi^K(x) expig \int_x^\infty A_\mu^K(x + \lambda e)e^\mu d\lambda,\ e^\mu e_\mu = -1 \tag{38}$$

which already appeared in publications of Jordan and Dirac during the 30s. But anybody who, besides playing formal games, tried to obtain a computational control of such composite stringlocal expressions knows that this is an impossible task.

The new SLF setting converts this problem from its head to its feet; instead of trying to represent physical charge-carrying fields in terms of pointlike fields, it bases renormalized perturbation theory direct on stringlocal fields. In this way one overcomes the clash between pointlike K-field with the Hilbert space structure [37][54]. Although for massive pointlike potentials (Proca potentials) a direct clash with the Hilbert space structure does not exist, the problem returns in a more hidden form through the lack of renormalizability of pointlike massive vectormeson interactions. Whereas the short distance dimension of stringlike vectorpotentials is $d = 1$, pointlike massive vectorpotentials have $d_{point} = 2$, too high for constructing renormalizable interactions. Hence in the massive case the localization aspect enters in a more discrete way through renormalizability.

A stringlocal massive vectorpotential $A_\mu(x,e)$ together with a Hermitian stringlocal scalar field $\phi(x,e)$ is defined in terms associated to a pointlike Proca field as follows

$$F_{\mu\nu}(x) := \partial_\mu A_\nu^P(x) - \partial_\nu A_\mu^P(x), \quad A_\mu(x,e) := \int_0^\infty F_{\mu\nu}(x+\lambda e)e^\mu d\lambda \tag{39}$$
$$\phi(x,e) := \int_0^\infty A_\mu^P(x+\lambda e)e^\mu d\lambda, \; e^2 = -1$$

All three covariant free fields are written in terms of the same basic Wigner $s = 1$ creation/annihilation operators $a^\#(p, s_3)$, $s_3 = -1, 0, 1;$ unlike in the BRST setting no additional Stückelberg degrees of freedom are introduced, so that the Hilbert space remains identical to that which the Proca field generates from the vacuum[39]. Unlike pointlocal scalar fields, *interacting massive stringlocal scalar fields can interpolate particles of any integer spin* [43], the usual relation between spinorial indices and physical spin for pointlike fields do not hold for massive stringlocal fields. The semiinfinite line integral in (39) lowers the dimension by one unit, so that the stringlocal potential and the stringlocal Stückelberg field permit to define formal interaction polynomials within the power-counting restriction. The string-localization shows up in the commutation relation; bosonic strings commute if and only if the entire strings $x + \mathbb{R}_+ e$ are spacelike relative to each other.

Between the Proca field and its stringlike relatives there exists a (easy verified) linear relation

$$A_\mu(x,e) = A_\mu^P(x) + \partial_\mu \phi(x,e), \quad d_{sd}(A_\mu) = 1, \; d_{sd}(\phi) = 1, \; d_{sd}(A_\mu^P) = 2 \tag{40}$$

In contrast to the equivalence relations (36) in Krein space, these relations are bona fide equations in Hilbert space which (in case of free fields) are direct consequences of the above definitions. The similarity to (36) suggest to call them (physical) Stückelberg fields. Their stringlike nature imparts them with a new property: different from fundamental scalar pointlike fields which lead to bound states through the formation of composites, massive stringlike scalar fields can directly interpolate particles of any integer spin [43]. Without interactions the Stückelberg field creates the $s = 1$ massive vectormeson (40). But in the presence of interaction with matter there is no selection rule which forbids physical Stückelberg fields to create e.g. a scalar bound state.

In contrast to the role of the scalar Higgs field, which must be added to the zero order field content, physical Stückelberg fields which are Hermitian and "string-scalar" are intrinsic objects of massive s=1 interactions since they are inexorable companions of renormalizable massive vectormesons. Together with the Proca field they disappear in the massless limit in which the relation (40) breaks down and only stringlocal vectorpotentials remain.

[39] This renders the SLF setting more similar to the Ginsberg-Landau phenomenological theory of superconductivity tnan the relation of the latter to the Higgs mechanism for which the "fattened" vectormeson need the presence of the Higgs particle.

Before presenting illustrative second order perturbative model calculations in the new SLF Hilbert space formulation, one needs to extend the local equivalence class relation between point- and string-local fields to the matter fields. Looking at the "gauge theoretic appearance"[40] of (40) it is not surprising that this relation takes the form of a gauge transformation

$$\psi(x) = \ e^{-ig\phi(x,e)}\psi(x,e) \tag{41}$$

The coupling-dependent exponential dependence on the physical Stückelberg field changes the renormalizable stringlocal matter field; the result is a very *singular pointlike field* with unbounded short distance dimensions (non-polynomial increase in momentum space). Such fields have been introduced in [61]; they are more singular[41] than operator-valued Schwartz distributions ("Wightman fields") and any attempt to calculate them directly (without using the relation to their stringlocal renormalizable siblings) will lead to a nonrenormalizable perturbation theory with infinitely many counterterm parameters.

In the new SLF Hilbert space setting the results obtained by imposing the BRST gauge formalism as the relation between couplings of different types of matter with the same vectorpotential (including the coupling between Y-M fields and their coupling to quarks) are now following from the causal localization setting in Hilbert space. More specifically they are structural properties following from the affiliation of the different stringlocal matter fields together with the vectorpotentials to one shared (string-extended) local field-class (Borchers-class). There is no other principle than quantum causal localization in Hilbert space, in particular no gauge-principle which separates $s<1$ interactions from from $s\geq 1$. What is is different are the consequences; in the latter case there are relations between s=1 couplings to different matter and "induced counterterms" (see later) which have no counterpart for $s<1$, i.e. the higher the spin the more dynamical restrictions.

The intrinsic nature of the stringlocal *physical* Stückelberg fields strengthens the analogy with the massive gauge fields in the Ginsberg-Landau theory of superconductivity. In contradistinction to the Higgs mechanism, which *adds additional degrees of freedom* (namely the extrinsic Higgs fields which are allegedly needed to create masses), the SLF setting describes massive vectorpotentials coupled to charged matter without adding degrees of freedom, just as the theory of superconductivity.

In fact the intrinsic Stückelberg field QFT shares many of the properties ascribed to a H-field. Instead of being a part of the zero order field content, it is an inseparable companion of any renormalizable interacting massive vectormeson, independent of the kind of matter to which it couples. Its bound states could account for the LHC result apart from the age old unsolved problem to relate bound particle states to the interacting field content.

It is interesting to note that the local equivalence class picture permits a generalization in which the linear relation between $s=1$ free fields is a special case a more general relation for integer spin $s>1$ fields

$$A_{\mu_1..\mu_n}(x,e) = A^P_{\mu_1..\mu_n}(x) + \partial_{\mu_1}\phi_{\mu_2..\mu_n} + \partial_{\mu_1}\partial_{\mu_2}\phi_{\mu_3..\mu_n} + ... + \partial_{\mu_1}...\partial_{\mu_{n_n}}\phi$$

[40] Beware that this is not a gauge transformations between fields of the same kind, but rather an equation which connects string-and point-like fields which are members of the same localization class.

[41] In fact they only allow smearing with a dense class of localized testfunctions.

The left hand side represents a stringlocal spin $s = n$ tensor potential associated to a pointlike tensor potential with the same spin. The $\phi' s$ $s = n - i,\ i = 1, .., n$ tensorial Stückelberg fields of dimension $d = n - i + 1$. Each ϕ "peels off" a unit of dimension so that at the end one is left with the desired spin s stringlocal $d = 1$ counterpart of the tensor analog of the Proca field. The main problem of using such generalizations is to identify those couplings which guaranty the existence of sufficiently many observables generated by pointlike Wightman fields (operator-valued Schwartz distributions). This may be important in attempts to generalize the idea of gauge theories in terms of SLF couplings involving massive $s > 1$ fields.

The two-point functions of the above $s = 1$ stringlocal fields are e-dependent and also include mixed functions. Writing

$$\langle \Phi_1(x,e)\Phi_2(x',e')\rangle = \frac{1}{(2\pi)^{3/2}} \int e^{-ip(x-x')} M_{\Phi_1,\Phi_2}(p;e,e') \frac{d^3p}{2p_0} \tag{42}$$

$$M_{A_\mu^P,A_\nu^P} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2},\ M_{\phi,\phi} = \frac{1}{m^2} - \frac{ee'}{(pe - i\varepsilon)(pe' + i\varepsilon)}$$

$$M_{A_\mu,A_\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{(pe - i\varepsilon)(pe' + i\varepsilon)} + \frac{p_\mu e_\nu}{pe - i\varepsilon} + \frac{p_\nu e'_\mu}{pe' + i\varepsilon}$$

Besides these three diagonal expectations there are also mixed e-dependent two-point functions of which only

$$M_{A_\mu,\phi} = -i\left(\frac{e'_\mu}{pe' + i\varepsilon} - \frac{p_\mu ee'}{(pe - i\varepsilon)(pe' + i\varepsilon)}\right) \tag{43}$$

will be needed later on. The ε-prescription defines the distributions as boundary values of analytic functions. The appearance of e-dependent time-ordered correlation complicates analytic perturbative calculations as compared to the BRST setting. But the extra work is unavoidable because it is the only possibility to construct correlation function involving zero mass matter fields since the latter only exist as stringlocal objects. Such constructions are unavoidable if one wants to show that confinement is a property of zero mass gluon-matter interactions.

One should also note that the simplicity of the pointlike BRST perturbation theory is somewhat deceiving; the difficult part is not the perturbation theory itself but rather the extraction of the physical results. Physical operators as the S-matrix inevitably contain unphysical fields and to compute their matrixelements between physical particle states is a nontrivial task since the physical space is not simply a subspace but rather results from a cohomological construction.

From a conceptual viewpoint theories with mass gaps are much simpler than models containing massless fields. Massless theories as QED and QCD lead to electrically charged infraparticles and confinement whereas their massive versions fit perfectly into the world of particles and fields as described by scattering theory. This was not always seen this way; the Higgs mechanism originated at a time when the dominant opinion was the converse; nowadays we know that this resulted from confusing the simplicity of calculations with pointlike gauge-dependent fields with the conceptual problems of extracting physical quantities. It was precisely this misunderstanding of simplicity of computational recipes with physical content which led to the Higgs mechanism.

6.1 SLF perturbation theory involving massive vectormesons

For the perturbative study of interactions of massive vectorpotentials with charged matter, one needs to establish the validity of relations as (4041) in every order of perturbation theory. The zero order matter fields are pointlike but, as a result of their interaction with the stringlocal vectorpotential, they become stringlike in higher orders . The important idea which permits to establish these relation in every order within the Stückelberg-Bogoliubov-Epstein-Glaser (SBEG) setting of renormalized perturbation theory will be referred to as "adiabatic equivalence" (AE) since it involves the adiabatic limit in which the spacetime dependent compact supported coupling $g(x)$ of the SBEG functional formalism approaches the time independent everywhere constant physical coupling strength g; this will be explained in the sequel.

Before we turn to some concrete model illustrations of perturbation theory in terms of stringlike fields, a historical remark about the origin of these ideas may be appropriate. It had been known for a long time that Wigner's infinite spin representations of the Poincaré group cannot be generated by pointlike wave functions [46]. Further progress had to await the concept of modular localization, which first appeared in the context of integrable models [6]. Of significant importance was the systematic application of modular localization to positive energy Wigner representations in [44]. In that paper it was shown that all such representations permit a causal localization in (arbitrary narrow) spacelike cones. Since the core of such a conic region is a semi-infinite spacelike string, it was suggestive that the only remaining computational problem was the construction of covariant fields $\Psi(x,e)$ which are causally localized on $x+\mathbb{R}_{+}e,\ e^2=-1$ [43]. This finally led to a solution of the age old problem concerning the field content of Wigner's "infinite spin" representation class.

It then turned out that the construction of stringlocal fields is also useful for the pointlike localizable representations since it resolves the *clash between pointlike localization and the Hilbert space positivity for zero mass $s\geq 1$ fields* which one encounters in passing from pointlike field strength to their associated potentials[42]. It turned out that the use of stringlike potentials also lowers the short distance dimension for massive fields; instead of $d_{sd}=s+1$ for pointlike spin s fields, one can always construct a free stringlike field with $d_{sd}=1$ for all s.

Although "modular localization" was important for the discovery of stringlocal fields and their role in the reformulation of gauge theory, the renormalization theory for stringlike fields can nowadays be carried out without direct use to modular localization. The latter remains present in the background; it furnishes the conceptual-mathematical fundament for the ongoing changes in QFT. It shows in particular, that the perturbative use of SLF in Hilbert space is more than a computational substitute of the BRST gauge formulation. The SLF setting is the only perturbative formulation in which the full field content complies with the physical principle of causal localization in a Hilbert space.

After having explained the philosophy behind SLF, we will now illustrate these ideas in three different models. As a preparatory step the reader is first reminded of the SBEG perturbation theory. Its central object is Bogoliubov's operator-S-functional which generates the *time-ordered functions* associated with the scalar interaction density $\mathcal{L}(x)$.

[42] A corresponding result holds for massless higher halfinteger spin fields.

The scattering matrix S_{scat} and the quantum fields are then defined in terms of the adiabatic limit of the following definitions

$$S(g\mathcal{L}) \equiv \sum_n \frac{i^n}{n!} T_n(\mathcal{L}, \ldots, \mathcal{L})(g, \ldots, g) =: Te^{i\int \mathcal{L}(x)g(x)},\ S_{scat} = \lim_{g(x)\to g} S(g\mathcal{L}) \tag{44}$$

$$\psi_g(f) := S(g\mathcal{L})^{-1} \sum_n \frac{i^n}{n!} T_{n+1}(\mathcal{L}, \ldots, \mathcal{L}, \psi)(g, \ldots, g, f),\ \psi(f) = \lim_{g(x)\to g} \psi_g(f)$$

Here $g(x) \to g$ is the adiabatic limit in which the spacetime dependent coupling approaches the coupling constant. A sufficient condition is the existence of mass-gaps, which is satisfied if all fields in the Lorentz-invariant interaction density are massive. Since quantum fields are not operator-valued functions but rather operator-valued distributions, the definitions of the S-matrix and quantum fields must be subjected to renormalization which has to be carried out order by order.

In the case of massive scalar QED [77][78] we have two $\mathcal{L}'s$, a pointlike interaction $\mathcal{L}^P$ and its stringlike counterpart $\mathcal{L}$

$$\begin{aligned} &\mathcal{L}^P(x) = j^\mu(x) A_\mu^P(x) \overset{(c\mathcal{L}ass1)}{=} \mathcal{L}(x,e) - \partial^\mu V_\mu \\ &\mathcal{L}(x,e) = j^\mu(x) A_\mu^S(x,e),\ V_\mu = j^\mu(x)\phi(x,e),\ j_\mu(x) =: \varphi^*(x) i \overleftrightarrow{\partial}_\mu \varphi(x) : \\ &S(g\mathcal{L}^P + f\psi) \simeq S(g\mathcal{L} + f\psi^S) \\ &A_\mu^P(x) = A_\mu^S(x,e) - \partial_\mu \phi(x,e),\ \psi^P(x) = e^{ig(x)\phi(x,e)}\psi^S(x,e) \end{aligned} \tag{45}$$

The $\mathcal{L}^P$ is the singular pointlike Proca interaction, whereas $\mathcal{L}$ is the new stringlike interaction which, as a result of $d_{sd}(A_\mu^S) = 1$, stays within the power-counting limit of renormalizable couplings; both $\mathcal{L}$ act in Hilbert of the free fields which were used in the definition of $\mathcal{L}^P$. The vector V_μ contains the Stückelberg field ϕ, and $\partial^\mu V_\mu$ with $d_{sd}(\partial^\mu V_\mu) = 5$ plays a similar role with respect to $\mathcal{L}^P$ as $\partial_\mu \phi$ in (40) with respect to A_μ^P, namely it "peels off" the highest short distance dimension from $\mathcal{L}^P$ and converts it into the renormalizable $d_{sd} = 4$ interaction density $\mathcal{L}$. The highest divergence is now carried by the derivative terms which, integrated with $g(x)$, becomes a boundary term and hence vanishes (in massive theories) in the adiabatic limit $g(x) \to g$. In this way one arrives at the equality (up to problems of normalization) of the first order pointlike scattering matrix with its string counterpart

$$\int \mathcal{L}^P d^4x = \int d^4x \mathcal{L} \ \ or\ \mathcal{L}^P \overset{AE}{\simeq} \mathcal{L} \tag{46}$$

Here and in the following two expressions are called *adiabatic equivalent* (AE) if they only differ by derivative terms (which, in massive theories, vanish after integration).

For notational conveniences, and also in order to maintain formal analogy to the BRST formalism, one views $A_\mu(x,e)$ and $\phi(x,e)$ as zero forms in e, with d_e denoting the differential operator which maps n-forms into n+1 forms so that $d_e^2 = 0$. Then the basic relation of string-independence (40) reads

$$\begin{aligned} &d_e(A_\mu(x,e) - \partial_\mu \phi(x,e)) = 0,\ u := d_e \phi \\ &\curvearrowright d_e(\mathcal{L}(x,e) - \partial_\mu V^\mu(x,e)) = 0 \end{aligned} \tag{47}$$

and the second line, in which the d_e acts on composites, is a consequence of the d_e action on the basic free fields. For all interactions of massive vectormesons with matter such pairs $\mathcal{L}$, V_μ exist. The content of the bracket in the second line is simply the lowest order nonrenormalizable pointlike interaction; for massive QED see (45).

The differential calculus is *formally* similar to the nilpotent s-operation of the cohomological BRST gauge formalism (see below). Their conceptual role remains however quite different; in the case at hand the differential formalism separates pointlocal observables from stringlocal fields in Hilbert space, whereas the main purpose of the BRST s-operation is to allow the return from an unphysical Krein space to a quantum theoretical Hilbert space in which the gauge invariant observables act. Operators as (38), which in the BRST terminology may be called "gauge invariant nonlocal matter fields", are outside the range of the perturbative gauge formalism, whereas in the SLF setting they define the basic renormalizable matter fields. In contrast to the nilpotent s-operation which is needed for the construction of a Hilbert space, the d_e acting on classical differential zero forms is directly related to the physical localization properties in Hilbert space.

If the T-products would not involve distributions with singularities at coinciding points as well at string crossings, higher order string independence relations as

$$(d_e + d_{e'})(T\mathcal{L}\ \mathcal{L}' - \partial_\mu T\ V^\mu \mathcal{L}^{'} - \partial_\nu' T\mathcal{L}\ V^{\nu\prime} + \partial_\mu \partial_v' T V^\mu V^{\nu\prime}) = 0 \tag{48}$$

would be an automatic consequence. This relation may be somewhat simplified by splitting it (using the symmetry in $x, e \leftrightarrow x', e'$) into:

$$d_e(T\mathcal{L}X' - \partial_\mu T V^\mu X') = 0, \quad X' = \mathcal{L}', V^{\mu\prime} \tag{49}$$

The ambiguities of time-ordering make the fulfillment of these relations a nontrivial renormalization problem. Their validity as distributional relations, including coalescent $x's$ and string crossings, would imply the string-independence of the second order scattering matrix, since all derivative terms lead to vanishing boundary terms in the AE limit.

The vanishing of the bracket in (48) also provides a second order definition of a T-product of singular "pointlike"[43] interactions $T\mathcal{L}^P(x)\mathcal{L}^P(x')$, which in the standard pointlike setting would be outside the range of renormalization theory.

$$T\mathcal{L}^P\mathcal{L}^{P\prime} \overset{AE}{\simeq} T\mathcal{L}\ \mathcal{L}', \;\; T\mathcal{L}^P\mathcal{L}^{P\prime} \equiv T\mathcal{L}\ \mathcal{L}' - \partial_\mu T\ V^\mu \mathcal{L}^{'} - \partial_\nu' T\mathcal{L}\ V^{\nu\prime} + \partial_\mu \partial_v' T V^\mu V^{\nu\prime} \tag{50}$$

The derivative terms, which in massive theories lead to vanishing surface contributions after integration over spacetime (see 46), account for the fact this e, e' independent definition of a second order pointlike interaction leads to the same scattering matrix as its stringlike counterpart. Renormalization means the construction of a time-ordering which fulfills e-independence in the sense of (50).

This is conveniently done in by decomposing the time-ordered products in terms of Wick-ordered products. Their resulting operator contributions are ordered according the number of contractions. The term with no contraction obviously fulfills the above identity. The so-called tree-contribution contains one contraction; for contractions containing

[43]The $TL^PL^{P\prime}$ is generally not pointlike as an interaction density, since there remain e-dependent contact terms which only vanish after integration (i.e. in the AE limit).

the time-ordering of derivative of fields this leads to a renormalization problem. The only massive vectormeson coupling in which this problem is absent is massive spinor QED [77]. Loop contributions are, as usual; absorbed in mass- and coupling- renormalization. In the following this problem and its solution will be sketched for three models: scalar massive QED, its chargeless counterpart (coupling to a Hermitian field H) and the massive Yang-Mills coupling (interacting massive gluons). In the following 3 subsection we will be content with the calculation of the second order S-matrix. The calculation of off-shell correlation of quantum fields and the relation between singular pointlike and renormalizable stringlike matter fields (41) will be left to a separate publication.

6.2 Scalar massive QED

The proof of e-independence (49) of the tree contribution in massive scalar QED (45) involves a renormalization problem for the two-point function

$$\langle T\partial_\mu\varphi^*\partial_\nu'\varphi'\rangle = \langle T_0\partial_\mu\varphi^*\partial_\nu'\varphi'\rangle + g_{\mu\nu}c\delta(x-x'), \quad \langle T_0\partial_\mu\varphi^*\partial_\nu'\varphi'\rangle \equiv \partial_\mu\partial_\nu'\langle T_0\varphi\varphi'\rangle \tag{51}$$

where the T_0 denotes the usual free field propagator without derivatives and c is a free renormalization parameter. As a result of the two derivatives, the two-pointfunctions on the left hand side involves fields of scaling dimension 2. The resulting scaling degree 4 of the 2-pointfunction leads to a delta function renormalization counterterm in $T\mathcal{L}\mathcal{L}'|_{1-contr}$ with undetermoned coefficient c. Our main interest is the fulfillment of the relations (49) in the tree approximation. It is clear that the use of the T_0 1-contraction contribution will not fulfill (49). In fact defining "anomalies" $\mathfrak{A}$ as

$$d_e\mathfrak{A} \equiv d_e(T_0\mathcal{L}\ \mathcal{L}' - \partial_\mu T_0 V^\mu\mathcal{L}^{'})_1, \quad d_e\mathfrak{A}_\nu \equiv d_e(\partial_\nu' T_0\mathcal{L}\ V^{\nu\prime} - \partial_\mu\partial_\upsilon' T_0 V^\mu V^{\nu\prime})_1 \tag{52}$$

(where the subscript 1 in the brackets refers to the 1-contraction component), it is easy to see that they consist of delta function contributions and their derivatives at coalescent $x = x'$ from the derivatives ∂_μ acting on contractions $T_0\langle\partial^\mu\varphi^*\varphi'\rangle$ in the $V^\mu\mathcal{L}'$ contribution

$$\begin{aligned} \mathfrak{A} &= \partial_\mu\partial^\mu\langle T_0\varphi^*\varphi'\rangle|_s\varphi\overleftrightarrow{\partial}'_\nu\varphi^{*\prime}\phi A^{\nu\prime} + h.c. = \\ &= -\delta(x-x')\varphi\partial_\nu'\varphi^{*\prime}\phi A^{\nu\prime} + \partial_\nu\delta(x-x')\varphi\varphi^{*\prime}\phi A^{\nu\prime} + h.c. = \\ &\overset{AE}{=} -\delta(x-x')\varphi\varphi^{*'}\partial_\upsilon\phi A^{\nu\prime} + h.c., \ d_e\mathfrak{A} \overset{AE}{=} -\delta(x-x')\varphi\varphi^{*'}A_\nu A^{\nu\prime} + h.c. \end{aligned} \tag{53}$$

Here the subscript s denotes the singular part of the wave operator applied to the time ordered propagator which is just a delta function. The equality in the sense of AE in the third line means that the integrals over x and x' are identical after omitting boundary terms in partial integration.

The anomaly of the total expression (50) is obviously symmetric under $x, e \longleftrightarrow x', e'$ interchange and it is easy to see the anomaly of this symmetric expression is obtained symmetrization of $\mathfrak{A}$

$$\mathfrak{A}_{sym} = -4\delta(x-x')\varphi\varphi^* A_\mu A'^\mu \tag{54}$$

where the dash on the A refers to the independent directional variable e'. This anomaly term, which violates the e-independence in (50), can be absorbed in a counterterm (51) of a contraction in T_0LL' by choosing $c = -4$ so that the second order net result for the tree contributions (1-contaction) can be written as

$$T\mathcal{L}^P\mathcal{L}^{P\prime}|_{1-c} = \mathcal{L}_2 + T_0\mathcal{L}\mathcal{L}'|_{1-c},\ \mathcal{L}_2 = \delta(x-x')4\varphi\varphi^*A_\mu A'^\mu \tag{55}$$

Taking into account the factor 1/2 in the second order contribution and combining it with the first order interaction, one finally obtaines for the local interaction contribution the (from gauge theory) expected form

$$\int \mathcal{L}_1 + \frac{1}{2}\int\int \mathcal{L}_2 = \int \varphi^* \overleftrightarrow{D}_\mu \varphi A'^\mu \tag{56}$$

The unsymmetric appearance on e, e' can be removed by integrating each e with a fixed compact supported test function $f(e)$ with $\int f = 1$, in which case the nth order has a nonlinear nth degree polynomial dependence on f. As a result of the e-independence , the f-dependence disappears in (50) and a fortiori in the S-matrix. The second order loop contribution leads (as in the pointlike case) to mass and wavefunction renormalization; this will be presented in more detail in forthcoming work.

The important message of this calculation is that *instead of imposing a gauge formalism*, whose formulation requires temporarily abandoning Hilbert space in terms of a Krein space and only recovering it incompletely (referring to the problems with *physical* matter fields) through a complicated ghost formalism, it is more physical to *maintain the Hilbert space of quantum theory and relax the formal pointlike localization requirement*[44]. The guiding idea is to let the theory itself decide about the tightest localization consistent with the Hilbert space positivity and renormalizability of interactions (i.e. the appearance of only a finite number of counterterms). In this way the first order massive scalar QED induces the second order string-dependent (quadratic in A) contact term which together with the remaining standard second order contribution results in a string-independent second order S-matrix. QED is obtained in the limit of massless vectormesons directly in terms of stringlocal physical matter fields. The gordic knot between pointlike localization and Hilbert space for interacting $s \geq 1$ fields has been cut in favor of stringlike localization in Hilbert space.

6.3 Couplings to Hermitian fields and the Higgs mechanism

Although having no counterpart in classical theory, one may ask how QFT models of *Hermitian scalar fields H* coupled to massive vectormeson (the charge-neutral counterpart of massive scalar QED) look like. Since a second order BRST operator gauge treatment which is suitable for a comparison with our SLF setting has been given by the University of Zürich group ([65] and references therein) and more recently in [68], it is appropriate to adapt their results to the present setting; this allows us to present the formal aspects of our SLF results [70] in terms of modifications on the BRST approach. The first order

[44] The pointlike fields of the quantization formalism clash with the Hilbert space structure of QT (which had no counterpart on the classical side).

pair $\mathcal{L}$, V_μ which corresponds to the lowest pointlike interaction with a Hermitian field H is[45] $(\phi_{Scharf} \sim m\phi)$

$$\mathcal{L}^P = m(A^P A^P H + cH^3) = \mathcal{L} - \partial_\mu V^\mu \ with: \tag{57}$$
$$\mathcal{L} = m(AAH + \frac{1}{2}A\phi \overleftrightarrow{\partial} H - \frac{m_H^2}{2}\phi^2 H + cH^3 + u\tilde{u}H)$$
$$V_\mu = m(A_\mu \phi H + \frac{1}{2}\phi^2 \overleftrightarrow{\partial_\mu} H)$$

where the superscripts K on $\mathcal{L}$ and V_μ have been omitted for notational convenience. The mass factor m (the vectormeson mass) has been introduced in order to keep track of the overall "engineering dimension" 4.

The appearance of a $\tilde{u}uH$ term (which are part of the ghost formalism and only vanishes on Ker$s$/Ims), which have no counterpart in the Hilbert space SLF setting and are simply absent there, the expressions are identical. Again one computes the anomalies of the one-contraction contributions (1-c) and compensates them with corresponding normalization terms by choosing the free normalization parameter in TLL' in such a way that they match the well-defined anomalies in the sense of AE (46) (50). They yield the induced counterterms C which together with the T_0-product define the renormalized T-product

$$s\mathfrak{A}^K = s(T_0\mathcal{L}\mathcal{L}'|_{1-c} - \partial^\mu T_0 V_\mu^K \mathcal{L}'|_{1-c} + (x \longleftrightarrow x')) \overset{AE}{=} s(C + C_\mu)$$
$$with\ T\mathcal{L}\mathcal{L}' = T_0\mathcal{L}\mathcal{L}'|_{1-c} + C, \quad TV_\mu\mathcal{L}'|_{1-c} = T_0 V_\mu \mathcal{L}'|_{1-c} + C_\mu$$
$$\curvearrowright s(T\mathcal{L}\mathcal{L}'|_{1-c} - TV_\mu L)|_{1-c} = 0$$

where the last relation results from absorbing the $C's$ (obtained from the calculation of the anomalies) into a redefinition of the As shown in [65] (page 147) this leads to 4 induced delta function normalization terms (counterterms)

$$\mathcal{L}_2 = AAH^2 + AA\phi^2 - \frac{1}{4}m^2 m_H^2 \phi^4 - \frac{1}{2}m_H^2 \phi^2 H^2 + c'H^4$$

Here the c' is an additional coupling which, although still free in second order, is needed for the compensation of anomalies in 3rd order which leads to the value $c' = -\frac{1}{4}\frac{m_H^2}{m^2}$. Again the sum of the local order terms $g\mathcal{L}_1 + \frac{1}{2}g^2\mathcal{L}_2$ is not physical by itself but together with the AE limit of $T_0\mathcal{L}(g)\mathcal{L}'(g)$ $g = const.$ represents the physical second order S-matrix $T\mathcal{L}(g)\mathcal{L}'(g)$ in the sense of BRST gauge invariance. As in (55) the form of the induced interaction $\mathcal{L}_2$ depends again on the definition of the T_0 with which the anomalies were computed. Independent of this choice, the the BRST induced second order Hermitian H field coupling to a massive vectormeson has the form of a Mexican hat potential. In contrast to the *Higgs mechanism* which assumes such a potential and as a result *runs into formal difficulties with maintaining the classical form of gauge invariance*, the induced Mexican hat potential is the result of the implementation of the BRST gauge formalism. Later on we have more to say on this important point.

[45] A term $A^P\partial H^2$ turns out to be a total derivative since $\partial A^P = 0$.

In the SLF setting the calculation proceeds in a similar fashion. But different from the case of scalar QED anomalies for which the wave operator only acted on pointlike propagators, its action on stringlike propagators contains besides pointlike delta functions also contributions from string-crossings. The same remark applies to the structure of counterterms in $T\mathcal{L}\mathcal{L}'$ in the 2nd order stringlocal interaction. Consider the counter-term renormalization of the scaling degree 4 propagator of the derivative of the Stückelberg field $\phi_\mu := \partial_\mu \phi$. Converting the Fourier representation of the stringlocal 2-point function of ϕ (42) formally into a convolution of an x-space propagator of a pointlike field with a stringlocal delta functions, we obtain

$$\langle T\phi_\mu(x,e)\phi_\nu(x',e')\rangle = \frac{1}{m^2}\langle T\phi_\mu(x)\phi_\nu(x')\rangle - \int \delta_{e,e'}(x-x'-y)\langle T\phi_\mu(y)\phi_v(0)\rangle\, d^4y \tag{58}$$

$$\delta_{e,e'}(\xi) = \int \frac{ee'e^{-ip\xi}}{(pe-i\varepsilon)(pe'+i\varepsilon)} d^4p = ee'\int_0^\infty ds \int_0^\infty ds' \delta(\xi + se - s'e')$$

Hence the counterterm renormalization can be reduced to that of pointlike fields convoluted with stringlocal delta functions. A similar argument holds for the anomalies

$$\partial^\mu \partial_\mu \langle T_0 \phi\phi'\rangle = f^{\phi\phi}(x,x';e,e') - m^2 \langle T_0\phi\phi'\rangle$$
$$f^{\phi\phi}(x,x';e,e') = \delta(x-x') + contr.\ from\ string\ crossings$$

In addition there are stringlike contributions from anomalies of mixed ϕ-A_μ propagators.Ignoring the stringlike delta contributions (which have to cancel among themselves)

As in the BRST calculation [65] there are 4 delta function contributions from the anomalies $d_e\mathfrak{A}$ which after partial integration (the AE property) can be brought into the form as in the BRST setting

$$A\cdot A\phi^2,\ A\cdot AH^2,\ \phi^4,\ \phi^2H^2 \tag{59}$$

As in [65] a 5th anomaly term $\sim H^4$ appears in the third order tree contribution which has to be compensated by the addition of a second order H^4 selfinteraction. The compensation requirement between the normalization terms and anomalies (which results from e-independence of the S-matrix) fixes the up to this point undetermined normalization parameters whose pointlike $\mathcal{L}_2$ contribution together with $T_0\mathcal{L}\mathcal{L}'$ define the renormalized e-independent second order S-matrix normalization e-independence matching of the yet undetermined parameters needed in order to The matching of these 4 anomaly terms with 4 corresponding normalization terms confirms the "Mexican hat induction mechanism" in [65]. The compensation of the stringlocal delta contributions together with a more details presentation of the SLF formalism will be left to a separate publication.

The central point of this section: *the relation of the coupling of massive vectormesons to neutral scalar matter with the the Higgs-mechanism.* A formal similarity between both follows from the fact that the induced 4th degree polynomial in H and the Stückelberg field ϕ has the form of the Mexican hat potential which is the starting point of the "spontaneous symmetry breaking" Higgs mechanism. But in the SLF setting of the neutral coupling model there is no such mechanism; it rather confirms the previously mentioned result of the BRST setting: the Mexican hat form of the potential is the results from "counterterm induction". The latter either follows from the imposition of the BRST gauge formalism in Krein space, or it is a direct consequence of causal localization in the Hilbert space (SLF setting).

This raises the question whether the Higgs mechanism is at all consistent with the principles of QFT. As mentioned at the beginning of this section, this problem led to conceptual stomachaches right from the time of the Higgs paper. How can a theory, as massless scalar QED with its quadrilinar counterterm in which the scalar charged field is unphysical (a carrier of nontrivial gauge transformations), be subjected to a symmetry breaking? A gauge symmetry is not a symmetry which can be broken, but rather a formal device which permits a return from a Krein space setting to a restricted description of local observables acting in a cohomologically defined Hilbert space. Or to ask this rhetorical question the other way around: how can a gauge dependent matter field in Krein space be the object which causes a Goldstone symmetry-breaking; after all Goldstone's mechanism as backed by a theorem [71] depends on pointlike locality in Hilbert space which is violated in both the BRST and the SLF setting. For anybody who has followed the historical discussions on this point it is quite ironic that it is precisely the apparent flaw of the Higgs mechanism with respect to gauge invariance which turns out to be the essential ingredient which leads to the induced Mexican hat form of the neutral vectormeson-matter coupling.

In order to avoid any misunderstanding of this point, the present critique is not against discoveries through metaphoric arguments; many discoveries, including Dirac's important idea of anti-particles were based on incorrect models or theories (the hole theory). This does not cause any harm as long as the metaphoric idea is recognized in time before it leads to conceptual confusions. That the situation concerning the Higgs mechanism went out of control has certainly nothing to with Higgs, who is an extremely modest scientist. It is the making of Big Science in its desperate search for finding a strong justification of ending a almost 40 year stagnation of the Standard Model (all the concepts and terminologies surrounding the Higgs phenomenon existed already more than 40 years ago). To justify the enormous effort which led to the LHC discovery of a neutral scalar particle, all precautions and breaks which normally surround experimental discoveries based on speculative theoretical proposals were dropped in this case. The selfjustification of Big Science stopped at nothing, not even at elevating a modest scientist with 8 publications to the level of Einstein, Heisenberg, Dirac and other great centennial discoverers.

One structural property which is not a metaphoric attribute but rather a valid intrinsic attribute of all couplings of massive vectormesons (independent of whether they are coupled to charge or neutral matter) is the fact that the Maxwell charge (the charge associated to the current obtains as the divergence of the field strength) is different from the global charge of the matter current. The Proca field of a free massive vectormeson is divergence free, and the charge corresponding to this "Maxwell current" $j_\mu^{Max} \sim m^2 A_\mu^P$ vanishes. The vanishing of the Maxwell current is characteristic of massive gauge theories. Schwinger was the first who conjectured this charge screening and Swieca presented a screening theorem [72][64]. In couplings to complex matter there exists also the conserved current which counts the (always unscreened) global charge of complex fields and exists also in the absence of vectormesons. In the massless limit both currents coalesce.

In the case of massive vectormesons coupled to Hermitian matter, the Maxwell current is the only conserved current and the theory is a free field theory in the massless limit.

The general situation with respect to conserved currents can be described in the following schematic way

$$\begin{aligned} &screening: \ Q = \int j_0(x) d^3x = 0, \ \partial^\mu j_\mu = 0 \\ &spont.symm. - breaking: \int j_0(x) d^3x = \infty \\ &symmetry: \int j_0(x) d^3x = finite \ \neq 0 \end{aligned} \tag{60}$$

This confirms again that the non-metaphoric part of the Higgs model has no intrinsic relation with spontaneous symmetry breaking contributions. There are only 3 types of renormalizable couplings of matter to massive vectormesons: couplings to charged fields, to Hermitian fields and among themselves, and the Higgs mechanism is an attempt to describe the Hermitian coupling by unsuitable means which led to the absurd idea that QFT is capable to describe the spontaneous creation of masses. In the next subsection the inverse direction will be explored, namely the construction of the conceptually much more subtle zero mass QED and Y-M models in terms of their much simpler (simple field-particle relation) massive counterparts. Here the vectormeson masses serve as covariant natural infrared "cutoffs". Massive models and their massless limit are different theories but the correlations of the latter may be obtained as massless limits of the former.

6.4 Selfinteracting massive gluons

For abelian massive gauge theories in the SLF Hilbert space formulation there are no structural reasons for enlarging the field content beyond the matter fields with which one wants to couple the massive vectormesons. This is less clear in case of selfinteracting massive gluons. Although the arguments against the consistency of the Higgs mechanism are generic (independent of the kind of vectormeson interactions), it is not clear that there could not be other consistency reasons why selfinteracting massive vectormesons cannot exist in the minimal form but may need the presence of independent chargeless scalar ($s = 0$) fields. It is well-known that the implementation of supersymmetry leads to spin multiplets, and although not expected in case of massive Y-M models, the author is not aware of a theorem which excludes such a possibility. In such a situation this problem can only be clarified by explicit low order perturbative calculations.

Such calculations have been carried out in the 90s by the University of Zürich group [65]; the result was that the consistent implementation of the BRST gauge property on massive selfinteracting gluons does indeed require the presence of Hermitian $s = 0$ fields. The authors emphasized that the appearance of additional scalar fields as a result of the BRST consistency has no relation with the spontaneous symmetry breaking mass creation of the metaphoric Higgs mechanism. This interesting situation has been a strong motivation for us[46] to apply our SLF Hilbert space setting to this problem. The appearance

[46] In collaboration of J. Mund with the author.

of additional contributions from string-crossings which have no counterpart in the BRST setting may lead to a compensation of amomalies without the introduction of additional compensating degrees of freedom; at the time of writing these calculations had not been completed.

Although none of the two possible outcomes has the importance for the credibility of the Standard Model which has been incorrectly attributed to the Higgs mechanism, the impact of either result on the future development of QFT will be considerable. In case of consistency without additional degrees of freedom this would show a limitation of Krein space methods as compared to the more physical setting of locality in Hilbert space. On the other hand the presence of s=0 fields for the consistency of selfinteracting massive $s = 1$ fields could be an important step towards a still little known perturbation theory of higher spin fields.

In the remainder of this subsection we will present some details of the new SLF Hilbert space setting for selfinteracting massive gluons. The starting point is the lowering of the dimension of the first order pointlike interaction by peeling off a derivative contribution which for the $O(3)$ Y-M model reads

$$\mathcal{L}^P = \sum \varepsilon_{abc} F_a^{\mu\nu} A_{b,\mu}^P A_{c,\nu}^P = \mathcal{L} - \partial^\mu V_\mu, \ or \ d_\varepsilon(\mathcal{L} - \partial^\mu V_\mu) = 0 \tag{61}$$

$$L = \sum \varepsilon_{abc} \left\{ F_a^{\mu\nu} A_{b,\mu} A_{c,\nu} + m^2 A_{a,\mu}^P A_b^\mu \phi^c \right\}, \ V_\mu = \sum \varepsilon_{abc} F_a^{\mu\nu} (A_{b,\nu} + A_{b,v}^P) \phi^c \tag{62}$$

The second order e-independence of the S-matrix follows from

$$d_e(\partial^\mu T V_\mu \mathcal{L}' - T\mathcal{L}\mathcal{L}') = 0 \tag{63}$$

Defining the T_0 ordering as before and considering only the pointlike delta function contribution to the normalization terms in $T_0\mathcal{L}\mathcal{L}'$ and anomaly contributions in $\partial^\mu T_0 V_\mu \mathcal{L}'$

field content why additional Hermitian fields besides the always present physical Before we return to the problem of whether massive Y-M need an additional coupling to neutral scalar fields in order to represent a renormalizable stringlocal theory with the classically expected pointlocal composites, some comments about the physical range of the SLF setting as compared to the BRST gauge formalism may be helpful. The main difference is in the understanding of the infrared aspects of physical fields. The BRST gauge invariant physical subalgebra is too small for the study of such problems. Already in massive QED the construction of pointlike physical matter fields can, if it is possible at all (presently there exist no such constructions), only be done within a setting which allows the use of more singular fields than those which lead to polynomial bounds in momentum space. This is because the formal connection between the unphysical pointlike fields to their imagined pointlike physical counterparts involves multiplications with exponential Stückelberg fields [79]. They cannot be Wightman fields (operator-valued tempered distributions) but could be singular field of the kind as considered by Jaffe [61][62]. The only Wightman fields are the stringlocal matter fields of the SLF setting. They are also the only fields which can survive in the $m \to 0$ infrared limit. It is precisely in this limit that Maxwell charges loose their screening aspect and become equal to the global charge.

The noncompact localization of Maxwell charges in abelian gauge theirs whose tightest generators are stringlocal matter fields has a long history. Structural (nonperturbative) arguments based on the quantum Gauss law have been known since the 80s [73]; already in the 70s arguments based on the use of the indefinite metric pointlike formulation of QED showed that this formalism leads to a vanishing Maxwell charge [74]. In fact the old perspective on gauge theory which consisted in believing that QED has been understood and it is the physics of massive vectormesons which should be analyzed in the light of its massless limit is misleading. Interactions of massive vectormesons fit into the standard particle framework and it is their infrared limit for vanishing vectormeson mass which lead to the unsolved foundational problem of gluon/quark confinement and the insufficiently understood spacetime aspects of infraparticles in QED and their spacetime descriptions in the scattering of charged particles. What may have contributed to this historical misunderstanding is the fact that certain technical renormalization aspects involving unphysical matter fields may appear simpler in the massless case. It is precisely this view which confuses computational rules with conceptual consistency within the setting of QFT which led to the Higgs mechanism as a surrogate for the coupling of neutral scalar particles. It also shows an interesting mechanism which was already effective in Dirac's anti-particle argument: the conceptual-mathematical nature asserts itself even against incorrect arguments which at the end of the day lead to correct results. Of course the analogy ends here since the question of whether the renormalization theory for massive gluons needs the presence of an additional real scalar field (in addition to the intrinsic stringlocal real scalar Stückelberg field) is still unsettled. It is clear that this is an important problem of particle theory which must be settled without reference to the LHC experimental findings. Without the belief that the Higgs mechanism is the only way to obtain gluon masses the tremendous experimental effort over many decades to find that particle would not have been undertaken, but to find a compelling reason within the foundational setting of QFT is a separate issue. If the presence of such a neutral scalar is necessary the Hilbert space setting of SLF should reveal the precise reasons; in a Hilbert space setting it is not easy to think of a mechanism which softens the high energy massive gluon interaction by adding terms while maintaining the original nonabelian interaction[47].

6.5 Zero mass limits and a perturbative scenario for confinement

The potentially most important consequence of the Hilbert space SLF formulation is the promise of a profound insight into hitherto incompletely or not understood infrared phenomena as "infraparticles" and confinement. Concerning the latter, the remarks on finds in the literature do not go beyond the statement that the perturbative expressions for the massless gauge-variant correlations of gluon- or quark- fields are infrared divergent and that this indicates the breakdown of perturbation theory. Short of a formula for a physical (according to (38) necessarily stringlocal) such a statement is void of physical meaning. The infraparticle situation is only slightly better. In this case there is at least the hope that the mass-shell restriction of correlations involving unphysical matter fields may be physical in which case the Yennie-Frautschi-Suura (YSF) recipe would be

[47] All known short distance improving mechanisms consist in not simply compensating interactions but rather softening the original Y-M interactions.

meaningful. The YFS proposal (generalizing previous model calculations by Bloch and Nordsiek) introduces an ad hoc infrared regularization λ in terms of which the scattering amplitudes involving charged particles are logarithmically divergent for $\lambda \to 0$. The leading logarithmic divergencies are the summed up to a coupling-dependent power $\lambda^{f(g)}$ which vanishes for $\lambda \to 0$. The vanishing of the scattering amplitude shows that the LSZ scattering theory is not the correct concept for obtaining nontrivial scattering information for "infraparticles". Low order perturbative calculations show that the vanishing can be prevented by passing from scattering amplitudes to photon inclusive cross sections before letting $\lambda \to 0$..

Clearly the SLF setting calls for a more physical reformulation of this YFS prescription based on the idea that the coupling of massive vectormesons with its standard field-particle relation is the physically simpler than its QED limit. Hence the starting point would be the correlation functions of the stringlocal matter fields which are expected to have finite QED limits for vanishing vectormeson mass $m \to 0$ (infrared divergencies in QED only affect on-shell objects). The problem is that the application of the LSZ scattering theory cannot be interchanged with the massless limit. The SLF setting permits to formulate the YFS prescriptions in terms of a Hilbert space setting and replaces the ad hoc infrared regulator by a natural covariant regularization in terms of the vectormeson mass m. In this way the logarithmic divergencies of scattering amplitudes are explained as in terms of an illegitimate interchange of the massless limit with the perturbative expansion. The use of physical matter fields preserves the hope that in a future more profound understanding of scattering of infraparticles the perturbative YMS prescriptions [81] could be replaced by spacetime localization properties of stringlocal fields.

This suggest a perturbative understanding of confinement along the following lines. In analogy to massive QED one starts from selfinteracting massive gluons in terms of renormalized stringlocal fields. The expected appearance of logarithmic mass divergences in the stringlocal *off-shell* Hilbert space gluon correlations would be the starting point for a generalized From the YFS resummation argument of the leading *logm* terms one expects *the vanishing of all correlations for* $m \to 0$ *containing at least one gluon field*; only correlations of pointlike composites need not be zero. Besides presenting a new way in which confinement becomes accessible by perturbative methods, this picture also contains for the first time a structural proposal (in the opinion of the author, the only one consistent with the foundational principles of QFT) concerning the meaning of confinement in terms of correlation of fields and its possible connection with perturbative logarithmic divergent off-shell correlations.

Although both QED and Y-M gluons couplings lead to stringlocal fields, their mathematical structure and physical manifestations are very different. Interacting vectorpotentials in QED are integrals over pointlike observable field strength whereas this is property is lost in Y-M interactions. We will refer to massless stringlike fields which cannot be approximated by local observables as *irreducible* strings. Such objects are inherently nonlocal i.e. unlike normal global objects as charges (integrals over pointlike currents) they cannot be approximated by compact localized matter. Inherently noncompact fields would create havoc with causality if they could create particles. Confinement in the sense of vanishing correlation functions (except those whose only observables are composites) containing irreducibly stringlocal basic fields prevents this clash with causality.

The idea allows a generalization to quark confinement. The existence of anti-quarks changes the physical consequences of string-localization. If the e of quark and anti-quark is chosen in the direction of the spacelike connecting direction of the endpoints, the infinite parts of the on top lying strings cancel so that only the finite "string-bridge" between x and x' remains. Such a pair defines a local observable. In the SLF formalism it is nothing else than the product of the elementary (no composite bridge construction) stringlocal quark-anti-quark operators with a special choice of their $e's$. What seemed to be out of reach in the BRST setting (38), is now part of renormalization theory of the basic fields in terms whose interactions define the field content.

The argument also contains an invitation to look behind the standard argument stating that "long distances are non-perturbative" which is used as an excuse for the omission of the long distance contribution in the derivation of asymptotic freedom from a beta function. Beta functions are part of Callan-Symanzik equations; these are in turn derived from the renormalization theory of correlation functions. The coefficients in these parametric differential equation are global quantities and it does not make sense to try to get informations about them from short distances only; strictly speaking it is not possible to base a calculation of a global quantity on short distance properties only[48]. Even if one has no doubts about the negative sign (indicating asymptotic freedom) of $\beta(g)$, it would be preferable to base such important physical conclusions on more solid mathematical arguments. The SLF setting for the massive Y-M theory offers a credible setting for a better calculation.

The above confinement scenario presents an interesting contrast to another kind of stringlocal matter: the QFT of Wigner's zero mass "infinite spin" positive energy representation class. Actually the understanding of the importance of string-localization for the conceptual progress of QFT started with a paper [43]; the main point of that work was the presentation of the QFT behind this mysterious 1939 Wigner representation. As a positive energy representation it shares properties as stability of matter and coupling to the gravitational field with the massive and massless finite helicity representation. It turns out that the Wigner representations contain no pointlike covariant wave functions at all and there are convincing arguments that the associated net of local algebras admits no compact localized subalgebras generated by composite pointlike fields; such representations describe noncompact matter par excellence.

Whereas gluon or quark matter cannot emerge from collisions of normal matter which interacts in a compact region, noncompact free infinite spin matter once inside our universe cannot be registered in earthly particle counters. In fact it is totally inert apart from gravitational manifestations [92]. This means that its omnipresence wold change the gravitational balance of normal matter in a galaxy. When Weinberg wrote his book on QFT he rejected this kind of matter because "nature does not make use of it"; at that time its strange noncompact localization properties were not yet known, apart from the fact that all attempts to describe this matter in terms of pointlike covariant fields had failed. Although its property of eluding registration in particle counters would still cause stomachaches with high energy physicists, it seem that astrophysicists like such inert matter whose only arena of action are galaxies.

[48]The best one can do with such incomplete knowledge is to show that the assumed $g \to 0$ behavior of beta is consistent with the perturbative short distance behavior.

It may be helpful for the reader to use again Galileo's method of codification in terms of a dialog between Sagredo and Simplicio in order to facilitate the acceptance of the rather heavy new foundational results by adding a light touch.

Sagredo: Dear friend Simplicio, are you really claiming that the Higgs mechanism is only a metaphor for the coupling of real scalar fields to a massive vectorpotential i.e. the neutral analog of the massive scalar QED? Does this mean that the Hermitian field and its related chargeless particle does not originate from a spontaneous symmetry breaking of the scalar two-parametric QED[49] by a vacuum expectation of the matter field which leads to a Mexican hat potential by applying a field shift to the selfinteracting quadrilinear QED counterterm (which carries the second parameter of scalar QED) ? Is the picture of a distinguished particle whose interaction does not only create the mass of the vectormeson but also its own mass (the "God" particle), inconsistent with the principles of QFT?

Simplicio: This is more or less my point of view, but I would suggest to look at the situation in a historical context and avoid to see disputes in science like sport events or as caused by past intellectual limitations. The idea to start from a massless situation and to generate masses by spontaneous symmetry-breaking appears natural at first sight, especially if one believes that the physical properties of zero mass theories were much better understood than those with a mass gap. This was the Zeitgeist at the time of the proposed "Higgs mechanism". Shortly after the proposal of the Higgs mechanism there were people who had doubts about the consistency of such a symmetry breaking with gauge invariance which, as a result of its different nature from that of global internal symmetries, cannot be broken. Nowadays we know that the physics behind Higgs models is that of a massive vectormeson coupled to a neutral scalar matter field. There are three kinds of renormalizable vectormeson couplings namely couplings to spinor matter (massive QED) or to scalar complex matter, couplings to Hermitian (charge-neutral) matter and self-couplings (Y-M models). This exhausts all possibilities of massive vectormeson couplings.

The reason why the understanding of this point took such a long time is that the neutral coupling has no classical counterpart; in fact it only exists for massive vectormesons and disappears (i.e. on is left with free fields) in the massless limit. This coupling to neutral matter is also related with a Mexican hat type potential, but its origin is totally different: it is not put into the interaction, but it is "induced" in second order perturbation by the implementation of locality in a Hilbert space setting (or by imposing the BRST gauge formalism in a Krein space setting).

Sagredo: But if what you call the metaphoric formulation leads at the end of the day to the same result as your coupling to neutral matter, can one not forget the incorrect interpretation and except it as a computational recipe?

Simplicio: Let me try to answer your question in the form of a parable. Start from the simplest massless Goldstone model (abelian sigma model) of a selfinteracting complex field and break the phase symmetry (or rotational symmetry in the real components) by a nonvanishing expectation value. The resulting interactions is a massive field accompanied by a massless field. The important observation here is that the charge of the conserved symmetry current, whose charge before the field shift was related to a *finite* global charge

[49]Different from spinor QED which only has one coupling parameter, the renormalization of a pointlike scalar gauge coupling leads in addition to a quadrilinear selfcoupling of the matter field.

operator, is infinite after the charge after the shift; as the result of the coupling of the massless field (the Goldstone boson) to the current, the integral over $j_0(x)$ diverges for large distances. *The important physical message* behind this formal manipulation *is the connection of the the divergent charge of the conserved current (the definition of spontaneous symmetry breaking) with the unavoidable appearance of a massless particle* and not the process of "massaging" an interaction (a Lagrangian) in a particular way. The Goldstone symmetry breaking (i.e. the inexorable connection between a model independent definition of spontaneous symmetry breaking and the appearance of a massless meson in the spectrum) is secured by an impressive structural theorem based on the use of the Jost-Lehmann-Dyson representation [71]. Lagrangian manipulation have a physical meaning if they lead to intrinsic properties in the expectation values of the observables. Calling the field shift "spontaneous symmetry-breaking" is justified since it leads to a conserved current whose charge integral diverges for large distances as a result of its coupling to a Goldstone boson. However to call it a mass-creation in any physical sense (apart from serving as a "pons asini" for the calculating theoretician) would be misleading, although in this case it had no harmful consequences.

Sagredo: So the mass-creating Higgs mechanism is a computational prescription? A pons asini for what kind of physics?

Simplicio: Its intrinsic physical content is the renormalizable coupling (either in the BRST gauge setting or in Hilbert space SLF) of a massive vectormeson to a Hermitian scalar matter field. The *intrinsic characterization of all massive vectormeson-to-matter couplings is this Schwinger-Swieca screening*; the main difference between complex (charged) matter and Hermitian (neutral) matter is that in the Hermitian case this is the only conserved current. In the massless limit of complex matter the Maxwell current and the charge counting current (complex, charge-anticharge) coalesce (the charge-screening disappears) and the interaction of Hermitian H-matter and massive vectormesons vanishes. This absence of a zero mass vectormeson-H coupling explains in a certain way why these couplings were found in this confused way. In the hierarchy $Q = 0,\ finite \neq 0,\ \infty$ the screening situation is furthest remove from spontaneous symmetry breaking with the normal internal symmetry situation is in the middle.

It is difficult to say to what degree Schwinger was aware about these facts, but certainly Swieca (who proved the Goldstone- and the screening- theorem) knew about the importance of clear intrinsic physical pictures was completely aware about this; in fact he tried to counteract the tide by referring in all of his papers to the "Schwinger-Higgs" screening in order to save what is intrinsic and valuable to the Higgs model as the first attempt to describe gauge theory of Hermitian matter (even against Higgs own view) from serious misunderstandings. Unfortunately he failed and his ideas were lost in the maelstrom of time. With Big Science determining the path of particle physics the chances to get back on track are minimal, unless a Nobel laureate takes notice of these facts and injects them into Big Science.

Returning to your question, calling the field shift in Goldstone's model a spontaneous symmetry breaking is a harmless pons asini, whereas in case of the Higgs model it leads to serious misunderstandings. Not only does it have nothing to do with any physically meaningful form of symmetry breaking (what is the meaning of a field shift for the unphysical (gauge-dependent) matter field of the two parametric scalar QED?), the picture of a neutral particle which creates masses of vectormesons (including its own) distracts from foundational problems: could it be that causal locality in Hilbert space for massive selfinteracting s=1 fields requires the presence of s=0 companions? This, if true, would be the first time that a given free field content cannot interact without the presence of additional degrees of freedom. This certainly does not happen for $s < 1$ interactions, but for $s \geq 1$ the restrictions from locality are more restrictive (see the appearance of induced interactions)

The parable reveals that the result of massaging of Lagrangian is not always what one naively expects. The application of this ideas to scalar QED was miscarried from the very start: a *gauge dependent scalar matter field* cannot be the starting point of a Goldstone-like argument. Whatever one expects as the physical content at the end, it can have nothing to do with spontaneous symmetry breaking. The intrinsic physical result is something (by most physicists) completely unexpected: the Schwinger-Swieca screening of the Maxwell charges in massive vectormeson couplings. Indeed the appearance of two different conserved currents (the standard "counting current" and that and the divergence of the field strength) is somewhat unusual since in (massless) QED they coalesce i.e. the Maxwell charge becomes unscreened. The Maxwell current is the only one in the case of neutral matter, and the coupling of Hermitian matter to massive vectormesons vanishes in the massless limit. If somebody is looking for an intrinsic experimentally verifiable consequence of massive vectormesons it is the Schwinger-Swieca screening of Maxwell charges. This important property of which remains of Higgs-like models of the end of the day, and which can be checked easily, has never been noticed in any of the calculations on Higgs-like models because it was hidden behind the cloud of the symmetry-breaking metaphor.

Sagredo: Are you implying that the foundational work of Glashow, Salam and Weinberg is flawed?

Simplicio: Saying that would show a complete misunderstanding of the way particle physics reveals itself to us. High energy physics is based on QFT and the latter is, in contrast to e.g. Einstein's special and general relativity, very far removed from its conceptual closure; its conceptual physical content only unfolds stepwise. *The important part of the GSW work is the QFT description of weak interactions involving massive Y-M gluons (W-Z particles) and its junction with electromagnetism.* The reason for their use of the Higgs mechanism was related to the widespread believe that interactions involving only massive vectormesons do not permit a description as renormalizable theories. This had some justification at the times of Higgs, since *the perturbative Gupta-Bleuler gauge formalism becomes only applicable to the massive case after its BRST ghost operator extension.*

The dismissal of the spontaneous symmetry breaking does however not mean that there could not be other foundational reasons for a presence of additional degrees freedom beyond the minimal field content of massive gauge theory; certainly GSW would have taken this into account if an alternative would have been available in the 60s. At the end of the day any necessary presence of additional degrees of freedom must come from the foundational causal locality principle in Hilbert space.

The consistency of the ghost charge formalism (and not any Higgs mechanism) seems to demand the presence of scalar neutral fields in addition to the massive Y-M fields, but one would not like to surrender such a foundational problem to a Krein space formalism. In the SLF Hilbert space setting the presence of an additional coupling of massive gluons to neutral matter fields would bring the (properly adjusted) Schwinger-Swieca charge screening into the nonabelian setting. For the purpose of the present discussion concerning the incompatibility of a Higgs symmetry-breaking mechanism with QFT the still (at the time of writing) unknown answer concerning the possibly required presence of charge-neutral couplings is without importance.

What a particle theorist raised in the Einstein tradition definite abhors (especially after the 40 years attempts to find a particle believed to be of fundamental significance) is a theory which can be simplified by application of Occam's razor.

Sagredo: Are the answers to these open questions decisive for the future of the Standard Model?

Simplicio: They are very important but they do not yet reduce the number of parameters to the minimal number which one could reasonably expect from a QFT, namely the field content (masses and spins of the field content interms of which the interaction is defined, but not those of boundstates which should come out, even if we presently do not know in what way) and a few coupling strengths. The most important unsolved conceptual problem to which the new SLF Hilbert space setting sheds new light is the gluon/quark confinement conundrum. On the one hand it provides a clear-cut definition in the form of vanishing of all correlation functions which contain would-be Hilbert space (and hence stringlocal) gluon and quark correlations so that only correlations of pointlike composites survive. On the other hand, and this is more important, it provides for the first time the tool how one can extract this property in the massless limit of a perturbative renormalizable theory of massive gluons and quarks from the logarithmically divergent correlation functions in the massless gluon limit with the help of an extended YFS perturbative resummation argument. Last and not least it provides a perturbative method to investigate $q-\bar{q}$ pairs which (by appropriate choice of the e-direction[50]) automatically contain (what in the gauge theoretic setting was referred to as the composite "gauge bridge") the elementary string bridge. What was hitherto called "string breaking" (related to hadronization), namely the behavior of $q-\bar{q}$ pairs under increasing spacelike separation, can now also also be studied within renormalized perturbation theory.

The real impact of the new setting is that it places the old ideas about the QED to massive vectormeson relation from its head to its feet: the conceptual and computational simplicity is on the side of massive vectormesons and their zero mass limits present the remaining challenge. Even in the case of QED the description of charged particle scattering ("infraparticles") has only been "understood" in terms of a photon-inclusive prescription for photon-inclusive cross sections, but not in terms of a process in spacetime. Physical charge-carrying particles only exist in the form of stringlocal "infraparticle fields", and a spacetime procedure which replaces the LSZ formalism in the presence of mass-gaps to infraparticle situation has still to be worked out. The SLF setting presents the means to do it.

The future of particle theory depends on whether it is possible to maintain the innovative democratic power it unfolded in the past or whether it slides even more into the

[50]Except q-$\bar{q}$ pairs whose $e'$are parallel to the spacelike x-x' line.

control of Big Science.

Sagredo: Thank you dear friend for sharing your thoughts, and I hope that your pessimistic assessment about particle theory remains a warning and does not become a prediction about its future. It will take some time to fully comprehend what you told me; lots of important issues to think about lie before me before I can meet you again.

7 The dual model, misunderstandings about particle crossing

The idea to avoid the use of singular fields, which led to the problem of ultraviolet divergencies, and instead formulate particle physics in terms of the S-matrix goes back to Heisenberg. It was abandoned soon afterwards when the success of renormalized perturbation theory in QED left no doubts that the conclusion of inconsistency of QFT based on the ultraviolet was premature. The problem which perturbative methods had with strong interactions led to adaptation of the Kramers-Kronig dispersion relations to particle physics. It was modest in scope[51] but after a decade it came to closure by achieving all its objectives (the only project in particle theory which came to a successful closure). This success encouraged several theoreticians to formulate a new constructive S-matrix setting in which the perturbative analytic particle crossing property for the S-matrix (and later formfactors) was the basis of the new setting. Together with unitarity and Poincaré invariance it became known as the "S-matrix bootstrap" but it soon ended as a result of the unmanageable nonlinear problems arising from simultaneously implementing these three properties "by hand". Another problem was the insufficient understanding of the conceptual origin of particle crossing; its derivation from the locality principle for some very special scattering amplitudes did not lead to sufficient insights and the prohibitively difficult method of analytic functions [23] of several complex variables led to an early end of these attempts.

Another attempt to obtain a constructive computational access to particle theory in terms of an on-shell project based on S-matrix properties was formulated by Mandelstam [29]. In analogy to the successful use of the Jost-Lehmann-Dyson spectral representation which led to a rigorous proof of dispersion relation, Mandelstam postulated the validity of a double spectral representation for the elastic scattering amplitude as a starting point for getting access to analytic on-shell properties including the crossing property.

The era of genuine misunderstanding of particle crossing started with Veneziano's [82] construction (based on properties Euler's beta function) of a meromorphic function of two variables which had an infinity of first order poles in the two variables which were related by an analytic crossing relation. Although his presentation did not contain any physical argument why the mathematically constructed function which is meromorphic in variables which he identified with the Mandelstam s,t,u variables should be identified with the elastic part of a scattering amplitude his construction created a lot of excitement within which a critical attitude had little chance. Apparently the results on integrable models,

[51] Its main aim was to make sure that the causal localitity principle of QFT continues to be valid at the energies of the newly emerging High Rnergy Physics.

which could have revealed that although scattering amplitudes can be meromorphic in the rapidity variables but not in the Mandelstam variables, were not known to the dual model community.

Instead of speculating about what went on the mind of peoples who excepted Veneziano's use of the dual model meromorphic function as an approximation of an elastic scattering amplitude (to be improved by "unitarization") it is much easier to understand what kind of quantum field theoretic idea leads precisely to such dual model function. This clarification is due to Mack [89], and his construction is here referred to as the "Mack-machine"; this name is chosen because it cannot only produce Veneziano's dual model and similar dual models constructed later, but in a certain sense it can produce all dual models beyond those which have not been constructed.

The construction uses conformal global operator expansions for pairs of operators which, in contrast to the Wilson-Zimmermann short distance expansions, are known to converge

$$A(x)B(y)\Omega = \sum_k \int d^4z \Delta_{A,B,,C_k}(x,y,z)C_k(z)\Omega \tag{64}$$

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4)\rangle \rightarrow 3\ different\ expansions \tag{65}$$

and applies them inside the 4-point function (second line) Each pair of operators has a converging expansion on the vacuum in which the resulting operators C_k stand for a list of composites which can be connected with the given pair through nonvanishing 3 point functions Δ. Used inside the 4-point function, this leads to three different ways of decomposing the 4-point function into a sum over two three-point functions multiplicatively connected by an integrated two-point-function. Mack showed that the Mellin transform of this infinite sum over $C's$ leads precisely to the pole representation of the meromorphic functions which define dual models; the position of the first order poles is given in terms of the spectrum of scale dimensions of the $C's$ which couple to the pairs. Veneziano's model corresponds to a certain chiral conformal model, but any conformal 4 point function in any spacetime dimension upon expansion of its 4-point function and Mellin transformation of the resulting series always leads to a dual model in the sense of defining a meromorphic function with first order poles which fulfills a crossing relation. What initially looked magic and unique[52] in the hands of Veneziano is "mass-produced" by the Mack-machine.

Graphically the relation is reminiscent of an identity between two types of infinite sums over Feynman graphs with particle exchanges either in Mandelstam's s or t variable but, as the underlying conformal QFT shows conformal, there is no conceptual relation to scattering of particles. Conformal theories are interesting quantum field theories from which one can learn a lot about the inner workings of the modular localization properties, but they certainly contain no information about scattering of particles; in fact *interacting conformal models contain no particles at all*, they are rather theories of anomalous scale

[52] The uniqueness, which was already expected to be follow from the bootstrap principles, was a precursor of the reductionist idea of a theory of everything (TOE) which originated in connection with ST.

dimensions which live on a covering of the compactified Minkowski space. Mellin transforms of their 4-point functions [89] may be called dual models, but this has no bearing on interactions between particles. It does not make sense to apply ideas of unitarization to dual models (as if they would define a kind of nonunitary approximation of an S-matrix).

This could have been the end of a misunderstanding and the closure of this unfortunate chapter of misguided particle theory and it probably would have been the end if not an even stranger twist would have greatly increased the mysterious aspects and with it the attractiveness of ST, a subject which is inseparably linked with the dual model (confusing metaphoric appearances with conceptual depth). This consisted in the observation that the oscillator algebra resulting from the Fourier decomposition of a certain chiral 10-component current algebra formally related to supersymmetric version of the Polyakov action

$$\int d\sigma d\tau \sum_{\xi=\sigma,\tau} \partial_\xi X_\mu(\sigma,\tau) g^{\mu\nu} \partial^\xi X_\mu(\sigma,\tau), \ \sigma,\tau = t \pm x \tag{66}$$
$$X = potential\ of\ conformal\ current\ j$$

permits the representation of a positive energy representation of the Poincaré group which decomposes into a discrete infinite sum of irreducible representation. (an infinite (m,s) "tower").

The construction of such a tower (an infinite component field equation) from an *irreducible algebraic structure* was one of Majorana's project which he formulated in 1932 with the idea to achieve something similar to what the O(4,2) group representation theory does for the hydrogen atom spectrum in QM. This project was revived in the 60s where it acquired some popularity under the name "dynamic infinite group representation project" (Fronsdal, Barut, Kleinert,..[83]). In fact Majorana's project as well as its later revival restricted this search to irreducible representations of extensions of the Lorentz group. But the only known solution is the representation *on the irreducible oscillator algebra of the supersymmetric 10 component current algebra,* the so-called superstring representation of the Poincaré group. This is a group theoretic fact which, although discovered by string theorists, has no relation to Mandelstam S-matrix based on-shell project.

To understand a more generic way the prerequisites one need to encounter the representation of a noncompact group as a kind of internal symmetry group on the component space of a multicomponent chiral conformal algebra, it is helpful to be reminded of same basic fact of LQP in which inner symmetries arise from the (generally assumed without inner symmetries) the local net of observable algebras in the vacuum representation. The other inequivalent local representation classes (superselection sectors) can in typical cases be combined with the vacuum representation within a larger *field algebra net.* There are convincing arguments why a continuous set of superselection sectors (in the presence of zero mass particles as QED one must pass to charge-classes [80]) and noncompact internal symmetries of the field algebras cannot occur in higher than two dimensions. The superselection analysis is very different in d=1+1 dimensions.

As an illustration let us look at a n-component current algebra

$$\partial\Phi_k(x) = j_k(x),\ \Phi_k(x) = \int_{-\infty}^{x} j_k(x),\ \langle j_k(x)j_l(x')\rangle \sim \delta_{k,l}\left(x - x' - i\varepsilon\right)^{-2} \tag{67}$$
$$Q_k = \Phi_k(\infty)\,,\ \Psi(x,\vec{q}) = \text{"} : e^{i\vec{q}\vec{\Phi}(x)\text{"}} :\ ,\ carries\ \vec{q} - charge$$
$$Q_k \simeq P_k,\ dim(e^{i\vec{q}\vec{\Phi}(x)}) \sim \vec{q}\cdot\vec{q} \simeq p_\mu p^\mu,\ (d_{sd}, s) \sim (m, s)$$

Here we have avoided the confusion notation X in favor of Φ for the multicomponent current potential because we want to avoid a notation which may suggest the wrong idea of an operator which embeds a chiral conformal theory on a lightray (or on its compactified circle) into a n-dimensional Minkowski spacetime so that its development in time it looks like a 2-dimensional surface (a tube, in case of a chiral theory on a circle). This picture of a covariant string sweeping through a tube-like world-sheet is incorrect inasmuch as it is incorrect to think that the classical covariant particle Lagrangian $\sqrt{ds^2}$ leads to a covariant quantum embedding described in terms of a covariant operator $x_\mu^{op}(\tau)$. In fact, ignoring Lagrangian quantization, there simply exists no covariant operator whose projectors in the spectral decomposition fulfill the requirements of covariant localization, a fact which certainly was already on the mind of Wigner when he constructed relativistic particles by representation theory and not by quantization.

In the book on string theory by Polchinski he used this classical relativistic particle Lagrangian as a "trailer" for a relativistic quantum theory of a strings based on the Nambu-Goto which is described by a replacing the ds^2 under the square root by the corresponding covariant surface differential. But instead of being helpful this analogy turns out to be a squid load. Indeed the quantization of the Nambu-Goto Lagrangian according to the correct rules for quantization in the presence of a parametrization invariance resembles that of quantizing the Einstein-Hilbert action. It is certainly non-renormalizable and has no natural relation to the Poincaré group which acts on the embedding Minkowski spacetime [84]. There is another approach to the square root N-G Lagrangian which is due to Pohlmeyer [85]; it is based on the observation that the classical system is integrable. So instead of confronting the problem of quantization of reparametrization invariant actions which inevitably leads to renormalization problems, he proposes to quantize the Poisson relations between the infinitely many conserved "charges". The problem with this quantization is that one looses the connection with localization in spacetime and Poincaré covariance.

On the other hand the Polyakov Lagrangian has a direct relation to chiral conformal QFT, so one believes to be on conceptually safe grounds. Here the problem is that the representation of the irreducible oscillator algebra behind the operator formalism (67) which serves for the representation of the Poincaré group (and the ensuing intrinsic localization concept which comes with positive energy representation of the Poincaré group [44]) is not the same as the one which localizes the chiral model on the lightray. With other words the Hilbert space representations of the oscillator algebra are different in both cases. The charge spectrum of the chiral theory is the whole $\mathbb{R}^n$ and the sigma-model fields Ψ in (67) are the charge carriers. On the other hand the spectrum of the representation of the Poincaré group is contained in the forward light cone and has mass gaps. On the other hand the spectrum of the zero mode multicomponent charge operator covers the full spectrum of the charge superselection structure. The treacherous nature of the analogy between the mass spectrum and the conformal dimensional spectrum

$$P_\mu \tilde{} Q_\mu .\ P^2 \tilde{} Q^2 \qquad (68)$$
$$\curvearrowright m^2 \tilde{} d_{scale}$$

is overlooked by string theorists. These analogies get even more seductive if one realizes that a particular discrete particle representation of the Poincaré group (the superstring representation) does appears on the oscillator algebra of a 10 component supersymmetric current model (unique up to a finite discrete "M-theoretic" variation). But what has this group theoretic coincidence, which represents the only known solution of the 1932 Majorana project, to do with Mandelstam's on-shell S-matrix project? The answer is nothing.

Majorana's projects is of a purely group theoretical kind, whereas Mandelstam aimed a dynamical particle theory which starts with the S-matrix and its analytic crossing property. In distinction to the string-localization of matter fields interacting with vectorpotentials in previous section, the representations occurring in the superstring representation are all *pointlike* generated. This was also what the calculations of the (graded) spacelike commutator of the putative string-fields by string-theorists in the 90s showed [86][87], but this is not how they interpreted their computed result; for them these points were located on a (invisible) string!

The fact that the dimensional spectrum which appears in the Mellin transform of global operator expansions of two sigma-model fields in a very special chiral current model contains the spectrum of a discrete unitary representation of the Poincaré group is quite amusing, but it has nothing to do with Mandelstam's constructive on-shell project notwithstanding that he still supports this unfortunate turn. None of the critical remarks in this section should be construed as diminishing the enormous importance of a correctly pursued constructive on-shell project as initiated by Mandelstam for the future of particle physics. Apart from the absence of any connection between ST and an S-matrix approach, there is also no embedding (as claimed by string theorists) of an n-component chiral current source theory into its internal symmetry target space; the localization concepts of the source theory cannot be realized simultaneously with that of the representation of the Poincaré group on the target theory since they use different (unitarily inequivalent) representations of the infinite oscillator algebra associated with conformal currents. In other words: we are not living in a (dimensionally reduced) target space of a chiral conformal QFT!

In fact a lower dimensional QFT can never be imbedded into a larger dimensional one, and neither is it possible to do the inverse (dimensional restriction). The Kaluza-Klein reduction can be implemented on classical Lagrangians and quasiclassical approximations of QFT, but the intrinsic modular localization structure of QFT does not allow to do this on its solution in terms of correlation functions or of nets of local algebras. Physical matter and its spacetime localization property are inseparably connected.

In most of the papers which were written under the influence of ST more than 2 decades ago, as those dealing with the Maldacena conjecture and the idea of branes inside a higher dimensional QFT, the "think as you computation moves along" attitude without a clear conceptual compass has led to confusions and stagnation of progress. Often the correct concepts which could have prevented wrong conclusions already existed but where lost in the maelstrom of time. One such subject is the holistic connection between the causal completion property and cardinality of degrees of freedom in LQP (see section 1). The wrong conclusions which result from ignoring it will be the topic of the next section.

8 Localization and phase-space degrees of freedom

In a course on QM one learns that the number of "degrees of freedom" (quantum states) per unit cell of phase space is finite. Already in the beginning of the 60s it became clear that this not compatible with the causal localization in QFT for which the cardinality cannot be finite. The first computation revealed that the infinity is not worse than that of a compact set [88] which in later work of Buchholz and Wichmann became sharpened to the cardinality of a *nuclear set* [7]; together with modular localization theory it led to the important concept of modular nuclearity [7].

The physical motivation of these investigations is the desire to understand the connection between field localization and the presence of particles. The ultimate aim to understand under what circumstances fields connect to particles with discrete masses and the validity of scattering theory including the important property of *asymptotic completeness*, remained only partially achieved up to this date. One remarkable result (whose importance should be seen in the context of more than 8 decades lasting attempts to verify the existence of models of QFT with interactions and obtain mathematically controlled approximations) is the before mentioned existence proof for certain strictly renormalizable models (i.e. models with realistic short distance behavior[53]) with the help of modular nuclearity.

Another important use of these ideas consists in the *exclusion* of models with unphysical causality properties. Lagrangian quantization leads to divergent renormalized perturbative series, and hence it is not suited for addressing problems of existence of models. Therefore it is important to maintain its intuitive causality properties in the better mathematically controlled LQP settings of QFT. Whereas the spacelike Einstein causality property is usually taken care of, the relevance of the causal completion (causal shadow) property is sometimes overlooked. This is particularly relevant which formally do not occur in a Lagrangian/functional quantization setting, but which one must be aware of in any attempt to formulate QFT in terms of intrinsic requirements, starting from modular localization.

It is very easy to write down generalized free fields which fulfill Einstein causality but violate the causal completeness property (which is the local version of the time-slice property [36]). A recent illustration of a violation of this important physical property is the conformal covariant generalized free field which results from a normal free field on a AdS spacetime through the AdS_{n+1}-CFT_n correspondence [90]. The physical defect of such the causal completion property violating fields is that they produce a "poltergeist effect" in the causal shadow region; as one "moves up" from the spacetime region $\mathcal{O}$ into its causal completions $\mathcal{O}''$ there are causality violating degrees of freedom apparently

[53] The d=1+1 superrenormalizable theory can still be treated within a measure-theoretic functional quantization setting [33].

coming from nowhere.

The LQP setting reveals that this effect is of a general nature and may be viewed as a manifestation of the holistic nature of spacetime localization. As the holistic nature of life needs the right amount of chemicals, the holistic nature of causal localization in spacetime needs the right cardinality of degrees of freedom which is appropriate to the spacetime dimensionality. For the case at hand, starting from a physical AdS theory, one obtains an "overpopulated" CFT model which has its outing in the poltergeist phenomenon. In the opposite direction a "physical healthy" CFT passes to an "anemic" AdS theory, in which one has to *look at noncompact regions in order to find any degrees of freedom* [91].

Such holistic properties are absent in QM; in fact the latter does not possess any intrinsic spatial notion at all. Whether one picture an oscillator chain in one or higher dimension and whether the imagine space is our living space or some abstract internal space is up to physicist who uses QM. This holistic aspect gets lost in quasiclassical approximations. It is interesting to note that this pathology is absent in holographic projections onto null-surfaces; unlike in isomorphic correspondences, holographic projections "thin out" (loss of imbedding information) degree of freedom by the right amount which fits the lower dimensional surface.

A similar phenomenon happens in case one passes to a "brane" by fixing one spatial; as Mack showed [89], the overpopulation in a brane causes even problems to distinguish spacetime- from inner- symmetries. Brane physics has been exclusively discussed in terms of quasiclassical approximation where these pathologies remain hidden.

It is interesting to take a closer look at a special misinterpretation which played an important role in ST. As mentioned before, the irreducible oscillator algebra of the 10 component chiral current admits 2 inequivalent representations, one which is important for the invariance under the conformal Möbius group and the pointlike localized fields on a lightray and the other which carries the mentioned 10 dimensional superstring positive energy superstring representation of the Poincaré group. Both representations are pointlike generated; this is a property shared by all positive energy representations. Bot there is a huge difference in the cardinality of freedom; the oscillator representation carries the superstring Poincaré group representation, but certainly not the superstring field representation which is canonically associated with it and hence there is no embedding of one QT into the other. Hence it is not possible to view the one as embedded into the other. The misplaced terminology "ST" which refers to a stringlocal object in a target spacetime probably arose from such an incorrect picture.

At best this terminolgy could refer to an internal oscillator chain (after taking out the zero mode degree of freedom) "over" a spacetime localization point which carries the (m,s) representation as well as additional operators which are not needed for the representation of the Poincaré group, but link the different levels of the (m,s) tower in order to complete the reducible superstring representation to an irreducible algebra. Such a tower of free fields piling up over one point leads to pointlike singularities which are beyond those of ordinary (Wightman) QFT. Perhaps this could have been the reason why, despite their correct calculation, the authors in [87][86] presented their result as a confirmation of stringlike spacetime localization by declaring the localization point to be the center point on a spacetime string. The pressure of the ST community to which they belong could also have contributed to draw such a weird conclusion (against Heisenberg's notion of quantum observables) from a correct computation.

As previously mentioned the embedding of lower dimensional QFTs into higher dimensional ones and its Kaluza-Klein inverse are also inconsistent with the holistic localization principle. Arguments based on quasiclassical approximations or by "massaging" Lagrangians do not count; An explicit argument in terms of correlation functions or nets of algebras does not exist for good reasons, since it would violate the holistic nature of matter in QFT.

What is however consistent within modular localization is a *degrees of freedom reducing holographic projection* onto null-hypersurfaces (which, as mentioned in a previous section, is related to the area proportionality of localization-entropy). It is also conceivable that certain aspects of compactifying a spacetime dimension can be achieved by converting the time into temperature by applying the rules of "thermalization" which introduce a compactification through a kind of periodicity on a circle which decreases with increasing temperature. But strictly speaking, the holistic aspect of quantum matter in QFT does not support a clear separation between quantum matter and its appearance in spacetime; rather spacetime is imprinted on quantum matter and Kaluza-Klein reductions and embedding are only possible in quasiclassical approximations to which the holistic relation between localization and degrees of freedom does not apply. Knowledge about the conceptual structure of QFT models was not around at the time of Kaluza and Klein; QFT in those days was often not separated from representations of QM in the "second quantization" setting.

These insights into the connection between the cardinality of degrees of freedom and localization immediately disproves the Maldacena conjecture which claims that both sides of the AdS_5-CFT_4 represent *physical* theories. It also delegates "brane physics" "extra dimensions", "dimensional reduction" and many other ideas which originated in the same frame of mind about particle physics as ST (shut up and compute) to the dustbin of history, except that in this case history is often still very present. As a coauthors of a 1962 paper [36] which led to the concept of the causal completion property (which later on was related with the degree of freedom issue [7] it is particularly distressing to look at the present situation in which globalized communities of particle theorists have fallen behind previously attained levels of knowledge about important concepts (and where members of these community receive prizes for results inconsistent with publications in the pre-electronic era).

Returning again to Galileo's method of avoiding ideological/religious conflicts by using of the artifice of an imagined dialog, the conversation between Sagredo and Simplicio on issues of this section may have taken the following path:

Sagredo: Dear Simplicio, some of our friends tell me that you claim that the dual model and ST led to a derailment of an important part of particle theory?

Simplicio: Well, although my attitude has been critical, I have good reasons to avoid expressing my critique in this way. What prevents me is the fact that a mass shell based alternative to the quantization approach to QFT represent in my view a project in particle physics which is second in importance and subtlety only to the successful project of renormalized perturbation theory started by Tomonaga, Schwinger, Feynman and Dyson. Indeed, after the successful closure of the dispersion relation project it was natural to look for a "from top to bottom on-shell setting" which on the one hand is close to scattering observables (especially in case of strong interactions), and on the other hand avoids the handicaps of perturbative series which, as a consequence of their divergence, do not contain informations about the mathematical existence of models with realistic short distance behavior. But it was clear, in particular to its protagonists as Stanley Mandelstam, that a foundational understanding of on-shell analytic properties of the S-matrix and formfactors was even more demanding than those of off-shell correlation functions, since their relation to spectral properties and causal locality is more hidden. This applies in particular to the particle crossing relation.

Saying simply that a project has been derailed may be misunderstood as claiming that particle theory would have been better off without it. The correction of interesting errors leads often to deeper insights than those one could have obtained (with more luck) in a direct way. Different from any other area of science, errors whose correction points into new directions are as important as new discoveries; historians of science should not suppress them.

Sagredo: Are you suggesting that this problem was too subtle for the generally extremely successful conduct of research which consisted of starting calculations built on educated guesses and correcting until internal consistency has been reached? After all this more playful way of conducting research without time-consuming conceptual investments has been very successful; among other things it led Dirac to the discovery of antiparticles on the basis of the inconsistent hole theory.

Simplicio: The derivation of on-shell properties of the S-matrix and formfactors from the causal locality properties poses more difficult conceptual problems than those of perturbation properties and the derivation of structural properties from the Wightman setting. As the derivation of crossing shows, the conceptual distance between the causality principles of QFT and properties of the S-matrix and formfactors is much greater than that between those principles and the TCP property, the spin-statistics theorem and all the other properties which can be derived from the Wightman setting of correlation functions without using nonlinear properties. The derivation of the analytic properties needed in the derivation of dispersion relations did not require knowledge about the crossing property; the use of the rigorously established Jost-Lehmann-Dyson representation was sufficient for the adaptation of the Kramers-Kronig dispersion relations to high energy particle physics. But Mandelstam's project was more ambitious, and the spectral representation which he used was guessed and not derived as a consequence of the quantum causal localization principle; hence his project was not protected against derailment.

Sagredo: Are you implying that this is what happened in ST and explains why this theory (although being considered by some mathematicians as an extremely useful construct) has within by now 5 decades not led to any theoretically trustworthy and experimentally realistic physical prediction?

Simplicio: One has to be careful on this issue; there are of cause no time limits on when a theory, which is claimed to generalize our most successful QFT, has to deliver

observational verifiable results. In retrospect it is clear that the project of an S-matrix based on-shell approach was started at a time when no trustworthy knowledge about the conceptual origin of analytic and algebraic properties about on-shell properties in QFT was available (beyond that which led to the dispersion relations). The dual model and ST resulted from an unhealthy mix of phenomenological beliefs with subtle mathematical observations; there was no conceptual guidance on the side of QFT. More conceptual investment could have revealed that one became the victim of a "picture puzzle" in which insufficiently understood aspects of chiral QFT were misinterpreted as new deep properties of on-shell particle theory.

Sagredo: Do you want to suggest that ST, quite independent of its lack of observational success, has serious conceptual flaws?

Simplicio: The claimed string-localization has nothing to do with spacetime localization. The author of this misleading terminology may have confused a quantum mechanical chain of oscillators with a stringlocal object in spacetime. The so called string field is really a pointlike field with consists of infinitely many (m,s) free fields piled on top of each other with operators which interlink the different (m,s) levels. The (m,s) spectrum is related to the first order poles of the dual model which originates through Mellin transformation from the spectrum of anomalous scale dimensions in a conformal global operator expansion. The crossing property of the meromorphic dual model function of conformal QFTs has no relation to the particle crossing of scattering amplitudes and formfactors. In fact the particle crossing is never related to meromorphic behavior in the Mandelstam variables since scattering always leads to cuts in the analytically continued; even in d=1+1 integrable models meromorphic scattering amplitudes and formfactors can only be attained in the momentum space *rapidities* which are the uniformization variables of such models.

Sagredo: Does this mean that ST has no relation to particle theory at all ?

Simplicio: Not quite, the theorem that the irreducible oscillator algebra of a 10 component supersymmetric abelian chiral current model leads to a positive energy representation of the Poincaré group (namely the so-called superstring representation) is certainly a theorem obtained by string theorists whose veracity is not disputed by anybody. But particle physics deals with *interactions* as embodied in the S-matrix and formfactors, and the mentioned group-theoretic theorem contains *no informations* on those issues, it is of an entirely kinematic nature. The highly reducible positive energy superstring representation obtained from a certain irreducible ocillator algebra which in turn resulted from the Fourier decomposition of a compactified supersymmetric 10- component chiral current algebra is the only known solution of Majorana's project to construct "natural" infinite component field equations. Majorana was inspired by the analogy with the O(4,2) hydrogen spectrum, but even the most hardened string theorist would not think that such project is relevant for our present understanding of modern particle theory. The irony is that, when some people in the 60s looked for "dynamic infinite component relativistic field equations" in terms of extensions of the Lorentz group, they were not aware that string theorist already found an irreducible algebraic structure (the irreducible oscillator algebra associated to a 10-dimensional current algebra) which admits a representation which solves their problem of "dynamical group representations" (i.e. Majorana's project). Their motives were simply too different.

Sagredo: But doesn't this show that at least there exists a close relation between the Moebius covariant chiral "source" representation which can be localized on the lightray, with the target representation of the Poincaré group in a 10-dimensional spacetime?

Simplicio: It depends on what you mean by "close"; certainly the chiral representation and the representation on the index space of inner symmetries of the chiral model (which is probably what you mean by target representation) are representations of the same abstract irreducible infinite oscillator algebra. But they are not unitarily equivalent, which makes it impossible to interpret this situation as an embedding.

Sagredo: ST led to many extremely popular derivatives; besides "embedding" of lower dimensional QFT into higher dimensional ones, people associated with the ST community like to talk about "extra dimensions", "dimensional reductions", "branes" and holographic projections.

Simplicio: The obstacle against most of these ideas is that in QFT the index space of charge-carrying quantum fields for $d > 1+1$ can only carry representations of compact groups which does not permit spacetime-like target embedding; any noncompact representation would violate causal locality and covariance [7]. Internal symmetry is not a concept of classical field theory, but one can read it back from quantum internal symmetry (which a consequence of localized representation theory) into the classical setting. But classical fields can also have index spaces on which representations of noncompact groups may act. In fact one can introduce classical symmetries which have no counterpart in quantum physics. The simplest illustration is provided by the classical Lagrangian of a free relativistic particle $\mathcal{L}\tilde{}\sqrt{ds^2}$ whose Euler-Lagrange equations describe the relativistically covariant trajectory of a classical particle (see section), but there are no associated covariant quantum particle variables; a formaly closely related situation is that of the square root of the surface differential. The counterpart of this problem n d>2 is the impossibility to construct finite component models on whose component space index noncompact internal symmetry group act. The only known exception occur if the noncompact groups happen to be the spacetime symmetry groups associated to the "living spacetime" of the fields live (i.e. spinor/tensor indices of the fields). However d=1+1 chiral conformal models can have continuously many superselection rules ("non-rational" chiral models) on which noncompact groups may be represented (the target space of ST).

In QM the "Born localization" (related to the spectral decomposition of the position operator) has no intrinsic significance; a linear oscillator chain can be pictured in any desired dimension. In QFT this is not possible; even in Wigner's representation theory the (positive energy) particle spaces *depend* (through the concept of the little group) *essentially on spacetime dimensions.*

Wilson used his idea of analytic continuation in spacetime dimension only for *scalar* particles (critical phenomena) as a technical tool; it does not work on fields associated with nontrivial Wigner little groups. The Kaluza-Klein dimensional reduction and branes have only been exemplified in classical or quasiclassical approximations, in QFT they are ruled out by the connection of the cardinality of degrees of freedom with spacetime causality (through the causal completion property).

My dear Sagredo, at this late hour I propose that we close our conversation.

9 Resumé and concluding remarks

QFT provides particle theory with an important conceptual structure: its causal localization principle. It results from the amalgamation of the Faraday-Maxwell-Einstein classical causality with the Hilbert space structure of quantum theory. Its conceptual strength is matched by its concise mathematical formulation: the adaptation of the Tomita-Takesaki operator theory in the form of modular localization. The main reason for submitting the present work to a history/philosophy oriented physics journal is the fact that this new framework of QFT sheds new light on famous debates in the history of QFT as the dispute between Einstein and Jordan which led Jordan to the discovery of QFT [3]. It shows the importance of modular localization in unravelling insuffiently understood conceptual relations at the time of the birth of QFT and its innovative power in resolving ongoing problems. It produces fascinating results which contrast those of "Big Science" on issues of "mass creation by spontaneous symmetry breaking" and clarifies the relation between localization of interacting higher spin fields and their Hilbert space description,

It also prevents that an unsuccessful theory as ST (which dominated particle physics for 50 years) vanishes from the scene without a trace; it achieves this by exposing its conceptual errors, so that the particle physics community can profit and future historians can explain to a learned and curious public what went on through more than 5 decades. The misunderstandings of quantum causal locality are not limited to ST, but they also affected areas which, although logically independent of ST, appeared in the wake of ST. Affected is the use of Kaluza-Klein dimensional reductions and embeddings in QFT (i.e. outside classical field theories or quasiclassical approximations). Matter in QFT, in contrast to QM, is inexorably linked with spacetime dimension. Even in those cases in those cases in which it is *mathematically* possible to transplant algebras or fields between different spacetimes as in the AdS-CFT isomorphism or in the construction of branes in QFT, the breakdown of the causal completeness property prevents to attribute simultaneously physical properties to both sides (the impossibility of Madacena's physical interpretation). If particle physics continues to maintain its strong position within the cultural development which it enjoyed in the past, these facts will certainly leave footprints in the philosophy and history of physics.

Of immediate interest for the ongoing research are those consequences which affect the Higgs mechanism i.e. the claim that masses are generated from a massless theory by spontaneous symmetry-breaking. Although critique of such a physical interpretation can be traced back to the times of the original papers on the Higgs mechanism (section 6, subsection 3), the full understanding of the physics behind the metaphor of a mass creating Higgs mechanism was only possible in the recent Hilbert space formulation which replaces (and in a certain way extends) gauge theory in Krein space. The result is quite interesting: behind the Higgs model is nothing else than the uniquely defined renormalizable coupling of a massive vectormeson with a neutral (Hermitian field H) scalar matter field (instead of a charged (complex) scalar field i.e. massive scalar QED). There is a corresponding "Mexican hat potential" but instead of being a symmetry breaking input, it results from *induced* counter-terms whose numerical coefficients depend in addition to the vectormeson coupling also on mass ratios m_H/m.

As the induced counterterms are fixed by the new SLF Hilbert space setting for $s \geq 1$ couplings, there is the possibility that higher spin couplings are only consistent if they are accompanied by additional lower spin contributions (a dynamical counterpart of supersymmetry) in addition to the intrinsic stringlocal scalar Stueckelberg field which is a massive vectormesons inexorable escort. For abelian couplings this is not the case and for selfcouplings of vectormesons this question is presently under investigation. In QFT a property is understood if it can be reduced to its only foundational modular (quantum causal) localization principle and the issue of masses certainly does not belong o such properties. For nonabelian couplings the induction not only of new counterterms, but also of interactions involving new fields which were not part of the original Y-M field content, has not yet been excluded but does not appear plausible. It is of course always possible to add a coupling of massive Y-M gluons (their stringlocal Stueckelberg scalars always included) to a pointlike scalar neutral field (the Higgs.Kibble model) but a situation in which the coupled scalar Hermitian field H can be removed by Occam's razor is only of phenomenological interest and does not justify the for 4 decades claimed "fundamental significance" of H for the consistency of the Standard Model.

An unsolved problem of at least comparable importance is the derivation of gluon/quark confinement from the QCD coupling. As explained in the text, the problem amounts to establish the vanishing of *all correlation functions which contain would be physical stringlocal gluon or quark operators* in the limit of vanishing vectormeson mass[54]. Using the vectormeson mass m as a natural covariant infrared cutoff and the fact that the $m \to 0$ limes comes with logarithmic divergencies in perturbation theory, this result is very plausible. The computation with stringlocal covariant fields is only slightly more complicated than the corresponding (extended) YFS calculation for the scattering amplitude in massive QED in the mass-shell vectormeson limit (has also not been done in this natural infrared regularized way). The author encourages young physicists with sufficient computational stamina and perturbative experience to do these calculations.

The path from the dawn of QFT (the E-J conundrum) to the ongoing fundamental changes QFT under the roof of modular localizations has been a source of profound personal intellectual enjoyment, and hopfully the present article is able to transmit this to a wider audience.

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