

# Effect of hybridization in a two-band Hubbard superconductor: Strong coupling limit with extended s-wave gap

E.S. Caixeiro<sup>a,\*</sup>, A. Troper<sup>a,b</sup>

<sup>a</sup>Centro Brasileiro de Pesquisas Físicas, R. Xavier Sigaud 150, Rio de Janeiro 22290180, Brazil

<sup>b</sup>Universidade do Estado do Rio de Janeiro, R. São Francisco 524, Rio de Janeiro 20550013, Brazil

## Abstract

The critical temperatures  $T_c$ , for different hybridization strengths  $V$ , are obtained, within a Hubbard-I approximation, using an extended two-band Hubbard model. Here we considered an extended s-wave gap symmetry and a two-dimensional square lattice. The results show that for a fixed value of the attractive potential  $U$  and fixed hybridization  $V$  the gap raises for low temperatures, and diminishes as the temperature increases. Moreover, the gap behavior with hybridization is such that when  $V$  increases the gap diminishes. © 2007 Elsevier B.V. All rights reserved.

PACS: 75.30.-m; 75.30.Kz; 75.50.Ee; 77.80.-e; 77.84.Bw

Keywords: Hybridization; Superconductivity; Two-band

## 1. Introduction

The electronic hybridization has been extensively used to study the superconductivity in the framework of a BCS theory [1–4]. Since some high- $T_c$  materials, as well as superconducting heavy fermions, show strong electronic correlations [5,6], we apply here a Hubbard-I approximation [6] to obtain the zero temperature superconducting gap  $\Delta_0$  and the critical temperature  $T_c$  in a two-band Hubbard model, in the presence of a one-body hybridization. We consider an extended s-wave gap symmetry [7] and a constant and a  $k$ -dependent hybridization.

In order to study the dynamics of the carriers with correlations and the basic attractive interaction we consider an extended Hubbard Hamiltonian

$$H = \sum_{\langle ij \rangle \sigma} t_{ij}^d d_{i\sigma}^\dagger d_{j\sigma} + \sum_{\langle\langle ij \rangle\rangle \sigma} t_{ij}^s c_{i\sigma}^\dagger c_{j\sigma} - U \sum_{(ij)\sigma} n_{i,\sigma}^d n_{j,-\sigma}^d + \sum_{(ij)\sigma} V_{ij} (c_{i\sigma}^\dagger d_{j\sigma} + d_{i\sigma}^\dagger c_{j\sigma}), \quad (1)$$

\*Corresponding author. Tel.: +55 3121 2141 7285; fax: +55 3121 2141 7400.

E-mail address: caixa@if.uff.br (E.S. Caixeiro).

where  $c_{i\sigma}^\dagger(c_{i\sigma})$  and  $d_{i\sigma}^\dagger(d_{i\sigma})$  are the fermionic creation (annihilation) operator at site  $\mathbf{r}_i$  for the s and d bands, respectively. The lattice parameter for the square lattice is  $a = 1$ , and spin  $\sigma = \{\uparrow\downarrow\}$ .  $n_{i\sigma}^d = d_{i\sigma}^\dagger d_{i\sigma}$  is the density operator;  $t_{ij}^d$  and  $t_{ij}^s$  are the hopping integrals between sites  $i$  and  $j$  nearest-neighbors and next-nearest-neighbors for the s and d electrons.  $U$  is the attractive potential between the d electrons, which can be provided by the competition between on-site and nearest-neighbors site Coulomb interaction for some range of parameters [8].  $V_{ij}$  is the hybridization of the two bands, which may be  $k$ -dependent, arising from a non-local character of the mixing, or a constant one, representing an average hybridization over the Brillouin zone. Since the d-band density of states is much higher than the s one at the Fermi level, we assume throughout this work that the superconducting pairs originate at the d band. To obtain the superconductor order parameter, we calculate the equations of motion in the Wannier representation of the propagators  $\langle\langle d_{i\sigma}; d_{i\sigma}^\dagger \rangle\rangle_\omega$ ,  $\langle\langle d_{i,-\sigma}^\dagger; d_{i\sigma}^\dagger \rangle\rangle_\omega$ ,  $\langle\langle c_{i\sigma}; d_{i\sigma}^\dagger \rangle\rangle_\omega$  and  $\langle\langle c_{i,-\sigma}^\dagger; d_{i\sigma}^\dagger \rangle\rangle_\omega$  [6]. We calculate the equations of motion for the new generated Green's functions  $\langle\langle n_{j,-\sigma}^d; d_{i\sigma}^\dagger \rangle\rangle_\omega$  and  $\langle\langle n_{j\sigma}^d; d_{i,-\sigma}^\dagger \rangle\rangle_\omega$ , following the Hubbard-I approach and a mean-field treatment:  $(2U \sum_{jp} \langle\langle n_{p,-\sigma}^d; n_{j,-\sigma}^d; d_{i\sigma}^\dagger \rangle\rangle_\omega) \approx \tilde{U} \sum_p \langle\langle n_{p,-\sigma}^d; d_{i\sigma}^\dagger \rangle\rangle_\omega$

$d_{I\sigma}\rangle_{\omega} + 2\langle n^d \rangle \sum_p \Delta_{pi} \langle \langle d_{p,-\sigma}^\dagger; d_{I\sigma}^\dagger \rangle \rangle_{\omega}$ , where  $\tilde{U} = 2U\langle n^d \rangle$ , and  $\Delta_{ij} = U\langle d_i^\dagger d_{j,-\sigma}^\dagger \rangle$  is the superconducting order parameter. In the momentum space, and considering an extended s-wave gap symmetry, the order parameter is given by [7]:  $\Delta_{\mathbf{k}} = 2\Delta^{\max} |\cos(k_x) + \cos(k_y)|$ , where  $\Delta^{\max} = \Delta$  is the maximum gap amplitude and it is independent of momentum. From the above relations one obtains the gap self-consistent gap equation for an extended s-wave symmetry

$$\Delta = \frac{1}{N_s} \sum_{\mathbf{k}} 2\Delta\gamma_{\mathbf{k}} U \left[ \frac{F_{1\mathbf{k}} + G_{1\mathbf{k}} \tanh(\beta E_{1\mathbf{k}}/2)}{2E_{1\mathbf{k}}(E_{1\mathbf{k}}^2 - E_{2\mathbf{k}}^2)(E_{1\mathbf{k}}^2 - E_{3\mathbf{k}}^2)} \right] + \frac{1}{N_s} \sum_{\mathbf{k}} 2\Delta\gamma_{\mathbf{k}} U \left[ \frac{F_{2\mathbf{k}} + G_{2\mathbf{k}} \tanh(\beta E_{2\mathbf{k}}/2)}{2E_{2\mathbf{k}}(E_{2\mathbf{k}}^2 - E_{1\mathbf{k}}^2)(E_{2\mathbf{k}}^2 - E_{3\mathbf{k}}^2)} \right] + \frac{1}{N_s} \sum_{\mathbf{k}} 2\Delta\gamma_{\mathbf{k}} U \left[ \frac{F_{3\mathbf{k}} + G_{3\mathbf{k}} \tanh(\beta E_{3\mathbf{k}}/2)}{2E_{3\mathbf{k}}(E_{3\mathbf{k}}^2 - E_{1\mathbf{k}}^2)(E_{3\mathbf{k}}^2 - E_{2\mathbf{k}}^2)} \right], \quad (2)$$

where  $F_{i\mathbf{k}} = E_{i\mathbf{k}}^2 [E'_{1\mathbf{k}} + E'_{2\mathbf{k}} - \varepsilon_{s\mathbf{k}}] - \varepsilon_{s\mathbf{k}} E'_{1\mathbf{k}} E'_{2\mathbf{k}}$ ,  $G_{i\mathbf{k}} = E_{i\mathbf{k}} [E_{i\mathbf{k}}^2 - \varepsilon_{s\mathbf{k}}(E'_{1\mathbf{k}} + E'_{2\mathbf{k}}) + E'_{1\mathbf{k}} E'_{2\mathbf{k}}]$  and  $E_{1\mathbf{k}} = [-A_{\mathbf{k}}/3 + 2(|p_{\mathbf{k}}|/3)^{1/2} \cos \phi_{\mathbf{k}}/3]^{1/2}$ ,  $E_{2,3\mathbf{k}} = [-A_{\mathbf{k}}/3 - 2(|p_{\mathbf{k}}|/3)^{1/2} \cos(\phi_{\mathbf{k}} \pm \pi)/3]^{1/2}$ ,  $E'_{1,2\mathbf{k}} = -(\tilde{\varepsilon}_{\mathbf{k}} + \tilde{U})/2 \pm [(\tilde{\varepsilon}_{\mathbf{k}} + \tilde{U})^2 + 4\zeta_{\mathbf{k}}]^{1/2}/2$ ,  $\cos \phi_{\mathbf{k}} = -q_{\mathbf{k}}/[2(|p_{\mathbf{k}}|/3)^{3/2}]$ , and  $N_s$  is the number of sites in the lattice,  $\gamma_{\mathbf{k}} = |\cos(k_x) + \cos(k_y)|$ ,  $p_{\mathbf{k}} = (3B_{\mathbf{k}} - A_{\mathbf{k}}^2)/3$ ,  $q_{\mathbf{k}} = C_{\mathbf{k}} + 2A_{\mathbf{k}}^3/27 - A_{\mathbf{k}}B_{\mathbf{k}}/3$ ,  $\zeta_{\mathbf{k}} = 2\tilde{U}\varepsilon_{s\mathbf{k}} - \varepsilon_{d\mathbf{k}}\varepsilon_{s\mathbf{k}}$ ,  $A_{\mathbf{k}} = 2\tilde{V}_{\mathbf{k}} - \tilde{\varepsilon}_{\mathbf{k}}^2$ ,  $B_{\mathbf{k}} = \tilde{V}_{\mathbf{k}}^2 + 4\Delta_{\mathbf{k}}^2 \tilde{U}^2$ ,  $C_{\mathbf{k}} = 4\Delta_{\mathbf{k}}^2 [2\tilde{U}\varepsilon_{s\mathbf{k}} V_{\mathbf{k}}^2 - (\tilde{U}^2 \varepsilon_{s\mathbf{k}}^2 + V_{\mathbf{k}}^4)]$ ,  $\tilde{V}_{\mathbf{k}} = \varepsilon_{s\mathbf{k}}\varepsilon_{d\mathbf{k}} - V_{\mathbf{k}}^2 - \tilde{U}\varepsilon_{s\mathbf{k}}$ ,  $\tilde{\varepsilon}_{\mathbf{k}} = \tilde{U} - (\varepsilon_{s\mathbf{k}} + \varepsilon_{d\mathbf{k}})$ , and  $\beta = 1/k_B T$ .

To obtain  $\Delta_0$  and  $T_c$  for a specific value of  $V$  and  $U$ , Eq. (2) is solved self-consistently in the first Brillouin zone of the momentum space, of a square lattice, for  $\langle n^d \rangle = \langle n^s \rangle = 1.0$ , together with the dispersion relation:  $\varepsilon_{s\mathbf{k}} = -2t[\cos(k_x) + \cos(k_y)] + 4t_2 \cos(k_x) \cos(k_y) + \varepsilon_0$ . We start with initial values of  $\Delta$  and  $T$ , then Eq. (2) is iterated, until the limit  $T \rightarrow T_c$  is obtained. Here  $t$  is the hopping integral for the nearest-neighbors and  $t_2$  the hopping integral for the next-nearest-neighbors.  $\varepsilon_0$  is an adjustable parameter. Also, we introduce now the homothetic relation concerning the dispersion relation for s and d electrons [6]:  $\varepsilon_{d\mathbf{k}} = \alpha\varepsilon_{s\mathbf{k}}$ , where  $\alpha$  is a phenomenological parameter which plays the role of the effective band masses, and gives a fair approximation for the description of the two-band system. For the symmetric case,  $\varepsilon_{s\mathbf{k}}$  and  $\varepsilon_{d\mathbf{k}}$  are centered at the Fermi level.

In Fig. 1 we exhibit the behavior of the gap as a function of the hybridization for half-filled bands, and strong coupling  $U = 8t$ . Here we plot the case of a  $k$ -dependent hybridization, the results for a constant  $V$  being quite similar. We observe that, for small  $T$ , the gap increases slightly, whereas when  $T$  increases the gap decreases abruptly. Fig. 2a shows the behavior of  $\xi = 2\Delta_0/k_B T_c$  for the same set of parameters of Fig. 1, for a  $k$ -dependent hybridization. One sees that when  $U$  increases,  $\xi$  seems to stabilize in the region of validity of the approximation for  $U$ . Fig. 2b shows the behavior of  $\xi$  for different  $V$ , for both: a  $k$ -dependent and a constant hybridization, for

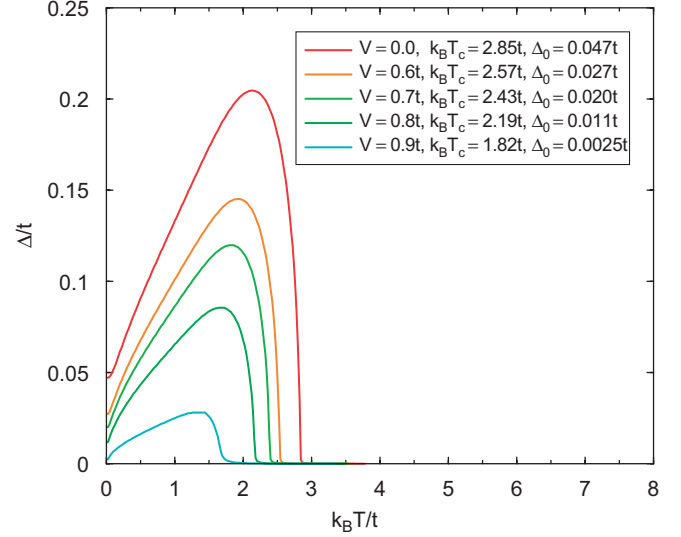


Fig. 1. Gap curves for different  $V$ . For a critical value  $V_c$  ( $\approx 1.0t$ ), the gap disappears.

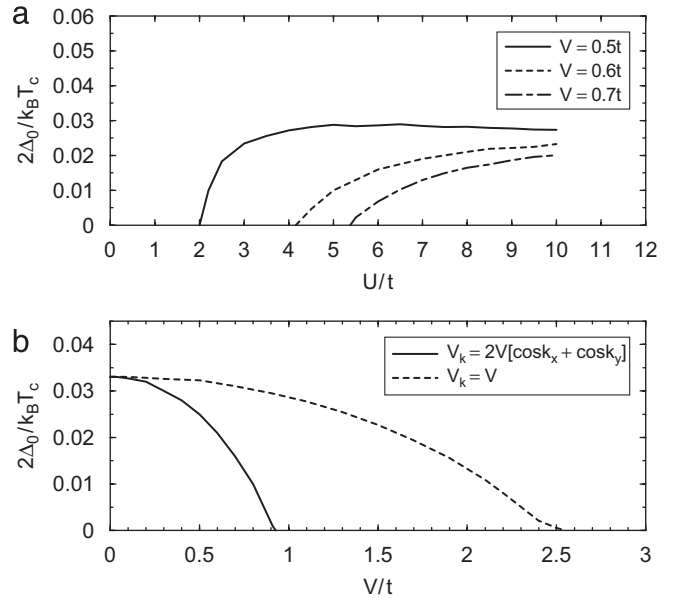


Fig. 2. In (a) we show the behavior of  $2\Delta_0/k_B T_c$  when  $U$  changes. In (b) we show the results of  $2\Delta_0/k_B T_c$  for two cases: a  $k$ -dependent and a constant  $V_{\mathbf{k}}$ .

$U = 8.0t$ . We want to point out that in general the hybridization increases with applied pressure. So, some of our results imply that  $\Delta_0$  decreases with increasing  $V$ . Therefore, our theoretical calculations can be used to understand the behavior of  $T_c$  under pressure of some two-band systems, e.g., heavy fermion intermetallic compounds.

## References

- [1] J.E. Hirsch, Phys. Lett. A 136 (1989) 163.
- [2] E. Dagotto, et al., Phys. Rev. B 49 (1994) 3548.

- [3] G.M. Japiassu, M. Continentino, A. Troper, *Phys. Rev. B* 45 (1992) 2986.
- [4] V.P. Ramunni, G.M. Japiassu, A. Troper, *Phys. C* 364 (2001) 190.
- [5] E.J. Calegari, S.G. Magalhães, A.A. Gomes, *European Phys. J.* 45 (2005) 485.
- [6] L.G. Sarasua, M. Continentino, *Phys. Rev. B* 65 (2002) 184503.
- [7] G.G.N. Angilella, et al., *Phys. Rev. B* 54 (1996) 15471.
- [8] J.E. Hirsch, D.J. Scalapino, *Phys. Rev. B* 32 (1985) 5639.