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Anisotropy of thermal stresses in confined dusty plasmas

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Abstract
Anisotropy of thermal stresses in confined dusty plasmas is considered. It is shown that in a multi-component low-temperature plasma containing electrons, ions and dust, the complicated dependence of the ion viscosity on ion temperature gradients leads to a plasma equilibrium state with anisotropic pressure. This pressure anisotropy can be of the order of the ion pressure in some limiting cases, in which the ion Larmor radius or the ion mean free path are of the order of the characteristic length of the plasma nonuniformity. For a sufficiently large dust number density, they contribute to the plasma pressure anisotropy and to its spatial dependence. Currently, it is not yet clear whether this equilibrium state is stable or not. Under these conditions, some convective plasma flows can arise in confinement devices. Therefore, this question needs special consideration.

1. Introduction

There are a number of electronic devices, e.g. thermal emission converters, plasma diodes and plasma surface deposition and etching devices, in which plasma is confined in magnetic configurations with various geometries [1–7]. In axially symmetric devices such as coaxial thermal emission converters, there are some special plasma flows related to the dependence of the plasma viscosity not only on spatial derivatives of plasma velocities (the Navier–Stokes kind of plasma viscosity), but also on the spatial derivatives of heat fluxes (the Burnett kind of plasma viscosity) [8,9]. These flows are analogous to the so-called residual poloidal and toroidal rotation in tokamaks [10–13], which have been confirmed experimentally on many occasions.

If there is no axial symmetry in such devices, e.g. in the case of the plane geometry of closed devices, such as parallelepipeds, it is clear that these flows are absent in the equilibrium state. Nevertheless, the viscosity dependence on the spatial derivatives of heat fluxes and, consequently, on spatial derivatives of plasma temperature, can lead to an anisotropy of thermal stresses in confined plasmas. Moreover, the complicated dependence of the ion viscosity on ion temperature gradients can lead to a plasma equilibrium state with anisotropic pressure. The pressure anisotropy can be of the order of the ion pressure in some limiting cases.

An important feature of plasma technological devices is the presence of natural contamination or dust [14, 15]. The presence of dust modifies plasma properties [16–18], especially in inhomogeneous plasma regions near walls [19, 20]. It was previously shown [21] that dust contamination of devices with pulsed external current circuits (e.g. thermal emission converters and plasma diodes) can induce the temperature-gradient driven flows of ions.

In this paper, we show that dust can also contribute to the anisotropy of thermal stresses in plasmas produced in such devices. This contribution appears through the ion viscosity dependence on the ion–dust collision frequencies. We found an equilibrium state for a magnetized dusty plasma with the anisotropic pressure; however, it is at present unclear whether this equilibrium state is stable. Indeed, under the physical conditions required for the equilibrium, convective flows can arise in the plasma, bringing up the necessity of further investigations of the problem.

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2. Starting equations

To show the origin of the induced viscosity forces in plasma dynamics, we proceed from the momentum equations [22–26] for particles of α-species

$$M_\alpha n_\alpha \frac{\mathrm{d}_\alpha \boldsymbol{V}_\alpha}{\mathrm{d}t} = -\nabla p_\alpha - \nabla\cdot\hat{\pi}_\alpha + e_\alpha n_\alpha \times \left(\boldsymbol{E} + \frac{1}{c}[\boldsymbol{V}_\alpha \times \boldsymbol{B}]\right) + \boldsymbol{R}_{u\alpha} + \boldsymbol{R}_{T\alpha}, \quad (1)$$

where the common notations are used, and α = e, i and n, stands for the plasma electrons, ions and neutrals, respectively. We take into account that the plasma quasineutrality condition is strongly affected by the presence of dust particles, and is given by $-en_\mathrm{e} + eZ_\mathrm{i}n_\mathrm{i} - eZ_\mathrm{d}n_\mathrm{d} = 0$, where n_e, n_i, n_d and $-e$, eZ_i, $-eZ_\mathrm{d}$ are the electron, ion and dust grain densities and charges, respectively. For the majority of dusty plasmas, the dust particles are highly (negatively) charged, $Z_\mathrm{d} \sim 10^2$–10^4, and their diameter d is approximately of a micrometer size, $d \sim 1\mu m$ [14, 15]. It follows from the quasineutrality condition that the density of the dust component may vary within the wide range $0 \leqslant Z_\mathrm{d}n_\mathrm{d} \leqslant Z_\mathrm{i}n_\mathrm{i}$ depending on the specific laboratory conditions. Thus, the electron density can also be changed within the range $Z_\mathrm{i}n_\mathrm{i} \geqslant n_\mathrm{e} \geqslant 0$.

The friction forces $\boldsymbol{R}_{\mathrm{i,e}}$, affecting electrons and ions, include collisions of these particles with neutrals and dust particles; we also take into account the thermal forces $\boldsymbol{R}_{T\alpha}$. The ion–dust collision frequencies are given by [21]

$$\nu_{\mathrm{id}} = \frac{4\sqrt{2\pi}\lambda e^4 Z_\mathrm{i}^2 Z_\mathrm{d}^2 n_\mathrm{d}}{3\sqrt{M_\mathrm{i}}T_\mathrm{i}^{3/2}}, \quad (2)$$

where the standard Braginskii [22] notations are used (in particular, λ is the Coulomb logarithm).

We consider a plasma nonuniform in the x, y-directions, which is confined by plane conducting walls at the coordinates $x = 0$ and $x = x_0$, such that the sheath temperatures are $T(0) = T_1$ and $T(x_0) = T_2 > T_1$, respectively. In contrast to [21], we take into account here the anisotropy of plasma pressures. The magnetic field is directed along the axis z, $\boldsymbol{B} = B\boldsymbol{e}_z$. The particle mean free paths $\lambda_{\mathrm{ii}} = v_{T_\mathrm{i}}/\nu_{\mathrm{ii}}$, $\lambda_{\mathrm{in}} = v_{T_\mathrm{i}}/\nu_{\mathrm{in}}$, and $\lambda_{\mathrm{id}} = v_{T_\mathrm{i}}/\nu_{\mathrm{id}}$ are assumed to be smaller than the system characteristic size x_0, namely, $\lambda_{\mathrm{i(i,n,d)}} < x_0$. For simplicity, we consider a two-dimensional problem with $\partial/\partial z = 0$ and neglect plasma collisions with neutrals. In this case, the equilibrium equations, summarized for electrons and ions, are

$$\frac{\partial}{\partial x}\left(p + \pi^\mathrm{i}_{xx}\right) + \frac{\partial}{\partial y}\pi^\mathrm{i}_{xy} - eZ_\mathrm{d}n_\mathrm{d}E_x - R_{T\mathrm{d}x} = 0, \quad (3)$$

$$\frac{\partial}{\partial y}\left(p + \pi^\mathrm{i}_{yy}\right) + \frac{\partial}{\partial x}\pi^\mathrm{i}_{xy} - eZ_\mathrm{d}n_\mathrm{d}E_y + \frac{1}{c}j_x B - R_{T\mathrm{d}y} = 0, \quad (4)$$

where j_x is the plasma current density component and $\boldsymbol{R}_{T\mathrm{d}}$ is the thermal force affecting ions as a result of their collisions with the dust. Generally speaking, the dust charge and number density can also be functions of the corresponding coordinates, namely $Z_\mathrm{d}(x, y)$ and $n_\mathrm{d}(x, y)$ [16–18]; for simplicity we neglect such a dependence here.

3. Thermal forces

We consider the thermal force $\boldsymbol{R}_{T\mathrm{d}}$ in the Braginskii form [22], with the replacement of electrons by ions and ions by dust, i.e. we change in the corresponding Braginskii formulas indices 'e' → 'i' and 'i' → 'd'. Thus, we have

$$\boldsymbol{R}_{T\mathrm{d}} = -\beta^{uT}_\perp \nabla_\perp T_\mathrm{i} - \beta^{uT}_\wedge \boldsymbol{h} \times \nabla_\perp T_\mathrm{i}, \quad (5)$$

where $\beta^{uT}_\perp = Z_\mathrm{i}n_\mathrm{i}(3.80\delta_\mathrm{d}^2 + 0.15)/\Delta_\mathrm{d}$, $\beta^{uT}_\wedge = Z_\mathrm{i}n_\mathrm{i}\delta_\mathrm{d}(1.5\delta_\mathrm{d}^2 + 0.88)/\Delta_\mathrm{d}$, $\delta_\mathrm{d} = \omega_{\mathrm{ci}}/\nu_{\mathrm{id}}$, $\omega_{\mathrm{ci}} = e_\mathrm{i}B/M_{\mathrm{ic}}$ is the ion cyclotron frequency and $\Delta_\mathrm{d} = \delta_d^4 + 7.48\delta_d^2 + 0.10$. We neglect in (5) the electron terms, which are small in comparison with that of the ion, at least as $\sqrt{M_\mathrm{e}/M_\mathrm{i}}$. Thus in (5) we have only two components of the thermal force, $R_{T\mathrm{d}x} = -\beta^{uT}_\perp \partial T_\mathrm{i}/\partial x + \beta^{uT}_\wedge \partial T_\mathrm{i}/\partial y$ and $R_{T\mathrm{d}y} = -\beta^{uT}_\perp \partial T_\mathrm{i}/\partial y - \beta^{uT}_\wedge \partial T_\mathrm{i}/\partial x$. For consistency with the nonuniformity assumptions, we consider $y_0 \gg x_0$, where y_0 is the characteristic scale of the system along the y direction. Such a situation can appear in plane thermal emission converters.

The components of the electric field, E_x and E_y, can be estimated from the ion momentum equation (1) $E_x \sim (1/e_\mathrm{i}n_\mathrm{i})\partial p_\mathrm{i}/\partial x$ and $E_y \sim (1/e_\mathrm{i}n_\mathrm{i})\partial p_\mathrm{i}/\partial y$. Taking into account the relation $Z_\mathrm{d}n_\mathrm{d} \ll Z_\mathrm{i}n_\mathrm{i}$, we can rewrite equations (3) and (4) in the form

$$\frac{\partial}{\partial x}\left(p + \pi^\mathrm{i}_{xx}\right) + \frac{\partial}{\partial y}\pi^\mathrm{i}_{xy} - R_{T\mathrm{d}x} = 0 \quad (6)$$

and

$$\frac{\partial}{\partial y}\left(p + \pi^\mathrm{i}_{yy}\right) + \frac{\partial}{\partial x}\pi^\mathrm{i}_{xy} + \frac{1}{c}j_x B - R_{T\mathrm{d}y} = 0. \quad (7)$$

In equations (6) and (7), the terms π^i_{xx} and π^i_{yy} contribute to the pressure anisotropy. Thus our goal is to calculate these terms. The components of the thermal forces $R_{T\mathrm{d}x}$ and $R_{T\mathrm{d}y}$ and the viscosity tensor component π^i_{xy} are responsible for the spatial dependence of the components of the pressure stress tensor $p + \pi^\mathrm{i}_{xx}$ and $p + \pi^\mathrm{i}_{yy}$.

4. Ion viscosity

The expression for the ion viscosity is rather cumbersome in the general case of the arbitrary parameter $\delta_\mathrm{i} = \omega_{\mathrm{ci}}/\nu_{\mathrm{ii}}$ [24–26]. It is presented in the appendix (see for the viscosity components equations (A1)–(A7), and for the viscosity coefficients equations (A8)–(A13)). The complicated structure of the ion viscosity is also explained in the appendix.

In the chosen plane geometry, the needed components of the viscosity tensor can be taken in the form [26]

$$\pi^\mathrm{i}_{xx} = \frac{1}{2}\eta^{(0)}_{(m)}\left(W^{(m)}_{xx} + W^{(m)}_{yy}\right) + \frac{1}{2}\eta^{(1)}_{(m)}\left(W^{(m)}_{xx} - W^{(m)}_{yy}\right) - \eta^{(3)}_{(m)}W^{(m)}_{xy}, \quad (8)$$

$$\pi^\mathrm{i}_{yy} = \frac{1}{2}\eta^{(0)}_{(m)}\left(W^{(m)}_{xx} + W^{(m)}_{yy}\right) - \frac{1}{2}\eta^{(1)}_{(m)}\left(W^{(m)}_{xx} - W^{(m)}_{yy}\right) + \eta^{(3)}_{(m)}W^{(m)}_{xy}, \quad (9)$$

and

$$\pi^{\rm i}_{xy} = \eta^{(1)}_{(m)} W^{(m)}_{xy} + \frac{1}{2}\eta^{(3)}_{(m)}\left(W^{(m)}_{xx} - W^{(m)}_{yy}\right). \quad (10)$$

Here

$$W^{(1)}_{xx} = \frac{2}{3}\left(2\frac{\partial V_{\rm ix}}{\partial x} - \frac{\partial V_{\rm iy}}{\partial y}\right) + \frac{4}{15 n_{\rm i} T_{\rm i}}\left(2\frac{\partial q_{\rm ix}}{\partial x} - \frac{\partial q_{\rm iy}}{\partial y}\right) - \frac{3}{50}\frac{\nu_{\rm ii} M_{\rm i}}{n_{\rm i}^2 T_{\rm i}^3}\left[\left(2q_{\rm ix}^2 - q_{\rm iy}^2\right) + \frac{7}{4}\left(2q_{\rm ix}q^*_{\rm ix} - q_{\rm iy}q^*_{\rm iy}\right) + \frac{63}{64}\left(2q^{*2}_{\rm ix} - q^{*2}_{\rm iy}\right)\right], \quad (11)$$

$$W^{(1)}_{yy} = \frac{2}{3}\left(2\frac{\partial V_{\rm iy}}{\partial y} - \frac{\partial V_{\rm ix}}{\partial x}\right) + \frac{4}{15 n_{\rm i} T_{\rm i}}\left(2\frac{\partial q_{\rm iy}}{\partial y} - \frac{\partial q_{\rm ix}}{\partial x}\right) - \frac{3}{50}\frac{\nu_{\rm ii} M_{\rm i}}{n_{\rm i}^2 T_{\rm i}^3}\left[\left(2q_{\rm iy}^2 - q_{\rm ix}^2\right) + \frac{7}{4}\left(2q_{\rm iy}q^*_{\rm iy} - q_{\rm ix}q^*_{\rm ix}\right) + \frac{63}{64}\left(2q^{*2}_{\rm iy} - q^{*2}_{\rm ix}\right)\right], \quad (12)$$

$$W^{(2)}_{xx} = \frac{2}{3 n_{\rm i} T_{\rm i}}\left\{\left(2q_{\rm ix}\frac{\partial \ln p_{\rm i}}{\partial x} - q_{\rm iy}\frac{\partial \ln p_{\rm i}}{\partial y}\right) - \left[2\frac{\partial\left(q_{\rm ix} - q^*_{\rm ix}\right)}{\partial x} - \frac{\partial\left(q_{\rm iy} - q^*_{\rm iy}\right)}{\partial y}\right]\right\} - \frac{2}{3 n_{\rm i} T_{\rm i}}\left[2\left(2q_{\rm ix} - q^*_{\rm ix}\right)\frac{\partial \ln T_{\rm i}}{\partial x} - \left(2q_{\rm iy} - q^*_{\rm iy}\right)\frac{\partial \ln T_{\rm i}}{\partial y}\right] - \frac{89}{1400}\frac{\nu_{\rm ii} M_{\rm i}}{n_{\rm i}^2 T_{\rm i}^3}\left[\left(2q_{\rm ix}^2 - q_{\rm iy}^2\right) + \frac{865}{356}\left(2q_{\rm ix}q^*_{\rm ix} - q_{\rm iy}q^*_{\rm iy}\right) + \frac{5915}{5696}\left(2q^{*2}_{\rm ix} - q^{*2}_{\rm iy}\right)\right], \quad (13)$$

$$W^{(2)}_{yy} = \frac{2}{3 n_{\rm i} T_{\rm i}}\left\{\left(2q_{\rm iy}\frac{\partial \ln p_{\rm i}}{\partial y} - q_{\rm ix}\frac{\partial \ln p_{\rm i}}{\partial x}\right) - \left[2\frac{\partial\left(q_{\rm iy} - q^*_{\rm iy}\right)}{\partial y} - \frac{\partial\left(q_{\rm ix} - q^*_{\rm ix}\right)}{\partial x}\right]\right\} - \frac{2}{3 n_{\rm i} T_{\rm i}}\left[2\left(2q_{\rm iy} - q^*_{\rm iy}\right)\frac{\partial \ln T_{\rm i}}{\partial y} - \left(2q_{\rm ix} - q^*_{\rm ix}\right)\frac{\partial \ln T_{\rm i}}{\partial x}\right] - \frac{89}{1400}\frac{\nu_{\rm ii} M_{\rm i}}{n_{\rm i}^2 T_{\rm i}^3}\left[\left(2q_{\rm iy}^2 - q_{\rm ix}^2\right) + \frac{865}{356}\left(2q_{\rm iy}q^*_{\rm iy} - q_{\rm ix}q^*_{\rm ix}\right) + \frac{5915}{5696}\left(2q^{*2}_{\rm iy} - q^{*2}_{\rm ix}\right)\right], \quad (14)$$

$$W^{(1)}_{xy} = \frac{\partial V_{\rm ix}}{\partial y} + \frac{\partial V_{\rm iy}}{\partial x} + \frac{2}{5 n_{\rm i} T_{\rm i}}\left(\frac{\partial q_{\rm ix}}{\partial y} + \frac{\partial q_{\rm iy}}{\partial x}\right) - \frac{9}{100}\frac{\nu_{\rm ii} M_{\rm i}}{n_{\rm i}^2 T_{\rm i}^3} \times\left[2q_{\rm ix}q_{\rm iy} + \frac{7}{4}\left(q_{\rm ix}q^*_{\rm iy} + q_{\rm iy}q^*_{\rm ix}\right) + \frac{63}{32}q^*_{\rm ix}q^*_{\rm iy}\right], \quad (15)$$

and

$$W^{(2)}_{xy} = \frac{1}{n_{\rm i} T_{\rm i}}\left[q_{\rm ix}\frac{\partial \ln p_{\rm i}}{\partial y} + q_{\rm iy}\frac{\partial \ln p_{\rm i}}{\partial x} - \frac{\partial\left(q_{\rm iy} - q^*_{\rm iy}\right)}{\partial x} - \frac{\partial\left(q_{\rm ix} - q^*_{\rm ix}\right)}{\partial y}\right] - \frac{1}{n_{\rm i} T_{\rm i}}\left[\left(2q_{\rm ix} - q^*_{\rm ix}\right)\frac{\partial \ln T_{\rm i}}{\partial y} + \left(2q_{\rm iy} - q^*_{\rm iy}\right)\frac{\partial \ln T_{\rm i}}{\partial x}\right] - \frac{267}{2800}\frac{\nu_{\rm ii} M_{\rm i}}{n_{\rm i}^2 T_{\rm i}^3}\left[2q_{\rm ix}q_{\rm iy} + \frac{865}{356}\left(q_{\rm ix}q^*_{\rm iy} + q_{\rm iy}q^*_{\rm ix}\right) + \frac{5915}{2848}q^*_{\rm ix}q^*_{\rm iy}\right]. \quad (16)$$

Heat fluxes $q_{\rm i}$ and $q^*_{\rm i}$ are presented in the appendix, equations (A19) and (A20), respectively.

5. Approximation for the strong magnetic field, $\nu_{\rm ii} \ll \omega_{\rm ci}$

To simplify these expressions, we use them in two approximations; first, when the ion–ion collision frequency $\nu_{\rm ii}$ is much smaller than the ion cyclotron frequency $\omega_{\rm ci}$, i.e. the parameter $\delta_{\rm i}$ is large. In this case we have, equations (A8)–(A12),

$$\eta^{(0)}_{(1)} = -0.96\frac{n_{\rm i} T_{\rm i}}{\nu_{\rm ii}}, \quad \eta^{(1)}_{(1)} = -\frac{3 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{10\omega^2_{\rm ci}}, \quad \eta^{(2)}_{(1)} = -\frac{6 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{5\omega^2_{\rm ci}},$$
$$\eta^{(3)}_{(1)} = \frac{n_{\rm i} T_{\rm i}}{2\omega_{\rm ci}}, \quad \eta^{(4)}_{(1)} = \frac{n_{\rm i} T_{\rm i}}{\omega_{\rm ci}}, \quad (17)$$

$$\eta^{(0)}_{(2)} = 0.24\frac{n_{\rm i} T_{\rm i}}{\nu_{\rm ii}}, \quad \eta^{(1)}_{(2)} = -\frac{9 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{100\omega^2_{\rm ci}}, \quad \eta^{(2)}_{(2)} = -\frac{9 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{25\omega^2_{\rm ci}},$$
$$\eta^{(3)}_{(2)} = 0, \quad \eta^{(4)}_{(2)} = 0. \quad (18)$$

Correspondingly, we find, equations (A21)–(A23),

$$\kappa^{\rm i}_{\perp} = \frac{2 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{M_{\rm i}\omega^2_{\rm ci}}, \quad \kappa^{\rm i}_{\wedge} = \frac{5 n_{\rm i} T_{\rm i}}{2 M_{\rm i}\omega_{\rm ci}},$$
$$\kappa^{*\rm i}_{\perp} = \frac{6 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{7 M_{\rm i}\omega^2_{\rm ci}}, \quad \kappa^{*i}_{\wedge} = 0. \quad (19)$$

The heat fluxes in the necessary approximation are

$$\boldsymbol{q}_{\rm i} = \boldsymbol{q}_{\rm i\perp} = -\frac{2 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{M_{\rm i}\omega^2_{\rm ci}}\nabla_{\perp} T_{\rm i} + \frac{5 n_{\rm i} T_{\rm i}}{2 M_{\rm i}\omega_{\rm ci}}\boldsymbol{h}\times\nabla T_{\rm i} \quad (20)$$

and

$$\boldsymbol{q}^*_{\rm i} = \boldsymbol{q}^*_{\rm i\perp} = -\frac{6 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{7 M_{\rm i}\omega^2_{\rm ci}}\boldsymbol{\nabla}_{\perp} T_{\rm i}. \quad (21)$$

From the Braginskii equations in the case of the quasistationary plasma in the absence of macroscopic flows, we have an equation for the spatial dependence of the ion temperature following from $\boldsymbol{\nabla}_{\perp}\cdot\boldsymbol{q}_{\rm i\perp} = 0$,

$$\boldsymbol{\nabla}_{\perp}\cdot\left[\frac{n_{\rm i} T_{\rm i}}{M_{\rm i}\omega_{\rm ci}}\left(\frac{2\nu_{\rm ii}}{\omega_{\rm ci}}\boldsymbol{\nabla}_{\perp} T_{\rm i} - \frac{5}{2}\boldsymbol{h}\times\boldsymbol{\nabla}_{\perp} T_{\rm i}\right)\right] = 0. \quad (22)$$

Assuming $x_0 \ll \partial/\partial y$, we find

$$\frac{\partial^2 T_{\rm i}}{\partial x^2} + \frac{\partial T_{\rm i}}{\partial x}\left(2\frac{\partial \ln n_{\rm i}}{\partial x} - 2\frac{\partial \ln B}{\partial x} - \frac{1}{2}\frac{\partial \ln T_{\rm i}}{\partial x}\right) = 0. \quad (23)$$

In addition, we suppose that the ion Larmor radius, $\rho_{\rm i} = v_{T\rm i}/\omega_{\rm ci}$, is smaller than the characteristic length x_0, $\rho_{\rm i} < x_0$. Thus we have a substantial simplification of equations (8)–(10).

$$\pi^{\rm i}_{xx} = -0.48\frac{n_{\rm i} T_{\rm i}}{\nu_{\rm ii}}\left(W^{(1)}_{xx} + W^{(1)}_{yy}\right) + 0.12\frac{n_{\rm i} T_{\rm i}}{\nu_{\rm ii}}\left(W^{(2)}_{xx} + W^{(2)}_{yy}\right) - \frac{3 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{20\omega^2_{\rm ci}}\left(W^{(1)}_{xx} - W^{(1)}_{yy}\right) - \frac{9 n_{\rm i} T_{\rm i}\nu_{\rm ii}}{200\omega^2_{\rm ci}}\left(W^{(2)}_{xx} - W^{(2)}_{yy}\right)$$

$$- \frac{n_i T_i}{2\omega_{ci}} W_{xy}^{(1)}, \qquad (24)$$

$$\pi_{yy}^{i} = -0.48 \frac{n_i T_i}{\nu_{ii}} \left(W_{xx}^{(1)} + W_{yy}^{(1)}\right) + 0.12 \frac{n_i T_i}{\nu_{ii}} \left(W_{xx}^{(2)} + W_{yy}^{(2)}\right) + \frac{3 n_i T_i \nu_{ii}}{20\omega_{ci}^2} \left(W_{xx}^{(1)} - W_{yy}^{(1)}\right) + \frac{9 n_i T_i \nu_{ii}}{200\omega_{ci}^2} \left(W_{xx}^{(2)} - W_{yy}^{(2)}\right) + \frac{n_i T_i}{2\omega_{ci}} W_{xy}^{(1)}, \qquad (25)$$

$$\pi_{xy}^{i} = -\frac{3 n_i T_i \nu_{ii}}{10\omega_{ci}^2} W_{xy}^{(1)} - \frac{9 n_i T_i \nu_{ii}}{100\omega_{ci}^2} W_{xy}^{(2)} + \frac{n_i T_i}{4\omega_{ci}} \left(W_{xx}^{(1)} - W_{yy}^{(1)}\right), \qquad (26)$$

which leads to

$$\pi_{xx}^{i} = -\frac{n_i T_i}{2M_i\omega_{ci}^2} \frac{\partial T_i}{\partial x} \left[0.74 \frac{\partial \ln T_i}{\partial x} + \frac{\partial}{\partial x} \left(\ln \frac{\partial T_i}{\partial x}\right) + \frac{\partial \ln n_i}{\partial x} - \frac{\partial \ln B}{\partial x}\right], \qquad (27)$$

$$\pi_{yy}^{i} = \frac{n_i T_i}{2M_i\omega_{ci}^2} \frac{\partial T_i}{\partial x} \left[1.26 \frac{\partial \ln T_i}{\partial x} + \frac{\partial}{\partial x} \left(\ln \frac{\partial T_i}{\partial x}\right) + \frac{\partial \ln n_i}{\partial x} - \frac{\partial \ln B}{\partial x}\right], \qquad (28)$$

$$\pi_{xy}^{i} = -\frac{0.1\nu_{ii} n_i T_i}{M_i\omega_{ci}^3} \frac{\partial T_i}{\partial x} \left[\frac{\partial}{\partial x} \left(\ln \frac{\partial T_i}{\partial x}\right) + 3.8 \frac{\partial \ln n_i}{\partial x} - \frac{\partial \ln B}{\partial x} - 5.4 \frac{\partial \ln T_i}{\partial x}\right]. \qquad (29)$$

Expressing the derivative $\partial \ln B/\partial x$ from equation (23)

$$\frac{\partial \ln B}{\partial x} = \frac{1}{2} \frac{\partial}{\partial x} \left(\ln \frac{\partial T_i}{\partial x}\right) + \frac{\partial \ln n_i}{\partial x} - \frac{1}{4} \frac{\partial \ln T_i}{\partial x}, \qquad (30)$$

we find, from equations (27)–(29),

$$\pi_{xx}^{i} = -\frac{n_i T_i}{2M_i\omega_{ci}^2} \frac{\partial T_i}{\partial x} \left[\frac{\partial \ln T_i}{\partial x} + \frac{1}{2} \frac{\partial}{\partial x} \left(\ln \frac{\partial T_i}{\partial x}\right)\right], \qquad (31)$$

$$\pi_{yy}^{i} = \frac{n_i T_i}{2M_i\omega_{ci}^2} \frac{\partial T_i}{\partial x} \left[1.5 \frac{\partial \ln T_i}{\partial x} + \frac{1}{2} \frac{\partial}{\partial x} \left(\ln \frac{\partial T_i}{\partial x}\right)\right], \qquad (32)$$

$$\pi_{xy}^{i} = -\frac{0.1\nu_{ii} n_i T_i}{M_i\omega_{ci}^3} \frac{\partial T_i}{\partial x} \left[\frac{1}{2} \frac{\partial}{\partial x} \left(\ln \frac{\partial T_i}{\partial x}\right) + 2.8 \frac{\partial \ln n_i}{\partial x} - 5.2 \frac{\partial \ln T_i}{\partial x}\right]. \qquad (33)$$

Equations (31)–(33) define the equilibrium state with the pressure anisotropy, $p + \pi_{xx}^{i} \neq p + \pi_{yy}^{i}$, and the spatial dependence of the anisotropic pressure in equations (6) and (7) due to the term π_{xy}^{i} for the case of the disconnected external current j_x and the negligibly small dust number density. The pressure anisotropy, $\Delta p = p + \pi_{xx}^{i} - \left(p + \pi_{yy}^{i}\right) = \pi_{xx}^{i} - \pi_{yy}^{i}$, is

$$\Delta p = -\frac{n_i T_i}{2M_i\omega_{ci}^2} \frac{\partial T_i}{\partial x} \left[2.5 \frac{\partial \ln T_i}{\partial x} + \frac{\partial}{\partial x} \left(\ln \frac{\partial T_i}{\partial x}\right)\right]. \qquad (34)$$

For the sufficiently large ion Larmor radius, $\rho_i \lesssim x_0$, the anisotropy pressure, Δp, can be of the order of the ion pressure, $\Delta p \lesssim p_i$, as can be seen from equation (34).

6. Approximation for the strong collision frequency, $\nu_{ii} \gg \omega_{ci}$

In the case $\nu_{ii} \gg \omega_{ci}$, we obtain from equations (A8)–(A12)

$$\eta_{(1)}^{(0)} = \eta_{(1)}^{(1)} = \eta_{(1)}^{(2)} = -0.96 \frac{n_i T_i}{\nu_{ii}}, \quad \left\{\eta_{(1)}^{(3)}; \eta_{(1)}^{(4)}\right\} = \{2.04; 1.02\} \frac{n_i T_i \omega_{ci}}{\nu_{ii}^2}, \qquad (35)$$

$$\eta_{(2)}^{(0)} = \eta_{(2)}^{(1)} = \eta_{(2)}^{(2)} = 0.24 \frac{n_i T_i}{\nu_{ii}}, \quad \left\{\eta_{(2)}^{(3)}; \eta_{(2)}^{(4)}\right\} = -\{0.82; 0.41\} \frac{n_i T_i \omega_{ci}}{\nu_{ii}^2} \qquad (36)$$

and from equations (A19)–(A23)

$$\kappa_{\parallel}^{i} = 3.91 \frac{n_i T_i}{\nu_{ii} M_i}, \qquad \kappa_{\perp}^{i} = \frac{3.91 n_i T_i}{\nu_{ii} M_i}, \qquad \kappa_{\wedge}^{i} = 6.84 \frac{n_i T_i \omega_{ci}}{\nu_{ii}^2 M_i}, \qquad (37)$$

$$\kappa_{\parallel}^{*i} = -1.04 \frac{n_i T_i}{\nu_{ii} M_i}, \qquad \kappa_{\perp}^{*i} = -1.04 \frac{n_i T_i}{\nu_{ii} M_i}, \qquad \kappa_{\wedge}^{*i} = -2.63 \frac{n_i T_i \omega_{ci}}{\nu_{ii}^2 M_i}, \qquad (38)$$

$$\boldsymbol{q}_i = -\frac{3.91 n_i T_i}{\nu_{ii} M_i} \nabla_{\perp} T_i + 6.84 \frac{n_i T_i \omega_{ci}}{\nu_{ii}^2 M_i} \boldsymbol{h} \times \nabla T_i, \qquad (39)$$

and

$$\boldsymbol{q}_i^{*} = 1.04 \frac{n_i T_i}{\nu_{ii} M_i} \nabla_{\perp} T_i - 2.63 \frac{n_i T_i \omega_{ci}}{\nu_{ii}^2 M_i} \boldsymbol{h} \times \nabla T_i. \qquad (40)$$

The temperature evolution equation is as follows:

$$\frac{\partial}{\partial x} \left(T_i^{5/2} \frac{\partial T_i}{\partial x}\right) = 0. \qquad (41)$$

Its solution is $T_i(x) = (C_1 x + C_2)^{2/7}$, with $C_1 = \left(T_2^{7/2} - T_1^{7/2}\right)/x_0$ and $C_2 = T_1^{7/2}$.

The components of the viscosity tensor are

$$\pi_{xx}^{i} = \frac{n_i T_i}{\nu_{ii}^2 M_i} \frac{\partial T_i}{\partial x} \left(1.92 \frac{\partial \ln T_i}{\partial x} - 1.26 \frac{\partial \ln n_i}{\partial x}\right), \qquad (42)$$

$$\pi_{yy}^{i} = -\frac{n_i T_i}{\nu_{ii}^2 M_i} \frac{\partial T_i}{\partial x} \left(1.75 \frac{\partial \ln T_i}{\partial x} - 0.63 \frac{\partial \ln n_i}{\partial x}\right), \qquad (43)$$

and

$$\pi_{xy}^{i} = \frac{n_i T_i \omega_{ci}}{\nu_{ii}^3 M_i} \frac{\partial T_i}{\partial x} \left(-4.90 \frac{\partial \ln B}{\partial x} + 8.13 \frac{\partial \ln n_i}{\partial x} - 17.82 \frac{\partial \ln T_i}{\partial x}\right). \qquad (44)$$

We conclude, analogously to the end of section 5, that the equilibrium state has a pressure anisotropy given by

$$\Delta p = \frac{n_i T_i}{\nu_{ii}^2 M_i} \frac{\partial T_i}{\partial x} \left(3.67 \frac{\partial \ln T_i}{\partial x} - 1.89 \frac{\partial \ln n_i}{\partial x}\right). \qquad (45)$$

In the case of the sufficiently large ion mean free path, $\lambda_{ii} \lesssim x_0$, the pressure anisotropy can be of the order of the ion pressure, $\Delta p \lesssim p_i$.

7. The role of dust in plasma equilibrium

The role of dust in equations (3) and (4) (or in equations (6) and (7)) is important only in the case when the ion–dust collision frequency, $\nu_{\rm id}$, is larger than the ion–ion collision frequency, $\nu_{\rm ii}$, $\nu_{\rm id} \gtrsim \nu_{\rm ii}$, i.e. for the sufficiently large dust number density, $n_{\rm d} \gtrsim n_{\rm i}Z_{\rm i}^2/Z_{\rm d}^2$, when ion collisions with dust are more relevant than ion–ion collisions. In this case, to obtain transport equations, in particular the ion viscosity, it is necessary to take into account ion–dust collisions in the collision term in the Boltzmann kinetic equation. This situation is similar to electron–ion collisions in the electron kinetic equation, see Braginskii [22]. These transport coefficients were not calculated taking into account the ion–dust collisions. However, according to Braginskii, the structure of viscosity in this case is the same except for the numerical coefficients. Then, for qualitative evaluations, we can use all expressions obtained above replacing $\nu_{\rm ii} \to \nu_{\rm id}$ and considering numerical coefficients to be of the order of unity for the case $n_{\rm d} \gtrsim n_{\rm i}Z_{\rm i}^2/Z_{\rm d}^2$.

Then, assuming $\omega_{\rm ci} \gg \nu_{\rm id}$, we find $\beta_{\perp}^{uT} = 3.80Z_{\rm i}n_{\rm i}\nu_{\rm id}^2/\omega_{\rm ci}^2$ and $\beta_{\wedge}^{uT} = 1.5Z_{\rm i}n_{\rm i}\nu_{\rm id}/\omega_{\rm ci}$. Correspondingly, we have

$$R_{T{\rm d}x} = -3.80Z_{\rm i}n_{\rm i}\frac{\nu_{\rm id}^2}{\omega_{\rm ci}^2}\frac{\partial T_{\rm i}}{\partial x}, \qquad R_{T{\rm d}y} = -1.5Z_{\rm i}n_{\rm i}\frac{\nu_{\rm id}}{\omega_{\rm ci}}\frac{\partial T_{\rm i}}{\partial x}. \tag{46}$$

For the case $\omega_{\rm ci} \ll \nu_{\rm id}$, we arrive at $\beta_{\perp}^{uT} = 1.5Z_{\rm i}n_{\rm i}$ and $\beta_{\wedge}^{uT} = 8.8Z_{\rm i}n_{\rm i}\omega_{\rm ci}/\nu_{\rm id}$. Consequently, we have

$$R_{T{\rm d}x} = -1.5Z_{\rm i}n_{\rm i}\frac{\partial T_{\rm i}}{\partial x}, \tag{47}$$

and

$$R_{T{\rm d}y} = -8.8Z_{\rm i}n_{\rm i}\frac{\omega_{\rm ci}}{\nu_{\rm id}}\frac{\partial T_{\rm i}}{\partial x}. \tag{48}$$

Thus the role of dust in equations (6) and (7) leads to replacing $\nu_{\rm ii} \to \nu_{\rm id}$ in the viscosity components $\pi_{xx}^{\rm i}$, $\pi_{yy}^{\rm i}$, and $\pi_{xy}^{\rm i}$, considering numerical coefficients to be of order unity in these components, and giving rise to the thermal force terms related to the ion–dust interaction in these equations. Estimations for the pressure anisotropy, equations (34) and (45), are qualitatively the same. It means that dust contributes to the pressure anisotropy and to the spatial dependence of the anisotropic pressure in the case of the sufficiently large dust number density, $n_{\rm d} \gtrsim n_{\rm i}Z_{\rm i}^2/Z_{\rm d}^2$.

Here, we provide some characteristic parameters for thermal emission converters [7] and make the corresponding estimations. Usually, the plasma in these converters consists of $\rm Cs_{55}^{133}$, $\rm K_{19}^{39}$ or $\rm Ru_{44}^{101}$. Typical parameters of converters are the following: temperatures of emitters T_2 and collectors T_1, the plasma pressure p, the magnetic field B are $T_2 = 1600$–$2400\,$K, $T_1 = 800$–$1100\,$K, $p = 0.15$–$70\,$Pa, $B \leqslant 10^2$. For example, we can find the ratio $\nu_{\rm ii}/\omega_{\rm ci}$ for the plasma which consists of $\rm K_{19}^{39}$: $\nu_{\rm ii}/\omega_{\rm ci} \geqslant 10$. Thus, to describe the plasma equilibrium state (the plasma pressure anisotropy) in thermal emission converters with the above-mentioned macroscopic parameters, we can use results of section 6 with the factor $\delta_{\rm i} \sim 0.1$. Furthermore, for devices where the typical ion number density is $n_{\rm i} \sim 10^{13}\,{\rm cm}^{-3}$ and the neutral gas density is of order $n_{\rm n} \sim 10^{16}\,{\rm cm}^{-3}$ [7], the influence of plasma–neutral collisions can be neglected. The ion $\nu_{\rm in}$ and electron $\nu_{\rm en}$ collision frequencies with neutrals can be obtained from the kinetic theory [28]. If the mass $M_{\rm n}$ of the neutral particle is greater than the ion mass $M_{\rm i}$, the frequency is given by

$$\nu_{\rm in} \approx \pi\sigma^2 n_{\rm n} v_{T_{\rm i}}, \tag{49}$$

where σ is the diameter of the neutral atom. These collisions, however, can become important when the ion number density falls lower than $10^{-5}\,n_{\rm n}$, as direct comparison of the (A13) and (49) can justify.

Turning to the possible applications of results obtained here, for example, we find a large variety of thermal emission converters such as the close-spaced high vacuum, low-pressure diodes, high-pressure cesium and others [7, 29]. The voltage–current characteristics of such devices can depend on plasma pressure in some conditions [3, 7, 29]. The effect of the natural anisotropy of plasma pressure on voltage–current characteristics of the thermal emission converters, and as a result on the outgoing characteristics such an efficiency, is a completely unknown subject. In addition, plasma convective flows resulting from the pressure anisotropy can also affect the characteristics of the thermal emission converters and analogous devices. Thus problems arise in finding out the natural pressure anisotropy in such devices, possible convective flows resulting from this anisotropy, and in studying these effects and their possible effect on outgoing characteristics of closed plasma devices with temperature gradients and transverse magnetic field.

8. Conclusion

To conclude, we demonstrated that in a multi-component low-temperature plasma containing electrons, ions and dust, the complicated dependence of the ion viscosity on the ion temperature gradients leads to a plasma equilibrium state with anisotropic pressure. This plasma pressure anisotropy can be of the order of the ion pressure in some limiting cases, in which the ion Larmor radius or the ion mean free path are of the order of the characteristic length of the plasma nonuniformity. The cases considered are (i) strong magnetic field, when the ion cyclotron frequency is larger than the ion–ion collision frequency, and (ii) strong collisional plasma, when these frequencies are in the contrary relation.

For sufficiently large dust number density, $n_{\rm d} \gtrsim n_{\rm i}Z_{\rm i}^2/Z_{\rm d}^2$, dust effectively contributes to the plasma pressure anisotropy and to its spatial dependence. For qualitative evaluations, one can use all expressions obtained only for the case only ion–ion collisions replacing the ion–ion collision frequency by the ion–dust collision frequency and considering numerical coefficients to be of the order of unity in the corresponding expressions.

The obvious next step after having established the equilibrium is to check its stability, especially with respect to perturbations of convective types. Also, it would be useful to proceed with a self-consistent calculation for some systems of interest. Since the x-directed electric field is a key element, it would seem that the physical sheath and flow to the electrodes might have to be handled in more detail. It might also be useful to examine the infinite uniform plasma with no electric field to see if any Hall-type instabilities are to be found. It seems likely that, without significant electric fields from the electrodes, any such instabilities would require rather high magnetic fields to reach their threshold. It seems physically reasonable that for sufficiently weak magnetic fields one might expect some anisotropy but not enough to force breaking of the assumed symmetry of uniformity in the y-direction. In that case a point that should be addressed is how these anisotropies might be measured experimentally. Indeed, measuring ion kinetics in the plasma is not easy; perhaps the video tracking of dust motion might provide a useful diagnostic. As we observe, with sufficient anisotropy there is the likelihood of a convective instability for which case no stationary state exists. With such symmetry breaking one would expect y-modulation of the ion Hall current, which ought to be measurable. A further theory/model might give threshold values and frequency and y-wavenumber for the symmetry-breaking instability.

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Appendix A. General expressions for the collisional ion viscosity and ion heat flux

Here we present general expressions for the ion viscosity and ion heat flux for the arbitrary parameter $\delta_{\rm i} = \omega_{\rm ci}/\nu_{\rm ii}$ (see [26]).

$$\pi^{\rm i}_{ik} = \eta^{(p)}_{(m)} W^{(m)}_{(p)ik}, \tag{A1}$$

where $m = 1, 2$ and $p = 0, 1, 2, 3, 4$.

$$W^{(m)}_{(0)ik} = \frac{3}{2}\left(h_i h_k - \frac{1}{3}\delta_{ik}\right)\left(h_\mu h_\nu - \frac{1}{3}\delta_{\mu\nu}\right) W^{(m)}_{\mu\nu}, \tag{A2}$$

$$W^{(m)}_{(1)ik} = \left(\hat{\delta}_{i\mu}\hat{\delta}_{k\nu} - \frac{1}{2}\hat{\delta}_{ik}\hat{\delta}_{\mu\nu}\right) W^{(m)}_{\mu\nu}, \tag{A3}$$

$$W^{(m)}_{(2)ik} = \left(\hat{\delta}_{i\mu} h_\nu h_k + \hat{\delta}_{k\nu} h_i h_\mu\right) W^{(m)}_{\mu\nu}, \tag{A4}$$

$$W^{(m)}_{(3)ik} = \frac{1}{2}\left(\hat{\delta}_{i\mu}\varepsilon_{k\gamma\nu} + \hat{\delta}_{k\nu}\varepsilon_{i\gamma\mu}\right) h_\gamma W^{(m)}_{\mu\nu}, \tag{A5}$$

$$W^{(m)}_{(4)ik} = \left(h_i h_\mu \varepsilon_{k\gamma\nu} + h_k h_\nu \varepsilon_{i\gamma\mu}\right) h_\gamma W^{(m)}_{\mu\nu}, \tag{A6}$$

$$\hat{\delta}_{i\mu} = \delta_{i\mu} - h_i h_\mu. \tag{A7}$$

$\delta_{i\mu}$ is the Kronecker symbol, and $\varepsilon_{k\gamma\nu}$ is the antisymmetric tensor with the components 0 or 1.

The coefficients $\eta^{(p)}_{(m)}$ in equation (13) are the following:

$$\eta^{(0)}_{(1)} = -0.96\frac{n_{\rm i}T_{\rm i}}{\nu_{\rm ii}}, \qquad \eta^{(1)}_{(1)} = \eta^{(2)}_{(1)}(2\delta_{\rm i}),$$

$$\eta^{(2)}_{(1)}(\delta_{\rm i}) = -\frac{n_{\rm i}T_{\rm i}}{\nu_{\rm ii}\Delta(\delta_{\rm i})}\left(\frac{6}{5}\delta_{\rm i}^2 + 2.23\right), \tag{A8}$$

$$\eta^{(3)}_{(1)} = \eta^{(4)}_{(1)}(2\delta_{\rm i}), \quad \eta^{(4)}_{(1)}(\delta_{\rm i}) = \frac{n_{\rm i}T_{\rm i}\delta_{\rm i}}{\nu_{\rm ii}\Delta(\delta_{\rm i})}\left(\delta_{\rm i}^2 + 2.38\right), \tag{A9}$$

$$\eta^{(0)}_{(2)} = 0.24\frac{n_{\rm i}T_{\rm i}}{\nu_{\rm ii}}, \qquad \eta^{(1)}_{(2)} = \eta^{(2)}_{(2)}(2\delta_{\rm i}),$$

$$\eta^{(2)}_{(2)}(\delta_{\rm i}) = -\frac{n_{\rm i}T_{\rm i}}{\nu_{\rm ii}\Delta(\delta_{\rm i})}\left(\frac{9}{25}\delta_{\rm i}^2 - 0.55\right), \tag{A10}$$

$$\eta^{(3)}_{(2)} = \eta^{(4)}_{(2)}(2\delta_{\rm i}), \qquad \eta^{(4)}_{(2)}(\delta_{\rm i}) = -0.96\frac{n_{\rm i}T_{\rm i}\delta_{\rm i}}{\nu_{\rm ii}\Delta(\delta_{\rm i})}, \tag{A11}$$

where

$$\Delta(\delta_{\rm i}) = \delta_{\rm i}^4 + 4.05\delta_{\rm i} + 2.33 \tag{A12}$$

and

$$\nu_{\rm ii} = \frac{4\sqrt{\pi}\lambda e^4 Z_{\rm i}^4 n_{\rm i}}{3\sqrt{M_{\rm i}}T_{\rm i}^{3/2}}. \tag{A13}$$

The ratio $\nu_{\rm ii}/\nu_{\rm id}$ is

$$\nu_{\rm ii}/\nu_{\rm id} = \frac{Z_{\rm i}^2 n_{\rm i}}{\sqrt{2}Z_{\rm d}^2 n_{\rm d}}, \tag{A14}$$

i.e. it can be small in the case

$$1 \ll \frac{Z_{\rm i}n_{\rm i}}{Z_{\rm d}n_{\rm d}} \ll \frac{Z_{\rm d}}{Z_{\rm i}}. \tag{A15}$$

The coefficients $\eta^{(p)}_{(m)}$ in equations (19)–(23) are obtained in the approximation $\nu_{\rm ii}/\nu_{\rm id} \gg 1$.

The functions $W^{(m)}_{\mu\nu}$ in equations (13)–(17) are

$$W^{(1)}_{\mu\nu} = \langle\nabla \boldsymbol{V}_{\rm i}\rangle_{\mu\nu} + \frac{2}{5n_{\rm i}T_{\rm i}}\langle\nabla \boldsymbol{q}_{\rm i}\rangle_{\mu\nu} - \frac{9}{100}\frac{\nu_{\rm ii}M_{\rm i}}{n_{\rm i}^2T_{\rm i}^3} \times \left(\langle\boldsymbol{q}_{\rm i}\boldsymbol{q}_{\rm i}\rangle_{\mu\nu} + \frac{7}{4}\langle\boldsymbol{q}_{\rm i}\boldsymbol{q}_{\rm i}^*\rangle_{\mu\nu} + \frac{63}{64}\langle\boldsymbol{q}_{\rm i}^*\boldsymbol{q}_{\rm i}^*\rangle_{\mu\nu}\right) \tag{A16}$$

and

$$W^{(2)}_{\mu\nu} = \frac{1}{n_{\rm i}T_{\rm i}}\left[\langle\boldsymbol{q}_{\rm i}\nabla\ln p_{\rm i}\rangle_{\mu\nu} - \langle\nabla(\boldsymbol{q}_{\rm i} - \boldsymbol{q}_{\rm i}^*)\rangle_{\mu\nu} - \langle(2\boldsymbol{q}_{\rm i} - \boldsymbol{q}_{\rm i}^*)\nabla\ln T_{\rm i}\rangle_{\mu\nu}\right] - \frac{267}{2800}\frac{\nu_{\rm ii}M_{\rm i}}{n_{\rm i}^2T_{\rm i}^3}\left(\langle\boldsymbol{q}_{\rm i}\boldsymbol{q}_{\rm i}\rangle_{\mu\nu} + \frac{865}{356}\langle\boldsymbol{q}_{\rm i}\boldsymbol{q}_{\rm i}^*\rangle_{\mu\nu} + \frac{5915}{5696}\langle\boldsymbol{q}_{\rm i}^*\boldsymbol{q}_{\rm i}^*\rangle_{\mu\nu}\right). \tag{A17}$$

Here

$$\langle\boldsymbol{AB}\rangle_{\mu\nu} = A_\mu B_\nu + A_\nu B_\mu - \frac{2}{3}\boldsymbol{A}\cdot\boldsymbol{B} \tag{A18}$$

and $\boldsymbol{q}_{\rm i}$ and $\boldsymbol{q}_{\rm i}^*$ are the ion heat flux and its analogue, respectively. Expressions for these values are

$$\boldsymbol{q}_{\rm i} = -\kappa^{\rm i}_{\parallel}\nabla_{\parallel}T_{\rm i} - \kappa^{\rm i}_{\perp}\nabla_{\perp}T_{\rm i} + \kappa^{\rm i}_{\wedge}\boldsymbol{h}\times\nabla T_{\rm i} \tag{A19}$$

and

$$\boldsymbol{q}_{\rm i}^* = -\kappa^{*\rm i}_{\parallel}\nabla_{\parallel}T_{\rm i} - \kappa^{*\rm i}_{\perp}\nabla_{\perp}T_{\rm i} + \kappa^{*\rm i}_{\wedge}\boldsymbol{h}\times\nabla T_{\rm i}, \tag{A20}$$

where

$$\kappa_{\parallel}^{\mathrm{i}}=3.91\frac{n_{\mathrm{i}}T_{\mathrm{i}}}{\nu_{\mathrm{ii}}M_{\mathrm{i}}},\qquad \kappa_{\perp}^{\mathrm{i}}=\frac{n_{\mathrm{i}}T_{\mathrm{i}}}{\nu_{\mathrm{ii}}M_{\mathrm{i}}}\frac{\left(2\delta_{\mathrm{i}}^{2}+2.65\right)}{\Delta_{q}},$$
$$\kappa_{\wedge}^{\mathrm{i}}=\frac{n_{\mathrm{i}}T_{\mathrm{i}}}{\nu_{\mathrm{ii}}M_{\mathrm{i}}}\frac{\delta_{\mathrm{i}}\left(2.5\delta_{\mathrm{i}}^{2}+4.65\right)}{\Delta_{q}}, \tag{A21}$$

$$\kappa_{\parallel}^{*\mathrm{i}}=-\frac{25}{24}\frac{n_{\mathrm{i}}T_{\mathrm{i}}}{\nu_{\mathrm{ii}}M_{\mathrm{i}}},\qquad \kappa_{\perp}^{*\mathrm{i}}=\frac{n_{\mathrm{i}}T_{\mathrm{i}}}{\nu_{\mathrm{ii}}M_{\mathrm{i}}}\frac{\left(\frac{6}{7}\delta_{\mathrm{i}}^{2}-0.71\right)}{\Delta_{\mathrm{q}}},$$
$$\kappa_{\wedge}^{*\mathrm{i}}=-1.79\frac{n_{\mathrm{i}}T_{\mathrm{i}}}{\nu_{\mathrm{ii}}M_{\mathrm{i}}}\frac{\delta_{\mathrm{i}}}{\Delta_{\mathrm{q}}}, \tag{A22}$$

and

$$\Delta_{\mathrm{q}}=\delta_{\mathrm{i}}^{4}+2.70\delta_{\mathrm{i}}^{2}+0.68. \tag{A23}$$

The structure of the tensors $W_{\mu\nu}^{(1)}$ and $W_{\mu\nu}^{(2)}$, equations (A16) and (A17), is the following. The term with spatial derivative of the ion velocity in equation (A16) is the Braginskii [22] (Navier–Stokes) viscosity. The terms in equations (A16) and (A17), which are linear with the heat flux $\boldsymbol{q}_{\mathrm{i}}$ or its analogue $\boldsymbol{q}_{\mathrm{i}}^{*}$, contribute to the Burnett kind of viscosity [23–26]. The terms in these equations that are nonlinear with heat fluxes $\boldsymbol{q}_{\mathrm{i}}$ and $\boldsymbol{q}_{\mathrm{i}}^{*}$ contribute to the so-called nonlinear viscosity [27].

The origin of the terms with the heat flux derivatives (with the temperature gradients) in equations (A16) and (A17) can be understood analogously to the similar terms in the electron momentum equation [22]. There is the electron–ion friction term $\boldsymbol{R}_{\mathrm{e}}=\boldsymbol{R}_{u}+\boldsymbol{R}_{T}$, which contains two contributions: the friction of the electron–ion particle fluxes $\boldsymbol{R}_{u}\sim M_{\mathrm{e}}n_{\mathrm{e}}\left(\boldsymbol{V}_{\mathrm{e}}-\boldsymbol{V}_{\mathrm{i}}\right)$, and the friction of the electron–ion heat fluxes (the thermal force) $\boldsymbol{R}_{T}\sim M_{\mathrm{e}}\left(\boldsymbol{q}_{\mathrm{e}}-\boldsymbol{q}_{\mathrm{i}}\right)/T_{\mathrm{e}}$. The last term can be expressed via the temperature gradients, as has been demonstrated in [15]. The terms with the velocity derivatives in equations (A16) and (A17) (i.e. the friction between the adjacent ion velocity fluxes) can be considered analogously to the term $\boldsymbol{R}_{u}$. Finally, the terms with the ion heat flux derivatives are analogous to the thermal force $\boldsymbol{R}_{T}$.

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