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Planar Supersymmetric Quantum Mechanics of a Charged Particle in an External Electromagnetic Field*

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Abstract

The supersymmetric quantum mechanics of a two-dimensional non-relativistic particle subject to both magnetic and electric fields is studied in a superfield formulation and with the typical non-minimal coupling of (2+1) dimensions. Both the N=1 and N=2 cases are contemplated and the introduction of the electric interaction is suitably analysed.

Key-words: SUSY; Quantum Mechanics; Planar Physics.

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1 Introduction

Since the pioneering papers on supersymmetric quantum mechanics (SQM) [1, 2, 3], a great deal of work on the subject has been done, including various reviews [4, 5, 6, 7, 8, 9] and books [10, 11, 12, 13, 14], the research in the field being still active. In particular, a very usual question in this field is the realization of supersymmetry (SUSY) in quantum-mechanical systems involving charged or neutral particles in interaction with magnetic fields, in various space dimensionalities. Not related to SQM, however, it is a well-known fact that in (2+1) dimensions a non-minimal coupling naturally arises [15, 16, 17] and allows for a magnetic moment interaction even in the case of spin-zero particles (scalar matter fields). These two aspects, SQM and non-minimal coupling, have not yet been contemplated simultaneously in the literature, and so the present work sets out to address this problem.

Here, from the very beginning, a superfield formulation is carried out that involves charged particles with magnetic moment subject to external electric and magnetic fields whose potentials are functions of the particle superfield coordinates. Both N=1 and N=2 cases are addressed to.

Another interesting question that remains open in the literature is whether it is possible an electric field interaction to be present without the explicit breaking of SUSY. For N=1-SQM, the traditional answer is no [12, p.51], but here this question is also reassessed and it is shown that, in a non-minimal coupling scheme, this indeed may occur: an N=1 supersymmetric quantum-mechanical system is proposed, where electric field interactions appear along with magnetic dipole moment–magnetic field couplings, and it is shown under which conditions this may take place. As for the N=2 case, Witten's model [2, 3] is the most celebrated and the one with more applications. The corresponding literature shows that an electric interaction (via a scalar potential) is possible within such supersymmetric models, but it occurs only in each of the two sectors ('bosonic' and 'fermionic') of the Hamiltonian: the two electric potentials (the 'bosonic' and the 'fermionic'), although deriving from the same superpotential, have different expressions in terms of it and thus do not refer simultaneously to the same particle, but rather refer to two almost isospectral systems (the 'almost' here refers to the ground state), typical of (unbroken) supersymmetric systems. Contrary, in the N=2-(N=1-)SUSY of Pauli equation in two (three) space dimensions [26], the two sectors of the Hamiltonian (the 'bosonic' and the 'fermionic' ones) refer to the two different spin states of the *same* spin-1/2 system. In the present work, a proposition is made about the possibility of a supersymmetric Pauli Hamiltonian in (2+1) dimensions including electric interactions, with a non-minimal coupling.

The outline of the present paper is as follows. In Section 2, a brief review of the (2+1)-dimensional non-minimal coupling is presented. Next, N=1- and N=2-SQM are discussed in Sections 3 and 4, respectively. Finally, in Section 5, the General Conclusions are drawn.

2 Non-minimal coupling in (2+1) dimensions

In (3+1) dimensions the dual $\widetilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\kappa\lambda}F_{\kappa\lambda}$ of the electromagnetic field $F_{\kappa\lambda}$ is a second-rank tensor; on the other hand, in (2+1) dimensions, it is a vector, $\widetilde{F}^{\mu} \equiv \frac{1}{2}\epsilon^{\mu\kappa\lambda}F_{\kappa\lambda}$, and thus the minimal covariant derivative

$$\mathcal{D}_\mu \equiv \partial_\mu + iqA_\mu \tag{1}$$

may be generalized to a non-minimal one,

$$\mathcal{D}_\mu \equiv \partial_\mu + iqA_\mu + ig\widetilde{F}_\mu, \tag{2}$$

where g is the planar analogue of the magnetic dipole moment, which couples non-minimally with the magnetic field¶. Indeed, since $\widetilde{F}^{\mu} = (-B, -E_y, E_x) \equiv (-B, -\widetilde{\vec{E}})$, the equation above splits in components as:

$$\mathcal{D}_0 \equiv \frac{\partial}{\partial t} + iq\Phi - igB \tag{3}$$

and

$$\mathcal{D}_i \equiv \partial_i - iq(\vec{A})_i + ig\widetilde{E}_i. \tag{4}$$

In order to obtain, say, the Schrödinger equation for an electron subject to an electromagnetic field, one proceeds as usual, starting from the free Hamiltonian and substituting $\partial_0 = \partial_t$ with $\mathcal{D}_0$, or, equivalently, adding the term

$$q\Phi - gB \tag{5}$$

to the Hamiltonian, and also substituting ∂_i with $\mathcal{D}_i$, or, equivalently, substituting the momentum, $\vec{p} = -i\vec{\nabla}$, with

$$\vec{p} - q\vec{A} + g\widetilde{\vec{E}}, \tag{6}$$

these substitutions being readily seen as equivalent to the minimal prescription, except for the following changes:

$$\Phi \to \Phi' = \Phi - \frac{g}{q}B \tag{7}$$

and

$$(\vec{A})_i \to (\vec{A}')_i = (\vec{A})_i - \frac{g}{q}\widetilde{E}_i, \tag{8}$$

which, due to the definitions‖:

$$B \equiv \vec{\nabla} \times \vec{A} \equiv \epsilon_{ij}\partial_i(\vec{A})_j \tag{9}$$

¶Notice that the derivative (2) behaves like a covariant one, since the non-minimal term is gauge covariant, by definition.

‖It is worthwhile to remark that Eq. (9) is a scalar one.

and

$$\vec{E} \equiv \frac{\partial}{\partial t}(-\vec{A}_i) - \vec{\nabla}_i \Phi, \tag{10}$$

imply that:

$$B \to B' = B + \frac{g}{q}\left(\vec{\nabla} \cdot \vec{E}\right) \tag{11}$$

and

$$\vec{E} \to \vec{E}' = \vec{E} + \frac{g}{q}\left(\vec{\nabla} B + \frac{\partial \widetilde{\vec{E}}}{\partial t}\right), \tag{12}$$

the Lorentz force preserving its form, which in (2+1) dimensions reads

$$\vec{F} = q\vec{E} + q\widetilde{\vec{v}} B. \tag{13}$$

The non-minimal coupling studied here may be considered as resulting from the dimensional reduction of a Lorentz-breaking Chern-Simons model in (3+1) dimensions [18], defined by the following derivative:

$$\nabla_\mu \equiv \partial_\mu + iqA_\mu + i\frac{\gamma}{2}\epsilon_{\mu\nu\kappa\lambda} v^\nu F^{\kappa\lambda}, \tag{14}$$

where γ is a constant (like q, a property of the particle), $\epsilon_{\mu\nu\kappa\lambda}$ is the (Levi-Civita) totally antisymmetric tensor in (3+1) dimensions and v^ν is a fixed (Lorentz-breaking) vector in spacetime. Indeed, performing the corresponding steps in order to obtain the Schrödinger equation for a charged particle, one obtains that it is equivalent to add the term

$$q\Phi - \gamma \vec{v} \cdot \vec{B} \tag{15}$$

to the Hamiltonian and substitute the momentum with

$$\vec{p} - q\vec{A} + \gamma v^0 \vec{B} - \gamma \vec{v} \times \vec{E}. \tag{16}$$

Thus, choosing $v^\nu = (0, \vec{v})$ and $\gamma\vec{v} = (0, 0, \gamma v^3)$, one immediately verifies that the redefinitions stated in Eqs. (5-6) are exactly recovered, with the (3+1)-dimensional quantity γv^3 playing the role of its (2+1)-dimensional counterpart g, and with only the third (z) component of the (3+1)-dimensional magnetic field $\vec{B}$ and the in-plane (x, y) components of the (3+1)-dimensional electric field $\vec{E}$ contributing to the Hamiltonian, just as it should be in (2+1) dimensions.

Since here the object under study is planar physics, it is natural to think of the most celebrated of such an effect, namely, the quantum Hall effect (QHE) [19, 20, 21, 22], especially the fractional one (FQHE). Indeed, in Ref. [23] a parallel was made between the particle with charge q and magnetic moment g as described in the present Section and the composite fermion (CF) of Jain's model for the FQHE [24]. In this context, the interpretation of g becomes more specific: it corresponds to the magnetic flux** attached

**In two space dimensions, the magnetic flux and the magnetic dipole moment have the same dimension, $\text{mass}^{-1/2}$ in a $\hbar = c = 1$ unit system, in contrast to the case of three space dimensions, where the former is mass^0 and the latter, mass^{-1}.

to the electron, thus performing a new 'particle', called composite fermion. According to Jain's model, the amount of flux attached to it is an even number of fluxons (the magnetic flux quanta, each one given by $\phi_0 = hc/e$): $g = 2n\phi_0$, with n an integer. However, according to the (3+1)-dimensional Lorentz-breaking origin of g as seen above, the CF flux, g, contains a contribution from the particle itself (by means of the parameter γ) and another from the peculiar condition of spacetime (by means of $|\vec{v}| = v^3$), which possesses a broken Lorentz symmetry that leaves only the planes perpendicular to $\vec{v}$ unaffected. Indeed, the construction of a CF is possible only in two space dimensions (space dimensions greater than three are not considered here). An interpretation is also possible for $\vec{v}$: it is responsible for the confinement of the electrons in the plane and therefore it is natural to relate $\vec{v}$ to the z-component of the three-dimensional magnetic field, which is very large in the FQHE (~10 T) and forbids the electrons to move in the z-direction, breaking in this way their (3+1)-dimensional Lorentz symmetry.

Now, if one wants to supesymmetrize this model, it is important to notice that this is not possible (in N=1-SQM)[††] for a scalar potential interaction such as the one given by expression (5). Therefore, in order to keep invariance under N=1-SUSY, it is necessary that:

$$gB(x, y) = q\Phi(x, y). \tag{17}$$

3 N=1-SQM

A charged planar particle non-minimally coupled to a magnetic field is described as an $N = 1$-SQM system by means of the superspace action below:

$$S_1 = \frac{iM}{2}\int dtd\theta(D\vec{X})\cdot\dot{\vec{X}} + iq\int dtd\theta D\vec{X}\cdot\vec{A}'(\vec{X}), \tag{18}$$

where $\vec{A}'(\vec{X})$ is the vector (super)potential in a non-minimal coupling scheme, given by Eq. (8), and $\vec{X}(t)$ is the real "superfield" (in fact, the supercoordinate of the particle), given by

$$X^j(t,\theta) = x^j(t) + i\theta\lambda^j(t), \quad j = (1, 2), \tag{19}$$

$x^j(t)$ being the two real coordinates of the planar particle, $\lambda^j(t)$ their Grassmannian supersymmetric partners and θ the real, Grassmannian supersymmetric coordinate that parametrizes the superspace, $(t;\theta)$. The supersymmetric covariant derivative D is given by:

$$D = \partial_\theta - i\theta\partial_t. \tag{20}$$

The action (18), $S_1 = \int dtL_1$, splits into components of the superfield as:

$$L_1 = \frac{M\dot{\vec{x}}^2}{2} - \frac{iM}{2}\dot{\vec{\lambda}}\cdot\vec{\lambda} + q\dot{\vec{x}}\cdot\vec{A} - g\dot{\vec{x}}\cdot\tilde{\vec{E}} - \frac{iq}{2}(\vec{\lambda}\times\vec{\lambda})B - \frac{ig}{2}(\vec{\lambda}\times\vec{\lambda})(\vec{\nabla}\cdot\vec{E}), \tag{21}$$

[††]See Ref. [12, p.51], where it is also shown that this is *not* the case when N=2.

where one notices that

$$\vec{\lambda}\times\vec{\lambda}\equiv\epsilon_{ij}\lambda_i\lambda_j=[\lambda_1,\lambda_2]. \tag{22}$$

A convenient change of variables will be performed:

$$\psi\equiv\sqrt{\frac{M}{2}}(\lambda_1+i\lambda_2) \tag{23}$$

$$\bar{\psi}\equiv\sqrt{\frac{M}{2}}(\lambda_1-i\lambda_2), \tag{24}$$

giving rise to the following expression for the Lagrangian:

$$L_1=\frac{M\dot{\vec{x}}^2}{2}-\frac{i}{2}(\dot{\psi}\bar{\psi}+\dot{\bar{\psi}}\psi)+q\dot{\vec{x}}\cdot\vec{A}-g\dot{\vec{x}}\cdot\widetilde{\vec{E}}+\frac{q}{2M}[\psi,\bar{\psi}]B+\frac{g}{2M}[\psi,\bar{\psi}](\vec{\nabla}\cdot\vec{E}). \tag{25}$$

The corresponding Hamiltonian will be obtained after a canonical quantization procedure following Ref. [11, p.46]. The Grassmannian momenta are defined as

$$\pi\equiv\frac{\partial L_1}{\partial\dot{\psi}}=-\frac{i}{2}\bar{\psi} \tag{26}$$

$$\bar{\pi}\equiv\frac{\partial L_1}{\partial\dot{\bar{\psi}}}=-\frac{i}{2}\psi, \tag{27}$$

leading to the following operator algebra ($\hbar=1$):

$$[x_i,p_j]=i\delta_{ij},\qquad \{\psi,\pi\}=\{\bar{\psi},\bar{\pi}\}=-\frac{i}{2} \tag{28}$$

$$\{\psi,\bar{\psi}\}=1,\qquad \{\pi,\bar{\pi}\}=-\frac{1}{4}, \tag{29}$$

besides $\pi^2=\bar{\pi}^2=\psi^2=\bar{\psi}^2=0$. These relations may be represented by:

$$\psi=\sigma_+,\qquad \bar{\psi}=\sigma_- \tag{30}$$

$$\pi=-\frac{i}{2}\sigma_-,\qquad \bar{\pi}=-\frac{i}{2}\sigma_+, \tag{31}$$

where the σ's are the Pauli matrices (and there is no other inequivalent representation [25]).

The quantized version of the Hamiltonian is:

$$H_1=\frac{\left(\vec{p}-q\vec{A}+g\widetilde{\vec{E}}\right)^2}{2M}-\frac{qB}{2M}\sigma_3-\frac{g(\vec{\nabla}\cdot\vec{E})}{2M}\sigma_3, \tag{32}$$

where the relation $[\sigma_+,\sigma_-]=\sigma_3$ was used. Notice that this Hamiltonian automatically reveals a spin-1/2 particle with magnetic dipole moment $q\sigma_3/2M$ and gyromagnetic ratio 2, as expected, and in agreement with Ref. [26], about SQM (but without the superfield formulation used here), and Refs. [27], with general arguments concerning particles in (2+1) dimensions.

It is interesting to compare this Hamiltonian with the one obtained in Ref. [23], as the non-relativistic limit of the non-minimal (2+1)-dimensional Dirac equation:

$$H = q\Phi + \frac{\left(\vec{p} - q\vec{A} + g\tilde{\vec{E}}\right)^2}{2M} - \frac{qB}{2M} - gB - \frac{g}{2M}\left(\vec{\nabla}\cdot\vec{E}\right). \tag{33}$$

As already mentioned above, the condition $gB = q\Phi$ is necessary in order to keep the N=1-SUSY. Thus, under such a condition,

$$H = \frac{\left(\vec{p} - q\vec{A} + g\tilde{\vec{E}}\right)^2}{2M} - \frac{qB}{2M} - \frac{g}{2M}\left(\vec{\nabla}\cdot\vec{E}\right). \tag{34}$$

Comparing Eqs. (32) and (34), one concludes that the spin-up component of the former equals the latter. The same occurs with the spin-down component when a representation different from Eqs. (30-31) is used, in which the matrices σ_+ and σ_- are interchanged.

The last term in Eq. (32) may be related to the magnetic field, in the case of Maxwell-Chern-Simons (MCS) theory [28, 23], in which the following field equations hold:

$$\vec{\nabla}\cdot\vec{E} - m_{cs}B = \rho \tag{35}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t} \tag{36}$$

$$\tilde{\vec{\nabla}}B - m_{cs}\tilde{\vec{E}} = \vec{J} + \frac{\partial\vec{E}}{\partial t}, \tag{37}$$

where, as above, $\tilde{\vec{\nabla}}_i \equiv \epsilon_{ij}\partial_j$, and m_{cs} is the Chern-Simons (topological) mass parameter, the (gauge-symmetry preserving) mass of the gauge field. Indeed, in the region outside external charges ($\rho = 0$), the Hamiltonian (32) turns into the following expression:

$$H_1 = \frac{\left(\vec{p} - q\vec{A} + g\tilde{\vec{E}}\right)^2}{2M} - \frac{q\sigma_3}{2M}\left(1 + \frac{gm_{cs}}{q}\right)B. \tag{38}$$

From this Hamiltonian, it is natural to define an *effective gyromagnetic ratio*, γ_{eff}, whose departure from 2 is given by:

$$\gamma_{\text{eff}} - 2 = \frac{gm_{cs}}{q} - 1, \tag{39}$$

which reinforces the well-known fact that g is to be interpreted as an *anomalous* magnetic dipole moment. In this context, the condition

$$gm_{cs}/q = 1 \tag{40}$$

is necessary in order to keep the effective gyromagnetic ratio in its standard value 2. Interestingly, such a condition was also obtained in field theoretical works, with other interpretations: it turns interacting MCS theory into a free one and relates it to pure-CS theory and anyons [15, 29]; it gives rise to no one-loop radiative corrections to the photon mass [29]; and it reduces the differential equations for the gauge fields from second- to first-order, allowing one to get vortex solutions [30].

4 N=2-SQM

The superfield formulation of Witten's (one space dimension, N=2-) SQM may be found in Refs. [14, 31, 32], in terms of a scalar superpotential (a function of the one-dimensional real supercoordinate). A generalization to d space dimensions is presented in Refs. [12, 32], also in terms of a scalar superpotential, but now as a function of d real superfield coordinates. A different approach to two space dimensions, using a vector superpotential instead of a scalar one, is outlined in Ref. [33], but without a superfield formulation.

On the other hand, the N=2-SQM of Pauli equation in two space dimensions is formulated in terms of (complex) chiral and anti-chiral superfield coordinates in Refs. [32, 34], by means of a (Kähler) super(pre)potential (a function of those superfield coordinates). The introduction of an electric interaction into the planar Pauli equation without the explicit breaking of SUSY was made in Ref. [35], but there a non-stationary magnetic field was considered. In Ref. [36], the Pauli operator (including an external scalar potential) in two space dimensions is identified with the 2×2 component of a total 4×4 super-Hamiltonian. An N=2-superfield formulation encompassing all these issues, viz., Pauli equation in (2+1) dimensions with electric interactions, and also considering the planar non-minimal coupling studied in Section 1, is lacking. The present Section is devoted to fill this gap. Non-stationary situations are not considered in this paper, and so the electric interaction is due only to a scalar potential. Also, the mentioned possibility of the Pauli operator to be a component of the total super-Hamiltonian will not be considered here, but rather it will always be regarded as the total super-Hamiltonian itself.

It has been seen in Section 2 that, in order to obtain the Schrödinger equation with a non-minimal coupling, it is necessary to add the term (5) to the free Hamiltonian, and also to perform the replacement expressed by Eq. (8). If condition (17) is valid, then there is no scalar potential interaction in the resulting Hamiltonian, which therefore becomes 'pure-magnetic', allowing one to derive it from the chiral superaction of Ref. [34]. Such a superaction contains, instead of $\vec{A}(x,y)$, the (real) Kähler prepotential $K(x,y)$ (as will be seen below), which satisfies the following relations (from now on, A_i stands for $(\vec{A})_i$):

$$A_j = \epsilon_{jk}\partial_k K \tag{41}$$

$$B \equiv \vec{\nabla} \times \vec{A} \equiv \epsilon_{ij}\partial_i A_j = -\nabla^2 K. \tag{42}$$

Therefore, it would be desirable to find out how to implement the non-minimal prescription of Eq. (8) in terms of the prepotential $K(x,y)$. This is done as follows:

$$\begin{aligned} A'_i &= A_i - \frac{g}{q}\widetilde{E}_i \\ &= \epsilon_{ij}\partial_j K - \frac{g}{q}\epsilon_{ij}E_j \equiv \epsilon_{ij}\partial_j K - \alpha\epsilon_{ij}E_j \\ &= \epsilon_{ij}\left[\partial_j K + \alpha\left(\partial_t A_j + \partial_j \Phi\right)\right] \\ &= \epsilon_{ij}\partial_j\left(K + \alpha\Phi\right) \equiv \epsilon_{ij}\partial_j K', \end{aligned} \tag{43}$$

where the stationary condition $\partial_t = 0$ was used. Thus, the required prescription may be considered as:

$$K \to K' = K + \alpha\Phi = K + \alpha^2 B = K - \alpha^2\nabla^2 K. \tag{44}$$

Turning now to the chiral superaction, and using a notation similar to that of Ref. [34], the superspace coordinates are the time, t, and the Grassmanian variables, θ and $\bar{\theta}$ (the bar over a quantity stands for its complex or Hermitian conjugate). In a non-minimal coupling scheme, the N=2 superaction for a planar particle with mass M and electric charge q in a magnetic field satisfying Eq. (42) is given by:

$$S_2 = \frac{M}{8}\int dt d\theta d\bar{\theta} D\bar{\phi}\bar{D}\phi + q\int dt d\theta d\bar{\theta} K'(\phi,\bar{\phi}), \tag{45}$$

where $K'(\phi,\bar{\phi})$ is the superpotential given by the redefined Kähler prepotential of Eq.(44), now in terms of the chiral and antichiral superfield coordinates of the particle, ϕ and $\bar{\phi}$:

$$\begin{aligned} \phi(t,\theta,\bar{\theta}) &= z(t) + \theta\xi(t) - i\theta\bar{\theta}\dot{z}(t) \\ \bar{\phi}(t,\theta,\bar{\theta}) &= \bar{z}(t) - \bar{\theta}\bar{\xi}(t) + i\theta\bar{\theta}\dot{\bar{z}}(t), \end{aligned} \tag{46}$$

satisfying $\bar{D}\bar{\phi} = D\phi = 0$, and with $z(t) = x(t) + iy(t)$ being the complex variable representing the real coordinates $x(t)$ and $y(t)$ of the particle, and $\xi(t)$ its Grassmanian supersymmetric partner. The supersymmetric derivatives are defined as

$$\begin{aligned} D &= \partial_{\bar{\theta}} - i\theta\partial_t \\ \bar{D} &= \partial_\theta - i\bar{\theta}\partial_t. \end{aligned} \tag{47}$$

The superaction (45) splits into components as $S_2 \equiv \int dt L_2$, with

$$L_2 = \frac{M\dot{\vec{x}}^2}{2} - i\frac{M}{8}(\dot{\xi}\bar{\xi} + \dot{\bar{\xi}}\xi) + q\dot{\vec{x}}\cdot\vec{A} - g\dot{\vec{x}}\cdot\widetilde{\vec{E}} + \frac{q}{8}[\xi,\bar{\xi}]B + \frac{g}{8}[\xi,\bar{\xi}](\vec{\nabla}\cdot\vec{E}), \tag{48}$$

which, as expected, is the same result that would be obtained if one had started with a minimal superaction, i.e., Eq. (45) with $K(\phi,\bar{\phi})$ replacing $K'(\phi,\bar{\phi})$, and the non-minimal prescription had been implemented only after the corresponding splitting in components, by means of Eqs. (8) and (11). Moreover, this Lagrangian is identical to the N=1 case, Eq. (25), provided the identification $\psi = \frac{\sqrt{M}}{2}\xi$ is made. Thus, all the quantization procedure carried out after Eq. (25) may be repeated, yielding the same results and attesting, in a superfield description, the fact that in (2+1) dimensions the Pauli equation possesses, rather than an N=1-, an N=2-SUSY [26] (note that this conclusion is valid independently whether the coupling is minimal or non-minimal).

5 Discussion and conclusions

Here, it has been shown the possibility, expressed by Eq. (17), for SUSY to be kept even with an electric field applied, provided a non-minimal coupling scheme holds. Moreover, since this work deals with planar physics, it may suggest a possible application to the quantum Hall effect (QHE) [19, 20, 21, 22]. Indeed, such a possibility was already pointed out in Ref. [23], where a parallel was made between the particle with charge and magnetic moment as described in Section 2 and the composite fermion of Jain's model for the fractional QHE [24]. Now, assuming the validity of such a parallel, the present work brings a SUSY to the system of composite fermions. Another result is Eq. (40), a condition also obtained in field theoretical works (with other interpretations) and which here guarantees the gyromagnetic ratio to be equal to its standard value, two. All the calculations are made in superfield formulation.

Finally, a more general possibility for the interacion will be discussed, in which the following terms are added to the superaction (45):

$$\int dt d\theta\, \Gamma(\phi) + \int dt d\bar{\theta}\, \bar{\Gamma}(\bar{\phi}), \tag{49}$$

where $\Gamma(\phi)$ and its complex conjugate $\bar{\Gamma}(\bar{\phi})$ are necessarily Grassmann external fields, in order to the action be bosonic. The corresponding components added to the Lagrangian (48) are:

$$\xi\Gamma'(z) - \bar{\xi}\bar{\Gamma}'(\bar{z}). \tag{50}$$

The (pseudo-)classical external field $\Gamma'(z)$, although not quantized, must anticommute with ξ as well as with itself. Therefore, it may be represented also by a 2×2-matrix. These anticommutation requirements, however, impose such severe restrictions on the matrix $\Gamma'(z)$ that, under the quantization procedure mentioned in Section 3, the contribution of the terms (50) to the Hamiltonian is zero. On the other hand, if the quantized Grassmannian coordinates ξ and $\bar{\xi}$ and their momenta are chosen to have a representation different from Eqs. (30–31) (remember that $\psi = \frac{\sqrt{M}}{2}\xi$), then the same does not occur. Indeed, considering for example

$$\xi = \sigma_+ \otimes \mathbf{1}_{2\times 2} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{51}$$

and

$$\Gamma'(z) \equiv \begin{pmatrix} f(z) & g(z) & h(z) & i(z) \\ j(z) & k(z) & l(z) & m(z) \\ n(z) & o(z) & p(z) & q(z) \\ r(z) & s(z) & t(z) & u(z) \end{pmatrix}, \tag{52}$$

the same anticommutation requirements for $\Gamma'(z)$ lead to the following total Hamiltonian:

$$H_2 = \frac{\left(\vec{p} - q\vec{A} + g\tilde{\vec{E}}\right)^2}{2M} - \frac{qB}{2M}\sigma_3 \otimes \mathbf{1}_{2\times 2} - \frac{g(\vec{\nabla}\cdot\vec{E})}{2M}\sigma_3 \otimes \mathbf{1}_{2\times 2} + \frac{2}{\sqrt{M}}G(z,\bar{z}), \tag{53}$$

where

$$G = \begin{pmatrix} 0 & -f(z) & 0 & -h(z) \\ -\bar{f}(\bar{z}) & 0 & \bar{f}^2(\bar{z})/\bar{h}(\bar{z}) & 0 \\ 0 & f^2(z)/h(z) & 0 & f(z) \\ -\bar{h}(\bar{z}) & 0 & \bar{f}(\bar{z}) & 0 \end{pmatrix}. \tag{54}$$

Notice that this interaction mixes the four components of the wave function, contrary to the original Hamiltonian. The Grassmann fields $\Gamma'(z)$ and $\bar{\Gamma}'(\bar{z})$ may be interpreted as photino-type (pseudo-)classical external fields, in the same way as the electromagnetic prepotential $K'(z,\bar{z})$ (or the potentials Φ and $\vec{A}$) is usually considered as a photon-type classical external field. The necessity of using 4-component wave functions is similar to what happens in (2+1)D field theory when one is forced to introduce 4-component massive fermions in order to make the mass compatible with the parity simmetry.

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