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# Spatial dust distribution and plasma dynamics in the tokamak edge

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**Abstract**
The role of dust, expected to be important in future large power wall loadings and long operation times, is now recognized for new devices such as ITER. In this context, we study the effect of dust poloidal distributions on plasma dynamics of the plasma edge. We show that in the considered range of dust densities, the poloidal distribution of dust affects only the toroidal velocity, while the poloidal velocity and the radial electric field are defined by the ion-dust friction and thermal forces.

It is well known that plasma dynamics, namely, poloidal $U_{i\theta}$ and toroidal $U_{i\varsigma}$ velocities and, consequently, the sheared radial electric field $E_r$, play an important role in tokamak plasmas, being responsible for improved confinement regimes (Burrell 1997). In conventional tokamaks, these values mainly depend on the ion viscosity or on the ion and electron friction with neutrals. Now it is well recognized that dust can be present in tokamak plasmas under some conditions and that the dust problem is assumed more worrisome for future large power wall loadings and long operation times expected for new devices such as ITER (Tsytovich and Winter 1998). Theoretical and experimental results also indicate that dust is mainly located in the tokamak bottom (divertor region).

Recently it was demonstrated (Tsypin *et al* 2004) that plasma dynamics in tokamaks with dust crucially depend on the parameter

$$\alpha = (R^2/\lambda_i^2) n_d Z_d^2/(n_i Z_i^2) \approx 1,$$

where $R$ is the major radius of the plasma column, $\lambda_i = v_{Ti}/\nu_i$ is the ion mean free path, $v_{Ti} = \sqrt{2T_i/M_i}$ is the ion thermal velocity, $\nu_i$ is the ion collision frequency, $T_i$ and $M_i$ are the ion temperature and mass and $n_d$, $Z_d$ and $n_i$, $Z_i$ are the dust number density and charge and the ion number density and charge, respectively. Accepting that at the plasma edge the conditions

$R^2 \gg \lambda_i^2$ and $Z_d/Z_i \gg 1$ are satisfied, we arrive at the conclusion that even a very small amount of dust changes the viscous dependence of the plasma dynamics to dust dependence. In this work, we consider the effect of a dust poloidal distribution on the plasma dynamics at the plasma edge. We conclude that there is a possibility of effectively acting on the plasma dynamics at the plasma edge by puffing dust into different regions of tokamaks.

We consider the quasineutrality condition of the form (Vladimirov *et al* 2005)

$$-en_e + eZ_in_i - eZ_dn_d \simeq 0, \tag{1}$$

where $-e \equiv e_e$ is the electron charge, $n_e$ is the electron number density, $eZ_i \equiv e_i$ and

$$-eZ_d \equiv e_d$$

are the ion and dust effective charges, respectively. In addition, we assume that

$$Z_dn_d \ll n_e \simeq Z_in_i,$$

$M_i \ll M_d$, $M_d$ is the dust particle mass and dust particles are motionless (the dust temperature is zero). From (1), we find that poloidal variations of the dust spatial distribution lead to the poloidal variations of ion and electron densities. Assuming

$$n_d = n_{d0} + n_d(\theta),$$
$$n_i = n_{i0} + n_i(\theta)$$

and

$$n_e = n_{e0} + n_e(\theta),$$

where

$$n_d(\theta) = -n_{ds}\sin\theta - n_{dc}\cos\theta,$$

$\theta$ is the poloidal angle and $n_{ds}$ and $n_{dc}$ are dust densities in the dust poloidal angle corresponding dependence, $n_d(\theta)$, we obtain by accepting

$$n_0 = n_{e0} \simeq Z_in_{i0}$$

such that

$$-n_e(\theta) + Z_in_i(\theta) - Z_dn_d(\theta) \simeq 0. \tag{2}$$

In the above equations and in the following the subscript '0' stands for equilibrium values not depending on the poloidal angle $\theta$.

All values $n_{d0}$, $n_{ds}$ and $n_{dc}$ can be functions of the radial coordinate $r$. For ions and electrons we accept the Boltzmann distribution,

$$n_i = n_{i0}\exp(-eZ_i\psi/T_i), \qquad n_e = n_{e0}\exp(e\psi/T_e),$$

where $\psi$ is the electric field potential. We assume $|eZ_i\psi/T_i| \sim |e\psi/T_e| \ll 1$. Consequently, we get from (2)

$$\psi(\theta) \simeq -Z_dn_d(\theta)\big/[en_{e0}f(T_\alpha)], \qquad n_i(\theta) \simeq -\frac{eZ_in_{i0}}{T_{i0}}\psi(\theta) = \frac{Z_dn_d(\theta)}{T_{i0}f(T_\alpha)}$$

and

$$n_e(\theta) \simeq -\frac{Z_dn_d(\theta)}{T_{e0}f(T_\alpha)}.$$

Here

$$f(T_\alpha) = (1/T_{e0}) + Z_i/T_{i0}$$

and $T_{e0}$ and $T_{i0}$ are the equilibrium electron and ion temperatures, respectively. Hence, we have

$$n_{i,e}(\theta) = n_{i,es} \sin\theta + n_{i,ec} \cos\theta,$$

where $n_{i,es}$ and $n_{i,ec}$ have obvious definitions.

For the following calculations we need the summed ion and electron momentum equations (Tsypin *et al* 2004, Mikhailovskii and Tsypin 1984):

$$-\nabla p - \nabla \cdot \boldsymbol{\pi}_{\parallel}^{i} - \nabla \cdot \boldsymbol{\pi}_{\perp}^{i} - eZ_{d}n_{d}\nabla\psi + \frac{1}{c}\left[\boldsymbol{j} \times \boldsymbol{B}\right] + \boldsymbol{R} \simeq 0. \quad (3)$$

Here $p = p_e + p_i$ is the plasma pressure, $\boldsymbol{\pi}_{\parallel}^{i}$ and $\boldsymbol{\pi}_{\perp}^{i}$ are the parallel and perpendicular tensor components of the ion viscosity tensor $\boldsymbol{\pi}^{i}$, $\boldsymbol{j}$ is the plasma current density, $\boldsymbol{B}$ is the magnetic field vector, $\boldsymbol{R}$ is the friction force between ions and dust including the thermal force $\boldsymbol{R}_T$, where (Tsypin *et al* 2004)

$$\boldsymbol{R} \simeq -M_i n_i \nu_{id} \left(0.51\boldsymbol{V}_{i\parallel} + \boldsymbol{V}_{i\perp}\right) + \boldsymbol{R}_T,$$

$$\boldsymbol{R}_T \simeq -2.69\frac{n_d Z_d^2}{Z_i^2}\boldsymbol{b}\nabla_{\parallel} T_i + \frac{3}{2}\frac{\nu_{id} n_i}{\omega_{ci}}\left[\boldsymbol{b} \times \nabla T_i\right], \qquad \boldsymbol{b}=\frac{\boldsymbol{B}}{B},$$

and

$$\nu_{id} = 4\sqrt{2\pi}\lambda e^4 Z_i^2 Z_d^2 n_d \Big/ \left(3\sqrt{M_i}T_i^{3/2}\right)$$

is the ion-dust collision frequency satisfying the inequalities $\nu_{id} \ll \omega_{ci}$ and $\nu_{id} \gg v_{Ti}/qR$, where $q = rB_{\zeta}/RB_{\theta}$ is the safety factor, $\omega_{ci}$ is the ion cyclotron frequency, $B_{\zeta}$ and $B_{\theta}$ are the toroidal and poloidal components of the magnetic field $\boldsymbol{B}$, respectively, and $\boldsymbol{V}_{i\parallel}$ and $\boldsymbol{V}_{i\perp}$ are the parallel and perpendicular vector components of the ion velocity $\boldsymbol{V}_i$.

The required relations can be found from the parallel and $\zeta$-covariant projections of (3). In the first case, we have

$$\frac{\partial p}{\partial\theta} + \frac{\boldsymbol{b}}{B^{\theta}}\cdot\left(\nabla \cdot \boldsymbol{\pi}_{\parallel}^{i}\right) - eZ_d n_d\frac{\partial\psi}{\partial\theta} + 0.51\frac{M_i n_i}{B^{\theta}}\nu_{id}V_{i\parallel} + 2.69\frac{n_d Z_d^2}{Z_i^2}\frac{\partial T_i}{\partial\theta} \simeq 0, \quad (4)$$

where the approximate expression $b^{\theta} \simeq 1/qR$ has been used for the $\theta$-contravariant projection of vector $\boldsymbol{b}$. Averaging (4) over $\theta$ we obtain

$$\left\langle \frac{\boldsymbol{b}}{B^{\theta}}\cdot\left(\nabla \cdot \boldsymbol{\pi}_{\parallel}^{i}\right)\right\rangle + 0.51M_i\frac{\nu_{id0}}{n_{d0}}\left\langle n_i n_d \frac{V_{i?}}{B^{\theta}}\right\rangle + \frac{2.69Z_d^2}{Z_i^2}\left\langle n_d(\theta)\frac{\partial T_i}{\partial\theta}\right\rangle \simeq 0, \quad (5)$$

where

$$\langle\cdots\rangle = \int_0^{2\pi}(\cdots)\,d\theta/2\pi$$

and (Tsypin *et al* , 2004)

$$\left\langle\frac{\boldsymbol{b}}{B^{\theta}}\cdot\left(\nabla\cdot\boldsymbol{\pi}_{\parallel}^{i}\right)\right\rangle = \frac{1.44}{R}\frac{p_i\epsilon}{\nu_i}\left[U_{i\theta} + 1.83U_{Ti} - \frac{n_{ic}}{\epsilon n_{i0}}\left(0.19U_{i\theta} + 1.52U_{Ti}\right)\right].$$

Here

$$U_{Ti} = (1/M_i\omega_{ci})\,\partial T_i/\partial r$$

and $\epsilon = r/R$.

We also get (Braginskii 1965, Tsypin *et al* 2004)

$$\frac{\partial T_i}{\partial\theta} = 1.28\frac{bT_i}{\nu_i d(b)}\left[2\frac{U_{Ti}}{r}\frac{\partial^2 \ln B}{\partial\theta^2} + \left(\frac{2}{5}\left\langle V_i^{\theta}\right\rangle - \frac{U_{Ti}}{r}\right)\frac{\partial^2 \ln n_i}{\partial\theta^2}\right],$$

where

$$d(b) = 1 + 2.2b\sqrt{M_e/M_i},$$
$$b = q^2R^2/\lambda_i^2$$

and $\lambda_i$ is the ion mean free path.

Now we turn to the $\zeta$-covariant projection of (3). For an axially symmetric tokamak, $\partial/\partial\zeta = 0$, we obtain from this equation

$$-\left(\nabla\cdot\boldsymbol{\pi}^i_{\parallel}\right)_\zeta - \left(\nabla\cdot\boldsymbol{\pi}^i_{\perp}\right)_\zeta + \frac{\sqrt{g}}{c} j^r B^\theta + R_\zeta \simeq 0, \qquad (6)$$

where

$$R_\zeta \simeq -0.51 M_i n_i \nu_{id} V_{i\zeta} - 2.69\frac{n_d Z_d^2}{Z_i^2 q}\frac{\partial T_i}{\partial\theta} - \frac{3}{2}\nu_{id} n_i M_i\sqrt{g} b^\theta U_{Ti},$$

$g$ is the determinant of the metric tensor and $j^r$ is the $r$ -contravariant projection of $\boldsymbol{j}$. It was shown (Tsypin *et al* 2004) that, using the ambipolarity condition,

$$\int_o^{2\pi} j^r\sqrt{g}\,\mathrm{d}\theta = 0,$$

we obtain

$$\left\langle\sqrt{g}\left(\nabla\cdot\boldsymbol{\pi}^i_{\parallel}\right)_\zeta\right\rangle = 0.$$

Multiplying (6) by $\sqrt{g}$ and using the Maxwell equation

$$\partial\left(\sqrt{g}B^\theta\right)/\partial\theta = 0,$$

we arrive at

$$\left\langle\sqrt{g}\left\{\left(\nabla\cdot\boldsymbol{\pi}^{\mathrm{i}}_{\perp}\right)_\zeta + \frac{\nu_{id0}}{n_{d0}} n_d n_i M_i\left(0.51 V_{i\zeta} + \frac{1.91}{\nu_i q M_i}\frac{\partial T_i}{\partial\theta} + \frac{3}{2}\sqrt{g} b^\theta U_{Ti}\right)\right\}\right\rangle \simeq 0. \qquad (7)$$

In addition, we should use the frozen-in condition,

$$\nabla\times[\boldsymbol{V}\times\boldsymbol{B}] = 0.$$

From this equation we find oscillating values

$$\tilde{V}_i^\zeta = q\tilde{V}_i^\theta.$$

We take the metric tensor components and their determinant as follows:

$$g_{rr} = 1, \qquad g_{\theta\theta} = \epsilon^2 g_{\zeta\zeta}, \qquad g_{\zeta\zeta} = R^2(1 - 2\epsilon\cos\theta), \qquad \sqrt{g} = \epsilon g_{\zeta\zeta}.$$

Hence,

$$\left\langle V_i^\zeta\right\rangle \simeq U_{i\zeta}/R$$

and

$$\left\langle V_i^\theta\right\rangle \simeq U_{i\theta}/r.$$

Then, from the ion continuity equation we obtain

$$\partial\left(\sqrt{g} n_i V_i^\theta\right)/\partial\theta \approx 0.$$

As a result, we have

$$V_i^\theta \approx \frac{U_{i\theta}}{r}\left[1 + 2\epsilon\cos\theta - \frac{Z_i Z_d n_d(\theta)}{n_0 T_{i0} f(T_\alpha)}\right]. \qquad (8)$$

We represent

$$\tilde{V}_i^{\theta,\zeta} = V_{ic}^{\theta,\zeta}\cos\theta + V_{is}^{\theta,\zeta}\sin\theta,$$

where definitions are obvious. Equations (5) and (7) with necessary substitutions are the basic equations for the analysis that follows.

*The conventional neoclassical case* $Z_d^2 n_{d0}/\left(Z_i^2 n_{i0}\right) \ll \rho_i^2/r^2$. We find from (5) and (7) the well-known equations (Hazeltine 1974, Mikhailovskii and Tsypin 1982, Claassen *et al* 2000)

$$U_{i\theta} + 1.83 U_{Ti} = 0$$

and

$$\frac{1}{n_i \nu_i} \frac{\partial}{\partial r} \left[ n_i \nu_i \rho_i^2 \left( \frac{\partial U_{i\zeta}}{\partial r} - 0.107 q^2 \frac{\partial \ln T_i}{\partial r} \frac{q}{\epsilon} U_{i\theta} \right) \right] = 0,$$

where $\rho_i = v_{Ti}/\omega_{ci}$ is the ion Larmor radius.

Substituting solutions of these equations into the equation for the radial electric field

$$E_r \approx \frac{1}{c} \left( -B_\zeta U_{i\theta} + B_\theta U_{i\zeta} + B U_{pi} \right), \qquad U_{pi} = \frac{c}{e_i n_i B} \frac{\partial p_i}{\partial r}, \tag{9}$$

we can obtain the well-known neoclassical result (Hazeltine 1974, Mikhailovskii and Tsypin 1982, Claassen 2000).

*The case of the negligible perpendicular viscosity* $\lambda_i^2/R^2 \gg Z_d^2 n_{d0}/\left(Z_i^2 n_{i0}\right) \gg \rho_i^2/r^2$. From (5) and (7) we find

$$U_{i\theta} = -1.83 U_{Ti}$$

and

$$U_{i\zeta} = -U_{Ti} \left[ \frac{\epsilon}{q} \left( 2.95 + 7.32 q^2 + 4.8 \frac{q^2}{\mathrm{d}(b)} \right) + q \frac{n_{dc}}{n_{d0}} \left( 1.83 + \frac{2.4}{d(b)} \right) \right].$$

Then we obtain

$$E_r \approx \frac{B}{c} U_{Ti} \left( 2.83 + \frac{\kappa_n}{\kappa_T} \right),$$

where $\kappa_n = \mathrm{d} \ln n / \mathrm{d} \ln r$ and $\kappa_T = \mathrm{d} \ln T_{i0} / \mathrm{d} \ln r$. We see that, in this case, the poloidal distribution of dust affects only the toroidal velocity. The poloidal velocity and the radial electric field are practically neoclassical.

*The case of the negligible plasma viscosity* $\epsilon Z_d/Z_i \gg Z_d^2 n_{d0}/\left(Z_i^2 n_{i0}\right) \gg \lambda_i^2/R^2$. We find from (5) and (7)

$$U_{i\theta} = 2.95 \left(1 + 2q^2\right)^{-1} U_{Ti} \left[1 + 1.63 q^2/d(b)\right],$$

$$U_{i\zeta} = -\frac{2.95 \epsilon U_{Ti}}{q\left(1 + 2q^2\right)} \left\{ \left[ 1 + \frac{1.63 q^2}{d(b)} \right] \left( 1 - 2q^2 - \frac{q^2}{\epsilon} \frac{n_{dc}}{n_{d0}} \right) + \frac{0.81 q^2 n_{dc} \left(1 + 2q^2\right)}{\epsilon d(b) n_{i0}} \right\}$$

and

$$E_r \approx -\frac{B}{c} U_{Ti} \left\{ 2.95 \left(1 + 2q^2\right)^{-1} \left[ 1 + \frac{1.63 q^2}{d(b)} \right] - 1 - \frac{\kappa_n}{\kappa_T} \right\}. \tag{10}$$

We again conclude that the poloidal distribution of dust affects only the toroidal velocity while the poloidal velocity and the radial electric field are defined by the ion-dust friction and by the ion-dust thermal forces. Nevertheless, it is clear that in spherical tokamaks, where the parameter $\epsilon \sim 1$, the poloidal distribution of dust can also affect the plasma poloidal velocity and the radial electric field.

*The large dust density* $1 > Z_{\rm d}\left\{n_{{\rm d}c}, n_{{\rm d}s}\right\}/(Z_{\rm i}n_{\rm i0}) > \epsilon$. It follows from (5) and (7) the same equation for the poloidal velocity and

$$U_{{\rm i}\zeta} = \frac{qZ_{\rm d}\left(n_{{\rm d}s}^2 + n_{{\rm d}c}^2\right)}{2\epsilon n_0 n_{\rm d0} f(T_\alpha)} \frac{Z_{\rm i}}{T_{\rm i0}} \left\{U_{{\rm i}\theta}\left[1 + \frac{0.96}{d(b)}\right] - \frac{2.4}{d(b)} U_{\rm Ti}\right\}.$$

Substituting these equations into (9) we obtain (10). It means that the dust poloidal distribution has practically no effect on the plasma poloidal velocity and the radial electric field, affecting only the plasma toroidal velocity.

In conclusion, recognizing the potentially important role of dust in future large power wall loadings and long operation times expected for new devices such as ITER, we have studied the effect of the dust poloidal distribution on the plasma dynamics at the plasma edge. This dust spatial distribution can be natural, as evidence exists that dust can mainly be presented in the divertor region of future tokamaks, or artificial induced by dust puffing aiming to operate plasma dynamics. We have shown that, in the considered range of dust densities, the poloidal distribution of dust affects only the toroidal velocity while the poloidal velocity and the radial electric field are defined by the ion-dust friction and by the ion-dust thermal forces. Note that, as follows from our calculations, in spherical tokamaks the poloidal distribution of dust can also affect the plasma poloidal velocity and the radial electric field in addition to affecting the plasma toroidal velocity.

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## References

Braginskii S I 1965 *Reviews of Plasma Physics* vol 1 (New York: Consultants Bureau) p 205
Burrell K H 1997 *Phys. Plasmas* **4** 1499
Claassen H A, Gerhauser H, Rogister A and Yarim C 2000 *Phys. Plasmas* **7** 3699
Hazeltine R D 1974 *Phys. Fluids* **17** 961
Mikhailovskii A B and Tsypin V S 1982 *Sov. Phys.—JETP* **56** 75
Mikhailovskii A B and Tsypin V S 1984 *Beitr. Plasma Phys.* **24** 335
Tsypin V S *et al* 2004 *Phys. Plasmas* **11** 4138
Tsytovich V N and Winter J 1998 *Usp. Fiz. Nauk* **168** 899
Vladimirov S V, Ostrikov K and Samarian A A 2005 *Physics and Applications of Complex Plasmas* (London: Imperial College)