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BRIEF COMMUNICATION

A possible model for 'snakes'

F E M da Silveira and R M O Galvão[1]

Instituto de Física da Universidade de São Paulo, Caixa Postal 66318, CEP 05315-970, São Paulo, SP, Brazil

E-mail: feugenio@if.usp.br and rgalvao@if.usp.br

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Abstract
A simple helical magnetohydrodynamic equilibrium with flow, in which the vorticity is proportional to the current density, is presented. It is argued that the helical magnetic structures, known as snakes, can be described as a local transition to this equilibrium, triggered by the large temperature drop, observed when a pellet crosses the $q = 1$ rational magnetic surface.

One of the most interesting phenomena occurring in tokamak plasmas is the appearance of long-lived structures at the $q = 1$ rational magnetic surface, where q is the inverse rotational transform of the magnetic field lines. Usually, the appearance of these structures is triggered by the crossing of externally injected hydrogen/deuterium pellets through the rational surface. This phenomenon was first observed in JET and the structures were dubbed 'snakes', for the signature they impress on the soft x-ray emission profile of the plasma, as recovered by tomography [1]. The standard model to describe snakes is based on the formation of a $m = 1$ magnetic island at the resonant surface, where m is the poloidal mode number of the structure. Essentially, as the pellet crosses this surface, it should cause a strong local cooling, enhancing the plasma resistivity, thus implying a drop in the current density. This perturbation could lead to the formation of a magnetic island, depending on the local shear of the magnetic field lines [2].

This model, however, is not entirely consistent with experimental observations as well as with some physical arguments. First of all, the structure has a poloidal cross-section much narrower than that of a $m = 1$ island; indeed, it usually extends to no more than 10% of the poloidal circumference of the resonant surface. Secondly, it can persist much longer than the characteristic time for recovery of the initial temperature drop and survive many sawtooth crashes, in some discharges, in which a strong temperature pulse passes through the reconnecting island [1,2]. Furthermore, later experimental work has shown that snakes can be spontaneously driven in ohmic discharges without pellet injection, and also in electron runaway discharges [3,4]. From these results, and other observations presented in the literature, it seems

[1] Also at: Centro Brasileiro de Pesquisas Físicas, R. Dr. Xavier Sigaud, 150, CEP 22290-180, Rio de Janeiro, RJ, Brazil.

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that the physical mechanisms involved in the formation and sustainment of snakes are far from being satisfactorily understood.

In this brief communication, we present a simple helical equilibrium configuration with flow, and argue that it can be considered as a possible model to describe the long-lived magnetic structure of snakes. Essentially, our argument is that snakes are formed when a perturbation on the temperature gradient induces a localized strong flow parallel to the magnetic field lines, in the neighborhood of a rational magnetic surface. A bifurcation of the equilibrium can thus give rise to a local helical equilibrium with flow, in which the vorticity is provided by the current density. This model has been suggested by the experimental observations that the tail structure formed in the ionized cloud around an injected pellet is determined by the $\vec{E} \times \vec{B}$ drift and the local plasma rotation, and also inspired by the resemblance of snakes with rotating small regions with $m = 1$ helicity [5–8].

We start with the magnetohydrodynamic (MHD) approximation with flow,

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right) \rho = -\rho \nabla \cdot \vec{v},$$
$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right) \vec{v} = -\frac{\nabla p - \vec{j} \times \vec{B}}{\rho},$$
$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right) p = -\Gamma p \nabla \cdot \vec{v}, \tag{1}$$

where ρ, $\vec{v}$ and p are the plasma density, velocity and pressure, respectively, together with

$$\nabla \times \vec{B} = \mu_0 \vec{j}, \tag{2}$$

where $\vec{B}$, and $\vec{j}$ are the magnetic field, and current density, respectively. Let us then assume a homogeneous fluid in a state of stationary equilibrium with flow parallel to the magnetic field. In particular, we require $\vec{B}$ and $\vec{v}$ to be related through

$$\vec{B} = \alpha \sqrt{\mu_0 \rho} \vec{v}, \tag{3}$$

where α is a dimensionless constant. From equation (3) and Maxwell's equations, it follows that the flow has to be incompressible, i.e. $\nabla \cdot \vec{v} = 0$ (consistently with the first of equations (1)). Using this condition together with equations (2) and (3), the equilibrium equation can be obtained from the equation of motion (the second of equations (1)),

$$\nabla \left(\frac{p}{\rho} + \frac{v^2}{2}\right) = (\alpha^2 - 1) \vec{w} \times \vec{v}, \tag{4}$$

where we have introduced the vorticity field

$$\vec{w} = \nabla \times \vec{v}. \tag{5}$$

It is interesting to point out that the equilibrium equation does not depend on the sign of the magnetic field with respect to the velocity field. In addition, from the incompressibility condition and the equation of state (the third of equations (1)), it follows that $\vec{v} \cdot \nabla p = 0$, i.e. the equilibrium equation imposes a constraint on the velocity field,

$$\vec{v} \cdot \nabla \left(\frac{v^2}{2}\right) = (\alpha^2 - 1) \vec{v} \cdot (\vec{w} \times \vec{v}) \equiv 0. \tag{6}$$

To understand qualitatively the physical characteristics of the constrained equilibrium just described, it is sufficient to consider a cylindrical approximation to a toroidal configuration. Therefore, we adopt a cylindrical coordinate system (r, θ, z), with z standing along the toroidal

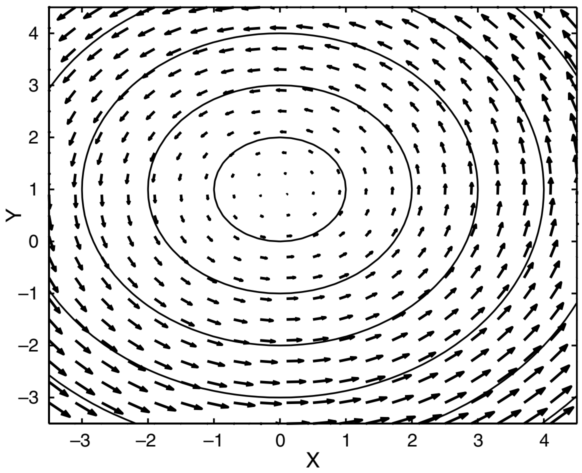

Figure 1. Flow lines of the transverse field given by equation (10). The velocity strength is normalized to $R\Omega$, and the X and Y Cartesian coordinates, to R.

direction, and assume that the longitudinal component B_z of the magnetic field is constant, and transverse component $\vec{v}_\perp$ of the velocity field can be written as

$$\vec{v}_\perp = v_r\hat{r} + r\omega\hat{\theta}, \tag{7}$$

with v_r and ω being functions of the coordinates. The equilibrium constraint then leads to

$$\left(v_r\frac{\partial}{\partial r} + \omega\frac{\partial}{\partial\theta}\right)\left(\frac{v_r^2 + r^2\omega^2}{2}\right) = 0, \tag{8}$$

while from the incompressibility condition it follows that

$$\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{\partial\omega}{\partial\theta} = 0. \tag{9}$$

Somewhat surprisingly, the two last equations have a simple solution with off-axis circular flow lines on the transverse plane resembling a $m = 1$ helical structure,

$$\vec{v}_\perp = R\Omega\left\{\cos(\theta - \kappa z)\hat{r} + \left[\frac{r}{R} - \sin(\theta - \kappa z)\right]\hat{\theta}\right\}, \tag{10}$$

with R, Ω and κ being constants. The convective cell represented by equation (10) is shown in figure 1. It is seen that, indeed, it has the required structure. The flow lines are circles centered at $r = R$, so that, in this model, the constant R is just the radial position of the center of the transverse cross-section of the helical structure. This configuration is, essentially, an 'aligned flow equilibrium' of the kind discussed in [9]. According to the proportionality between the current density and vorticity field, we arrive at the interesting result that the longitudinal component j_z of the former is a constant,

$$j_z = 2\alpha\Omega\sqrt{\frac{\rho}{\mu_0}}. \tag{11}$$

On the other hand, given the relation of the longitudinal plasma current to the toroidal loop voltage V_L, and plasma resistivity η, the product $\alpha\Omega$ can be expressed as

$$\alpha\Omega = \frac{\kappa V_\mathrm{L}}{4\pi\eta}\sqrt{\frac{\mu_0}{\rho}}. \tag{12}$$

Consequently, the strength Ω of the rotation of the convective cell constitutes a free parameter within the proposed model. Finally, to mock up a toroidal configuration, we must impose periodic conditions on the longitudinal direction, so that κ might be identified to the inverse of the major radius of the plasma column.

The possibility of linkage of field lines in the framework of the helical equilibrium can be examined through calculating the corresponding field helicities. The vector potential $\vec{A}$ is defined by $\nabla \times \vec{A} = \vec{B}$, so that the magnetic helicity K reads

$$K = \int_V \vec{A} \cdot \vec{B} \, \mathrm{d}V = \frac{\mu_0 j_z R B_z}{4} \int_V r \sin(\theta - \kappa z) \, \mathrm{d}V \equiv 0, \tag{13}$$

due to the periodicity of the sine function. This means that the magnetic lines are not linked. On the other hand, for the flow helicity H we obtain

$$H = \int_V \vec{v} \cdot \vec{w} \, \mathrm{d}V = -\kappa R^2 \Omega^2 \int_V \mathrm{d}V + \kappa R \Omega^2 \int_V r \sin(\theta - \kappa z) \, \mathrm{d}V = -\kappa R^2 \Omega^2 \int_V \mathrm{d}V, \tag{14}$$

which does not vanish. In other words, we arrive at the interesting result that the helical structure has an identically null magnetic helicity and a finite flow helicity.

The stability of the helical equilibrium can be roughly investigated on the basis of the electromagnetic induction. Indeed, through considering a (incompressible) small perturbation $\delta\vec{v}$ on the velocity field, the longitudinal component of the linearized (up to first-order terms) Faraday law yields

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} - \frac{\mu_0}{\eta} \frac{\partial}{\partial t} \right) \hat{z} \cdot \delta\vec{v} = 0. \tag{15}$$

Therefore, by regarding the perturbed velocity (whose amplitude is assumed to depend on r only) to fluctuate harmonically as $\exp[\imath(\gamma t - m\theta - kz)]$, equation (15) can be expressed as

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \left(c^2 + \frac{m^2}{r^2} \right) \right] \hat{z} \cdot \delta\vec{v} = 0, \tag{16}$$

provided the relation

$$c^2 = k^2 + \imath\gamma \frac{\mu_0}{\eta} \tag{17}$$

is satisfied. The solutions to equation (16) can be written in terms of conveniently chosen (in accordance with further boundary conditions to be imposed on the perturbed velocity) linear combinations of Bessel functions $J_m(\pm\imath x)$, where $x^2 = c^2 r^2$. In particular, by requiring $k^2 > c^2$, we must have $\imath\gamma < 0$ (for γ purely imaginary) and stability arises.

Let us now consider the possibility that the snakes observed at the $q = 1$ rational magnetic surface in tokamak discharges result from a local transition to this helical equilibrium. The parallel ion flow velocity in an axisymmetric equilibrium configuration is given by

$$v_{\mathrm{i}\|} = h_\theta v_{\mathrm{i}\theta} + h_\varphi v_{\mathrm{i}\varphi}, \tag{18}$$

where $h_\theta = B_\theta / B$, and $h_\varphi = B_\varphi / B$ are the metrics of the magnetic field lines, and $v_{\mathrm{i}\theta}$, and $v_{\mathrm{i}\varphi}$ are the poloidal, and toroidal components of the ion flow velocity, respectively. Within the neo-classical framework, the poloidal component is expressed as

$$v_{\mathrm{i}\theta} = \frac{\kappa_{\mathrm{coll}}}{m_{\mathrm{i}} \Omega_{\mathrm{i}}} \frac{\partial T_{\mathrm{i}}}{\partial r}, \tag{19}$$

where m_{i}, Ω_{i} and T_{i} are the ion mass, gyrofrequency and temperature, respectively, and κ_{coll} is a constant which depends on the collisionality regime [10]. The toroidal component, on the other hand, has a radial gradient which, in the absence of external momentum sources, is essentially proportional to $(\partial T_{\mathrm{i}} / \partial r)^2$.

When an externally injected pellet crosses the $q = 1$ rational magnetic surface, it is strongly ablated, causing a fast drop in the temperature inside a flux tube along the particular helical field line it traverses [11]. Therefore, a helical perturbation on the ion temperature gradient arises. The residual flow is thus disrupted and we argue that, since the effect on the toroidal component is stronger, the balance may be restored with a transition to a local helical equilibrium with flow, which would then be the snake.

We realize that the proposed model demands self-consistency. In particular, it is necessary to show that the helical magnetic flux function $\psi(r, \theta, z) = \kappa r A_\theta + A_z$ [12], which in the present context may be written as

$$\psi(r, \theta, z) = \frac{\kappa B_z r^2}{2} - \frac{\mu_0 j_z R r}{2}\left[\frac{r}{2R} - \sin(\theta - \kappa z)\right], \tag{20}$$

can match smoothly onto the magnetic equilibrium configuration surrounding the snakes. This is not a trivial problem given that in many cases of interest a $m = 1$ magnetic island may also be present. Even though, we believe that our model can motivate further investigations, both experimental and theoretical.

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