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Effect of up–down and left–right asymmetry of dust and/or heavy impurity distribution on plasma dynamics in the tokamak edge

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Abstract
The potentially important role of dust in future large power wall loadings and long operation times expected for new devices such as ITER is now recognized. In this context, we have studied the effect of the dust poloidal distribution on the plasma dynamics of the plasma edge. This dust spatial distribution can be natural or artificially induced by dust puffing aiming to act on the plasma dynamics. We show that, in the considered range of dust densities, in large-aspect-ratio tokamaks, the poloidal distribution of dust affects only the toroidal velocity. The poloidal velocity and the radial electric field are defined by the ion–dust friction and by the ion–dust thermal forces with the magnetic surface averaged dust density. It also follows from calculations presented in the paper that, in spherical tokamaks, the poloidal distribution of dust can also affect the plasma poloidal velocity and the radial electric field, in addition to affecting the plasma toroidal velocity. These results are also applicable to heavy impurities presented in tokamaks.

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1. Introduction

It is well-known that plasma dynamics, namely, poloidal $U_{i\theta}$ and toroidal $U_{i\varsigma}$ velocities and, consequently, the sheared radial electric field E_r, play an important role in tokamak plasmas, being responsible for improved confinement regimes [1]. In conventional tokamaks these values mainly depend on the ion viscosity or on the ion and electron friction with neutrals. Now it is well-recognized that dust can be presented in tokamak plasmas in some conditions, and that the dust problem becomes more worrisome for future large power wall loadings and long operation times expected for new devices such as ITER [2]. It is also noted in [2] that a strong radial profile of dust density can exist in tokamaks. Theoretical and experimental results also indicate that dust is mainly located in the tokamak bottom (divertor region) [2]. Other general information on dust in laboratory and space plasmas can be found, e.g. in [3].

Recently it was demonstrated [4] that plasma dynamics in tokamaks with dust crucially depends on the parameter $\alpha = (R^2/\lambda_i^2) n_d Z_d^2/(n_i Z_i^2) \gtrsim 1$, where R is the major radius of the plasma column, $\lambda_i = v_{Ti}/\nu_i$ is the ion mean free path, $v_{Ti} = \sqrt{2T_i/M_i}$ is the ion thermal velocity, ν_i is the ion collision frequency, T_i and M_i are the ion temperature and mass, n_d, Z_d and n_i, Z_i are the dust number density and charge and the ion number density and charge, correspondingly. Accepting that at the plasma edge the conditions $R^2 \gg \lambda_i^2$ and $Z_d/Z_i \gg 1$ are satisfied, we come at the conclusion that even a very small amount of dust, $n_d/n_i \gtrsim (\lambda_i^2/R^2)(Z_i^2/Z_d^2)$, changes the viscous dependence of the plasma dynamics to its dust dependence. In this work, distinct to [4], we consider the effect of a dust poloidal distribution on the plasma dynamics at the plasma edge. We mean that we study plasma dynamics inside the separatrix and not in the scrape off layer. As was mentioned above, there is a natural up–down asymmetry of the dust number density distribution in tokamaks. We model this distribution by the equation $n_d = n_{d0} - n_{ds}\sin\theta$, where $n_{d0} \geqslant |n_{ds}|$ and θ is the poloidal angle. In addition, the left–right asymmetry of dust in a tokamak can be artificially induced with the goal to act on the plasma dynamics. We model this distribution by the equation $n_d = n_{d0} - n_{dc}\cos\theta$, where $n_{d0} \geqslant |n_{dc}|$. We analyze the effect of such a dust spatial distribution on plasma dynamics at the plasma edge, finding the dependence of the poloidal $U_{i\theta}$ and toroidal $U_{i\varsigma}$ velocities and sheared radial electric field E_r on the parameters n_{d0}, n_{ds}, and n_{dc}, and determining conditions when the viscous dependence of the plasma dynamics is changed to its dust dependence. We conclude that there is a possibility to effectively act on the plasma dynamics at the plasma edge by puffing dust into different regions of tokamaks.

Although we mainly write about dust throughout the paper, all its results and conclusions are valid for heavy impurities in tokamaks satisfying the condition $M_i/M_I \ll 1$ and $Z_I n_I \ll \{n_e, Z_i n_i\}$, where M_I, Z_I and n_I are the impurity mass, charge and number density, respectively. In this case the subscript 'd' should be replaced by the subscript 'I'. The summation should be fulfilled over indices 'd' or 'I' in the presence of many dust or impurity species in the tokamak plasma.

2. Electrostatic potential

We consider the quasineutrality condition of the form

$$-en_e + eZ_i n_i - eZ_d n_d \simeq 0, \tag{1}$$

where $-e \equiv e_e$ is the electron charge, n_e is the electron number density, $eZ_i \equiv e_i$ and $-eZ_d \equiv e_d$ are the ion and dust effective charges, respectively. In addition, we assume that $Z_d n_d \ll \{n_e, Z_i n_i\}$, $M_i \ll M_d$, M_d is the dust particle mass, and dust particles are motionless (dust temperature is zero). From equation (1), we find that poloidal variations of the dust spatial distribution leads to the poloidal variations of ion and electron densities. Assuming $n_d = n_{d0} + n_d(\theta)$, $n_i = n_{i0} + n_i(\theta)$ and $n_e = n_{e0} + n_e(\theta)$, where

$$n_d(\theta) = -n_{ds}\sin\theta - n_{dc}\cos\theta, \tag{2}$$

we obtain by accepting $n_0 = n_{e0} \simeq Z_i n_{i0}$,

$$-n_{e0} + Z_i n_{i0} - Z_d n_{d0} \simeq 0, \tag{3}$$

and

$$-n_e(\theta) + Z_i n_i(\theta) - Z_d n_d(\theta) \simeq 0. \tag{4}$$

From equation (2) we find, in the case of simultaneous left–right and up–down asymmetries, the condition

$$n_{d0} \geqslant \sqrt{n_{ds}^2 + n_{dc}^2}. \tag{5}$$

We assume stationary charge dust, therefore for ions and electrons we accept the Boltzmann distribution, $n_{i,e} = n_{i,e0}\exp(-e_{i,e}\psi/T_{i,e})$, where ψ is the electric field potential. Consequently, we get from equation (4)

$$\psi(\theta) \simeq -Z_d n_d(\theta) \Big/ \left(\frac{en_{e0}}{T_{e0}} + \frac{eZ_i^2 n_{i0}}{T_{i0}}\right), \tag{6}$$

$$n_i(\theta) \simeq -\frac{eZ_i n_{i0}}{T_{i0}}\psi(\theta) = \frac{Z_d n_d(\theta)}{T_{i0}} \Big/ \left(\frac{1}{T_{e0}} + \frac{Z_i}{T_{i0}}\right), \tag{7}$$

and

$$n_e(\theta) \simeq -\frac{Z_d n_d(\theta)}{T_{e0}} \Big/ \left(\frac{1}{T_{e0}} + \frac{Z_i}{T_{i0}}\right). \tag{8}$$

3. Starting and magnetic surface averaged equations

For the following calculations we need the summed ion and electron momentum equations [4]–[6]

$$-\nabla p - \nabla\cdot\boldsymbol{\pi}_\parallel^i - \nabla\cdot\boldsymbol{\pi}_\perp^i - eZ_d n_d\nabla\psi + \frac{1}{c}\left[\mathbf{j}\times\mathbf{B}\right] + \mathbf{R} \simeq 0. \tag{9}$$

Here $p = p_e + p_i$ is the plasma pressure, $\boldsymbol{\pi}^i$ is the ion viscosity tensor, $\mathbf{j}$ is the plasma current density, $\mathbf{B}$ is the magnetic field vector, $\mathbf{R}$ is the friction force between ions and dust including the thermal force $\mathbf{R}_T$, where [4]

$$\mathbf{R} \simeq -M_i n_i \nu_{id}\left(0.51\mathbf{V}_{i\parallel} + \mathbf{V}_{i\perp}\right) + \mathbf{R}_T, \tag{10}$$

$$\mathbf{R}_T \simeq -2.69\frac{n_d Z_d^2}{Z_i^2}\mathbf{b}\nabla_\parallel T_i + \frac{3}{2}\frac{\nu_{id}n_i}{\omega_{ci}}\left[\mathbf{b}\times\nabla T_i\right], \quad \mathbf{b} = \frac{\mathbf{B}}{B}, \tag{11}$$

$$\nabla_\parallel = \mathbf{b}\cdot\nabla, \qquad \mathbf{V}_{i\parallel} = \mathbf{b}\left(\mathbf{b}\cdot\mathbf{V}_i\right) \equiv \mathbf{b}V_{i\parallel},$$
$$\mathbf{V}_{i\perp} = \left[\mathbf{b}\times\left[\mathbf{V}_i\times\mathbf{b}\right]\right], \tag{12}$$

and

$$\nu_{id} = \nu_{id0}\frac{n_d}{n_{d0}}, \tag{13}$$

is the ion–dust collision frequency satisfying the inequalities $\nu_{id} \ll \omega_{ci}$ and $\nu_{id} \gg \upsilon_{Ti}/qR$, where

$$\nu_{id0} = \frac{4\sqrt{2\pi}\lambda e^4 Z_i^2 Z_d^2 n_{d0}.}{3\sqrt{M_i}T_i^{3/2}}, \tag{14}$$

$q = rB_\zeta/RB_\theta$ is the safety factor, r is the minor radius of the plasma column, B_ζ and B_θ are the toroidal and poloidal components of the magnetic field $\mathbf{B}$, correspondingly, $\mathbf{V}_i$ is the ion velocity, ω_{ci} is the ion cyclotron frequency and λ is the Coulomb logarithm [5]. The latter inequalities correspond to the case of a collisional plasma relevant for the plasma edge.

The required relations can be found from the parallel and ζ-covariant projections of equation (9). In the first case, we have

$$\frac{\partial p}{\partial\theta} + \frac{\mathbf{b}}{B^\theta}\cdot\left(\nabla\cdot\boldsymbol{\pi}_\parallel^i\right) - eZ_d n_d\frac{\partial\psi}{\partial\theta}$$
$$+0.51\frac{M_i n_i}{B^\theta}\nu_{id}V_{i\parallel} + 2.69\frac{n_d Z_d^2}{Z_i^2}\frac{\partial T_i}{\partial\theta} \simeq 0, \tag{15}$$

where the approximate expression $b^\theta \simeq 1/qR$. has been used for the θ-contravariant projection of the vector $\mathbf{b}$. Averaging equation (15) over θ we obtain

$$\left\langle \frac{\mathbf{b}}{B^\theta}\cdot\left(\nabla\cdot\boldsymbol{\pi}^{\rm i}_{\parallel}\right)\right\rangle + 0.51 M_{\rm i}\frac{\nu_{\rm id0}}{n_{\rm d0}}\left\langle n_{\rm i} n_{\rm d}\frac{V_{\rm i\parallel}}{B^\theta}\right\rangle + \frac{2.69 Z_{\rm d}^2}{Z_{\rm i}^2}\left\langle n_{\rm d}\left(\theta\right)\frac{\partial T_{\rm i}}{\partial\theta}\right\rangle \simeq 0, \quad (16)$$

where $\langle\cdots\rangle = \int_0^{2\pi}(\cdots)\,{\rm d}\theta/2\pi$ and [4, 8]

$$\left\langle \frac{\mathbf{b}}{B^\theta}\cdot\left(\nabla\cdot\boldsymbol{\pi}^{\rm i}_{\parallel}\right)\right\rangle = \frac{3\cdot 1.92}{4R}\frac{p_{\rm i}\epsilon}{\nu_{\rm i}} \times\left[\frac{U_{{\rm i}\theta} + 1.83 U_{\rm Ti} - (0.19 U_{{\rm i}\theta} + 1.52 U_{\rm Ti})\left\langle \frac{\partial \ln B}{\partial\theta}\frac{\partial \ln n_{\rm i}}{\partial\theta}\right\rangle}{\left\langle\left(\frac{\partial \ln B}{\partial\theta}\right)^2\right\rangle}\right], \quad (17)$$

here $U_{\rm Ti} = (1/M_{\rm i}\omega_{\rm ci})\partial T_{\rm i}/\partial r$ and $\epsilon = r/R$. The last equation can be rewritten in the form

$$\left\langle \frac{\mathbf{b}}{B^\theta}\cdot\left(\nabla\cdot\boldsymbol{\pi}^{\rm i}_{\parallel}\right)\right\rangle = \frac{1.44}{R}\frac{p_{\rm i}\epsilon}{\nu_{\rm i}} \times\left[U_{{\rm i}\theta} + 1.83 U_{\rm Ti} - \frac{n_{\rm ic}}{\epsilon n_{\rm i0}}\left(0.19 U_{{\rm i}\theta} + 1.52 U_{\rm Ti}\right)\right]. \quad (18)$$

From [4, 5] we also get

$$\frac{1}{qR}\frac{\partial q_{\rm i\parallel}}{\partial\theta} + \nabla\cdot\mathbf{q}_{\rm i}\perp - T_{\rm i}V_{\rm i}^\theta\frac{\partial n_{\rm i}}{\partial\theta} + \frac{3M_{\rm e}n_{\rm e}\nu_{\rm e}}{M_{\rm i}}\left(T_{\rm i} - \langle T_{\rm i}\rangle_\theta\right) = 0, \quad (19)$$

and, consequently,

$$\frac{\partial T_{\rm i}}{\partial\theta} = 1.28\frac{bT_{\rm i}}{\nu_{\rm i} d(b)}\left[2\frac{U_{\rm Ti}}{r}\frac{\partial^2 \ln B}{\partial\theta^2} + \left(\frac{2}{5}\left\langle V_{\rm i}^\theta\right\rangle - \frac{U_{\rm Ti}}{r}\right)\frac{\partial^2 \ln n_{\rm i}}{\partial\theta^2}\right], \quad (20)$$

where $d(b) = 1 + 2.2b\sqrt{M_{\rm e}}/M_{\rm i}$ and $b = q^2R^2/\lambda_{\rm i}^2$.

4. Ambipolarity condition

Now we turn to the ζ-covariant projection of equation (9). For an axially-symmetric tokamak, $\partial/\partial\zeta = 0$, we obtain from this equation

$$-\left(\nabla\cdot\boldsymbol{\pi}^{\rm i}_{\parallel}\right)_\zeta - \left(\nabla\cdot\boldsymbol{\pi}^{\rm i}_{\perp}\right)_\zeta + \frac{\sqrt{g}}{c}j^r B^\theta + R_\zeta \simeq 0, \quad (21)$$

where

$$R_\zeta \simeq -0.51 M_{\rm i} n_{\rm i}\nu_{\rm id} V_{{\rm i}\zeta} - 2.69\frac{n_{\rm d}Z_{\rm d}^2}{Z_{\rm i}^2 q}\frac{\partial T_{\rm i}}{\partial\theta} - \tfrac{3}{2}\nu_{\rm id} n_{\rm i} M_{\rm i}\sqrt{g}b^\theta U_{\rm Ti}, \quad (22)$$

g is the determinant of the metric tensor, and j^r is the r-contravariant projection of $\mathbf{j}$. It was shown in [4] that, using the ambipolarity condition,

$$\int_o^{2\pi} j^r\sqrt{g}\,{\rm d}\theta = 0, \quad (23)$$

we obtain

$$\left\langle\sqrt{g}\left(\nabla\cdot\boldsymbol{\pi}^{\rm i}_{\parallel}\right)_\zeta\right\rangle = 0. \quad (24)$$

Multiplying equation (21) by $\sqrt{g}$ and using equations (22)–(24) and the Maxwell equation $\partial(\sqrt{g}B^\theta)/\partial\theta = 0$, we arrive at

$$\left\langle\sqrt{g}\left\{\left(\nabla\cdot\boldsymbol{\pi}^{\rm i}_{\perp}\right)_\zeta + \frac{\nu_{\rm id0}}{n_{\rm d0}}n_{\rm d}n_{\rm i}M_{\rm i}\left(0.51 V_{{\rm i}\zeta} + \frac{1.91}{\nu_{\rm i} q M_{\rm i}}\frac{\partial T_{\rm i}}{\partial\theta} + \frac{3}{2}\sqrt{g}b^\theta U_{\rm Ti}\right)\right\}\right\rangle \simeq 0. \quad (25)$$

In equation (25) the ratio of the viscous term to the friction term, in the neoclassical consideration, is of the order of $(n_{\rm i}Z_{\rm i}^2/n_{\rm d}Z_{\rm d}^2)\rho_{\rm i}^2/r^2$, where $\rho_{\rm i} = v_{\rm Ti}/\omega_{\rm ci}$ is the ion Larmor radius. This viscous term can be neglected for $n_{\rm d}/n_{\rm i} > (Z_{\rm i}^2/Z_{\rm d}^2)\rho_{\rm i}^2/r^2$, i.e. at a very small dust density. In absence of dust it produces the toroidal velocity $U_{{\rm i}\zeta} \sim 0.1(q^3/\epsilon)U_{{\rm i}\theta}$ (see [9]). Thus, we can approximate this term as [9]

$$\left\langle\sqrt{g}\left(\nabla\cdot\boldsymbol{\pi}^{\rm i}_{\perp}\right)_\zeta\right\rangle\langle\sqrt{g}\rangle \approx 0.6 R M_{\rm i}\frac{\partial}{\partial r} \times\left[n_{\rm i}\nu_{\rm i}\rho_{\rm i}^2\left(\frac{\partial U_{{\rm i}\zeta}}{\partial r} - 0.107 q^2\frac{\partial \ln T_{\rm i}}{\partial r}\frac{q}{\epsilon}U_{{\rm i}\theta}\right)\right]. \quad (26)$$

Note that gyroviscosity, $\nabla\cdot\boldsymbol{\pi}^{\rm i}_{\wedge}$, can also be included in equation (9) producing, at some conditions, terms of the same order as those in equation (25) [10]. It is related to substitution of the collisional heat flux into the gyroviscosity [10]. Nevertheless, in our case the gyroviscosity can be omitted in equations (9) and (25). We represent

$$\left\langle\frac{V_{\rm i\parallel}}{B^\theta}\right\rangle = q\left\langle g_{\zeta\zeta}\right\rangle\left\langle V_{\rm i}^\zeta\right\rangle + q\left\langle\tilde{g}_{\zeta\zeta}\tilde{V}_{\rm i}^\zeta\right\rangle + \langle g_{\theta\theta}\rangle\left\langle V_{\rm i}^\theta\right\rangle + \left\langle\tilde{g}_{\theta\theta}\tilde{V}_{\rm i}^\theta\right\rangle, \quad (27)$$

$$\widetilde{\left(\frac{V_{\rm i\parallel}}{B^\theta}\right)} = q\left\langle g_{\zeta\zeta}\right\rangle\tilde{V}_{\rm i}^\zeta + q\tilde{g}_{\zeta\zeta}\left\langle V_{\rm i}^\zeta\right\rangle + \tilde{g}_{\theta\theta}\left\langle V_{\rm i}^\theta\right\rangle + \langle g_{\theta\theta}\rangle\tilde{V}_{\rm i}^\theta, \quad (28)$$

and

$$\langle\sqrt{g}n_{\rm d}n_i V_{{\rm i}\zeta}\rangle = \left\langle\sqrt{g}g_{\zeta\zeta}n_{\rm d}n_i V_{\rm i}^\zeta\right\rangle = n_{\rm d0}n_{\rm i0}\langle\sqrt{g}\rangle\langle g_{\zeta\zeta}\rangle\left\langle V_{\rm i}^\zeta\right\rangle + 2n_{\rm d0}n_{\rm i0}\langle\sqrt{g}\rangle\left\langle\tilde{g}_{\zeta\zeta}\tilde{V}_{\rm i}^\zeta\right\rangle + n_{\rm i0}\langle\sqrt{g}\rangle\langle g_{\zeta\zeta}\rangle\left\langle n_{\rm d}\left(\theta\right)V_{\rm i}^\zeta\right\rangle. \quad (29)$$

5. Perturbed values

In addition, we should use the frozen-in condition,

$$\nabla\times[\mathbf{V}\times\mathbf{B}] = 0. \quad (30)$$

From this equation we find oscillating values

$$\tilde{V}_{\rm i}^\zeta = q\tilde{V}_{\rm i}^\theta. \quad (31)$$

We take the metric tensor components and their determinant as follows [8]

$$g_{rr} = 1, \quad g_{\theta\theta} = \epsilon^2 g_{\zeta\zeta}, \quad g_{\zeta\zeta} = R^2\left(1 - 2\epsilon\cos\theta\right), \quad \sqrt{g} = \epsilon g_{\zeta\zeta}. \quad (32)$$

Hence, $\langle V_{\rm i}^{\zeta}\rangle \simeq U_{\rm i\zeta}/R$ and $\langle V_{\rm i}^{\theta}\rangle \simeq U_{\rm i\theta}/r$. Then, from the ion continuity equation,

$$\nabla\cdot(n_{\rm i}\mathbf{V})\approx 0, \qquad (33)$$

we obtain

$$\frac{\partial}{\partial\theta}\left(\sqrt{g}n_{\rm i}V_{\rm i}^{\theta}\right)\approx 0. \qquad (34)$$

As a result, we have

$$V_{\rm i}^{\theta}\approx\frac{U_{\rm i\theta}}{r}\left[1+2\epsilon\cos\theta-\frac{Z_{\rm i}Z_{\rm d}n_{\rm d}(\theta)n_0T_{\rm i0}}{\left(\frac{1}{T_{\rm e0}}+\frac{Z_{\rm i}}{T_{\rm i0}}\right)}\right]. \qquad (35)$$

We represent

$$\tilde{V}_{\rm i}^{\theta}=V_{\rm ic}^{\theta}\cos\theta+V_{\rm is}^{\theta}\sin\theta, \qquad (36)$$

and

$$\tilde{V}_{\rm i}^{\zeta}=V_{\rm ic}^{\zeta}\cos\theta+V_{\rm is}^{\zeta}\sin\theta, \qquad (37)$$

where definitions are obvious.

Now equations (27) and (29) can be rewritten in the form

$$\left\langle\frac{V_{\rm i\parallel}}{B^{\theta}}\right\rangle=qRV_{\rm i\parallel 0}-qrRV_{\rm ic}^{\zeta}, \qquad (38)$$

and

$$\left\langle\sqrt{g}n_{\rm d}n_{\rm i}V_{\rm i\zeta}\right\rangle=n_{\rm d0}n_{\rm i0}rR^2U_{\rm i\zeta}-2n_{\rm d0}n_{\rm i0}r^2R^2V_{\rm ic}^{\zeta}$$
$$-\tfrac{1}{2}n_{\rm i0}rR^3(n_{\rm ds}V_{\rm is}^{\zeta}+n_{\rm dc}V_{\rm ic}^{\zeta}), \qquad (39)$$

where $V_{\rm i\parallel 0}=U_{\rm i\zeta}+(\epsilon/q)U_{\rm i\theta}$. In addition, we have from equation (16)

$$\left\langle n_{\rm i}n_{\rm d}\frac{V_{\rm i\parallel}}{B^{\theta}}\right\rangle=n_{\rm i0}n_{\rm d0}(qRV_{\rm i\parallel 0}-qrRV_{\rm ic}^{\zeta})$$
$$-\tfrac{1}{2}n_{\rm i0}q^2R^2\left(n_{\rm ds}V_{\rm is}^{\theta}+n_{\rm dc}V_{\rm ic}^{\theta}\right)+n_{\rm i0}qrV_{\rm i\parallel 0}n_{\rm dc}, \qquad (40)$$

from equations (15) and (20)

$$\left\langle n_{\rm d}(\theta)\frac{\partial T_{\rm i}}{\partial\theta}\right\rangle=1.28\frac{bT_{\rm i0}}{r\nu_{\rm i}d(b)}$$
$$\times\left[\epsilon n_{\rm dc}U_{\rm Ti}+\frac{1}{2n_{\rm i0}}\left(\tfrac{2}{5}U_{\rm i\theta}-U_{\rm Ti}\right)(n_{\rm ds}n_{\rm is}+n_{\rm dc}n_{\rm ic})\right], \qquad (41)$$

and from equations (20) and (25)

$$\frac{2.44}{qr}\frac{\nu_{\rm id0}}{\nu_{\rm i}}\frac{n_{\rm i}}{n_{\rm d0}}\frac{bT_{\rm i}}{\nu_{\rm i}d(b)}$$
$$\times\left\langle\sqrt{g}n_{\rm d}\left[2U_{\rm Ti}\frac{\partial^2\ln B}{\partial\theta^2}+\left(\tfrac{2}{5}U_{\rm i\theta}-U_{\rm Ti}\right)\frac{\partial^2\ln n_{\rm i}}{\partial\theta^2}\right]\right\rangle$$
$$=\frac{2.44}{q}\frac{\nu_{\rm id0}}{\nu_{\rm i}}\frac{bn_{\rm i0}T_{\rm i}R}{\nu_{\rm i}d(b)}\left[U_{\rm Ti}\epsilon\left(2\epsilon+\frac{n_{\rm dc}}{n_{\rm d0}}\right)+\left(\tfrac{2}{5}U_{\rm i\theta}-U_{\rm Ti}\right)\right.$$
$$\left.\times\left(\epsilon\frac{n_{\rm ic}}{n_{\rm i0}}+\frac{1}{2}\left(\frac{n_{\rm ds}}{n_{\rm d0}}\frac{n_{\rm is}}{n_{\rm i0}}+\frac{n_{\rm dc}}{n_{\rm d0}}\frac{n_{\rm ic}}{n_{\rm i0}}\right)\right)\right]. \qquad (42)$$

6. Plasma flow velocities

We transform equation (16)

$$U_{\rm i\theta}+1.83U_{\rm Ti}-\frac{n_{\rm ic}}{\epsilon n_{\rm io}}(0.19U_{\rm i\theta}+1.52U_{\rm Ti})$$
$$+\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\frac{R^2}{\lambda_{\rm i}^2}\frac{q}{\epsilon}\left[U_{\rm i}\zeta+\frac{\epsilon}{q}U_{\rm i\theta}-rV_{\rm ic}^{\zeta}-\tfrac{1}{2}qR\right.$$
$$\left.\times\left(\frac{n_{\rm ds}}{n_{\rm d0}}V_{\rm is}^{\theta}+\frac{n_{\rm dc}}{n_{\rm d0}}V_{\rm ic}^{\theta}\right)+\epsilon\left(U_{\rm i}\zeta+\frac{\epsilon}{q}U_{\rm i\theta}\right)\frac{n_{\rm dc}}{n_{\rm d0}}\right]$$
$$+2.39\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\frac{bR^2}{r^2d(b)}\left[\epsilon\frac{n_{\rm dc}}{n_{\rm d0}}U_{\rm Ti}+\frac{1}{2n_{\rm i0}n_{\rm d0}}\right.$$
$$\left.\times\left(\frac{2}{5}U_{\rm i\theta}-U_{\rm Ti}\right)(n_{\rm ds}n_{\rm is}+n_{\rm dc}n_{\rm ic})\right]=0 \qquad (43)$$

and equation (25)

$$\frac{1}{n_{\rm i}\nu_{\rm i}}\frac{\partial}{\partial r}\left[n_{\rm i}\nu_{\rm i}\rho_{\rm i^2}\left(\frac{\partial U_{\rm i\zeta}}{\partial r}-0.107q^2\frac{\partial\ln T_{\rm i}}{\partial r}\frac{q}{\epsilon}U_{\rm i\theta}\right)\right]$$
$$+1.2\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\left[U_{\rm i}\zeta-2rV_{\rm ic}^{\zeta}-\frac{1}{2}\frac{R}{n_{\rm d0}}\left(n_{\rm ds}V_{\rm is}^{\zeta}+n_{\rm dc}V_{\rm ic}^{\zeta}\right)\right]$$
$$+2.88\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\frac{q}{\epsilon d(b)}\left[U_{\rm Ti}\epsilon\left(2\epsilon+\frac{n_{\rm dc}}{n_{\rm d0}}\right)+\left(\frac{2}{5}U_{\rm i\theta}-U_{\rm Ti}\right)\right.$$
$$\left.\times\left(\epsilon\frac{n_{\rm ic}}{n_{\rm i0}}+\frac{1}{2}\left(\frac{n_{\rm ds}}{n_{\rm d0}}\frac{n_{\rm is}}{n_{\rm i0}}+\frac{n_{\rm dc}}{n_{\rm d0}}\frac{n_{\rm ic}}{n_{\rm i0}}\right)\right)\right]$$
$$+3.54\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\frac{\epsilon}{q}U_{\rm Ti}\simeq 0. \qquad (44)$$

Then, we substitute necessary values into equations (43) and (44)

$$U_{\rm i\theta}+1.83U_{\rm Ti}+\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\frac{R^2}{\lambda_{\rm i}^2}\frac{q}{\epsilon}\left\{U_{\rm i\zeta}+\frac{\epsilon}{q}U_{\rm i\theta}\left(1-2q^2\right)\right.$$
$$\left.-\frac{q^2}{\epsilon}\frac{\epsilon}{q}U_{\rm i\theta}\left[\frac{n_{\rm dc}}{n_{\rm d0}}+\frac{Z_{\rm d}(n_{\rm ds}^2+n_{\rm dc}^2)}{2\epsilon n_0n_{\rm d0}}\frac{Z_{\rm i}}{T_{\rm i0}}\Big/\left(\frac{1}{T_{\rm e0}}+\frac{Z_{\rm i}}{T_{\rm i0}}\right)\right]\right\}$$
$$+2.4\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\frac{1}{\epsilon d(b)}\frac{q^2R^2}{\lambda_{\rm i^2}}\left[\frac{n_{\rm dc}}{n_{\rm i0}}U_{Ti}-\frac{Z_{\rm d}\left(n_{\rm ds}^2+n_{\rm dc}^2\right)}{2\epsilon n_0n_{\rm d0}}\right.$$
$$\left.\times\left(\tfrac{2}{5}U_{\rm i\theta}-U_{Ti}\right)\frac{Z_{\rm i}}{T_{\rm i0}}\Big/\left(\frac{1}{T_{\rm e0}}+\frac{Z_{\rm i}}{T_{\rm i0}}\right)\right]=0 \qquad (45)$$

and

$$\frac{0.83}{n_{\rm i}\nu_{\rm i}}\frac{\partial}{\partial r}\left[n_{\rm i}\nu_{\rm i}\rho_{\rm i}^2\left(\frac{\partial U_{\rm i\zeta}}{\partial r}-0.107q^2\frac{\partial\ln T_{\rm i}}{\partial r}\frac{q}{\epsilon}U_{\rm i\theta}\right)\right]$$
$$+\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\left\{U_{\rm i\zeta}-4\epsilon qU_{\rm i\theta}-qU_{\rm i\theta}\right.$$
$$\left.\times\left[\frac{n_{\rm dc}}{n_{\rm d0}}+\frac{Z_{\rm d}\left(n_{\rm ds}^2+n_{\rm dc}^2\right)}{2\epsilon n_0n_{\rm d0}}\frac{Z_{\rm i}}{T_{\rm i0}}\Big/\left(\frac{1}{T_{\rm e0}}+\frac{Z_{\rm i}}{T_{\rm i0}}\right)\right]\right\}$$
$$+2.4\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\frac{q}{d(b)}\left\{U_{\rm Ti}\left(2\epsilon+\frac{n_{\rm dc}}{n_{\rm d0}}\right)\right.$$
$$\left.-\left(\tfrac{2}{5}U_{\rm i\theta}-U_{\rm Ti}\right)\frac{Z_{\rm d}\left(n_{\rm ds}^2+n_{\rm dc}^2\right)}{2\epsilon n_0n_{\rm d0}}\frac{Z_{\rm i}}{T_{\rm i0}}\Big/\left(\frac{1}{T_{\rm e0}}+\frac{Z_{\rm i}}{T_{\rm i0}}\right)\right\}$$
$$+2.95\frac{Z_{\rm d}^2n_{\rm d0}}{Z_{\rm i}^2n_{\rm i0}}\frac{\epsilon}{q}U_{\rm Ti}\simeq 0. \qquad (46)$$

Equations (45) and (46) are the basic equations for the analysis that follows.

7. Analyses of flow velocities and the radial electric field

7.1. The conventional neoclassical case $Z_d^2 n_d 0/(Z_i^2 n_i 0) \ll \rho_i^2/r^2$

We find from equations (45) and (46) the well-known equations [9, 11, 12]

$$U_{i\theta} + 1.83U_{Ti} = 0, \tag{47}$$

and

$$\frac{1}{n_i\nu_i}\frac{\partial}{\partial r}\left[n_i\nu_i\rho_i^2\left(\frac{\partial U_{i\zeta}}{\partial r} - 0.107q^2\frac{\partial \ln T_i}{\partial r}\frac{q}{\epsilon}U_{i\theta}\right)\right] = 0. \tag{48}$$

Substituting solutions of equations (47) and (48) into the equation for the radial electric field

$$E_r \approx \frac{1}{c}(-B_\zeta U_{i\theta} + B_\theta U_{i\zeta} + BU_{pi}), \qquad U_{pi} = \frac{c}{e_i n_i B}\frac{\partial p_i}{\partial r}, \tag{49}$$

we can obtain the well-known neoclassical result [9, 11, 12].

7.2. The case of the negligible perpendicular viscosity $\lambda_i^2/R^2 \gg Z_d^2 n_{d0}/(Z_i^2 n_{i0}) \gg \rho_i^2/r^2$

From equations (45) and (46) we find

$$U_{i\theta} = -1.83U_{Ti}, \tag{50}$$

and

$$U_{i}\zeta = -U_{Ti}\left[\frac{\epsilon}{q}\left(2.95 + 7.32q^2 + 4.8\frac{q^2}{d(b)}\right) + q\frac{n_{dc}}{n_{d0}}\left(1.83 + \frac{2.4}{d(b)}\right)\right]. \tag{51}$$

Then we obtain

$$E_r \approx \frac{B}{c}U_{Ti}\left(2.83 + \frac{\kappa_n}{\kappa_T}\right), \tag{52}$$

where $\kappa_n = d\ln n/d\ln r$ and $\kappa_T = d\ln T_{i0}/d\ln r$. We see that, in this case, the poloidal distribution of dust affects only the toroidal velocity. The poloidal velocity and the radial electric field are practically neoclassical.

7.3. The case of the negligible plasma viscosity $\epsilon Z_d/Z_i \gg Z_d^2 n_{d0}/(Z_i^2 n_{i0}) \gg \lambda_i^2/R^2$

We find from equations (45) and (46)

$$U_{i\theta} = 2.95(1+2q^2)^{-1}U_{Ti}\left[1 + \frac{1.63q^2}{d(b)}\right], \tag{53}$$

$$U_{i\zeta} = -\frac{\epsilon}{q}\left(1+2q^2\right)^{-1}U_{Ti}\left\{2.95\left[1 + \frac{1.63q^2}{d(b)}\right] \times\left(1 - 2q^2 - \frac{q^2}{\epsilon}\frac{n_{dc}}{n_{d0}}\right) + 2.4\frac{q^2}{\epsilon d(b)}\frac{n_{dc}}{n_{i0}}(1+2q^2)\right\}, \tag{54}$$

and

$$E_r \approx -\frac{B}{c}U_{Ti}\left\{2.95\left(1+2q^2\right)^{-1}\left[1 + \frac{1.63q^2}{d(b)}\right] - 1 - \frac{\kappa_n}{\kappa_T}\right\}. \tag{55}$$

We again conclude that the poloidal distribution of dust affects only the toroidal velocity while the poloidal velocity and the radial electric field are defined by the ion-dust friction and by the ion-dust thermal forces. Nevertheless, it is clear that in spherical tokamaks, where the parameter $\epsilon \sim 1$, the poloidal distribution of dust can also affect the plasma poloidal velocity and the radial electric field. Equations (53)–(55) correct the corresponding results of [4].

7.4. The large dust density $1 > Z_d\{n_{dc}, n_{ds}\}/(Z_i n_{i0}) \gtrsim \epsilon$

It follows from equations (45) and (46)

$$U_{i\theta} = 2.95\left(1+2q^2\right)^{-1}U_{Ti}\left[1 + \frac{1.63q^2}{d(b)}\right] \tag{56}$$

and

$$U_{i\zeta} = \frac{qZ_d(n_{ds}^2 + n_{dc}^2)}{2\epsilon n_0 n_{d0}}\frac{Z_i}{T_{i0}}\bigg/\left(\frac{1}{T_{e0}} + \frac{Z_i}{T_{i0}}\right) \times\left\{U_{i\theta}\left[1 + \frac{0.96}{d(b)}\right] - \frac{2.4}{d(b)}U_{Ti}\right\}, \tag{57}$$

i.e. the same equation for the poloidal velocity. Substituting equations (56) and (57) into equation (49), we obtain equation (55). It means that the dust poloidal distribution practically has no effect on the plasma poloidal velocity and the radial electric field, affecting only the plasma toroidal velocity.

8. Conclusion

Recognizing the potentially important role of dust in future large power wall loadings and long operation times expected for new devices such as ITER, we have studied the effect of the dust poloidal distribution on the plasma dynamics at the plasma edge. This dust spatial distribution can be 'natural', as evidence exists that dust can mainly be presented in the divertor region of future tokamaks, or 'artificial', induced by the dust puffing aiming to operate plasma dynamics. We have shown that, in the considered range of dust densities, the poloidal distribution of dust affects only the toroidal velocity while the poloidal velocity and the radial electric field are defined by the ion-dust friction and by the ion-dust thermal forces. The conclusion of [4] was confirmed that plasma dynamics in tokamaks with dust crucially depends on the parameter $\alpha = (R^2/\lambda_i^2)n_d Z_d^2/(n_i Z_i^2)$, changing the viscous dependence of the plasma dynamics to its dust dependence at $\alpha \gtrsim 1$. Note that, as follows from our calculations, in spherical tokamaks the poloidal distribution of dust can also affect the plasma poloidal velocity and the radial electric field in addition to affecting the plasma toroidal velocity. The results of the paper are also applicable to heavy impurities presented in tokamaks. We have considered only the spatial dust distribution of the form equation (2). Really, this distribution can be arbitrary. It seems that the general results of the paper are qualitatively valid for this arbitrary distribution if an amplitude of this distribution is of the order of amplitudes in equation (2). But only detailed numerical analysis can confirm or reject this assumption. Such an analysis could be a topic of future investigations.

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