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### From worldline to quantum superconformal mechanics with and without oscillatorial terms: $D(2, 1; \alpha)$ and $sl(2|1)$ models

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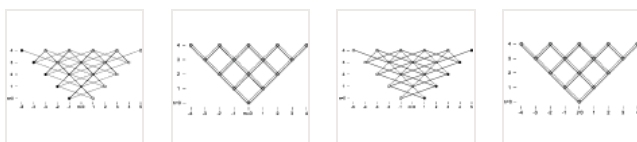
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#### ABSTRACT

In this paper, we quantize superconformal  $\sigma$  models defined by worldline supermultiplets. Two types of superconformal mechanics, with and without a De Alfaro Fubini Furlan (DFF) term, are considered. Without a DFF term (Calogero potential only), the supersymmetry is unbroken. The models with a DFF term correspond to deformed (if the Calogero potential is present) or undeformed oscillators. For these (un)deformed oscillators, the classical invariant superconformal algebra acts as a spectrum-generating algebra of the quantum theory. Besides the  $osp(1|2)$  examples, we explicitly quantize the superconformally invariant worldline  $\sigma$  models defined by the  $\mathcal{N} = 4$  (1, 4, 3) supermultiplet [with  $D(2, 1; \alpha)$  invariance, for  $\alpha \neq 0, -1$ ] and by the  $\mathcal{N} = 2$  (2, 2, 0) supermultiplet [with two-dimensional target and  $sl(2|1)$  invariance]. The parameter  $\alpha$  is the scaling dimension of the (1, 4, 3) supermultiplet and, in the DFF case, has a direct interpretation as a vacuum energy. In the DFF case, for the  $sl(2|1)$  models, the scaling dimension  $\lambda$  is quantized (either  $\lambda = \frac{1}{2} + \mathbb{Z}$  or  $\lambda = \mathbb{Z}$ ). The ordinary two-dimensional oscillator is recovered, after imposing a superselection restriction, from the  $\lambda = -\frac{1}{2}$  model. In particular, a single bosonic vacuum is selected. The spectrum of the unrestricted two-dimensional theory is decomposed into an infinite set of lowest-weight representations of  $sl(2|1)$ . Extra fermionic raising operators, not belonging to the original  $sl(2|1)$  superalgebra, allow (for  $\lambda = \frac{1}{2} + \mathbb{Z}$ ) to construct the whole spectrum from the two degenerate (one bosonic and one fermionic) vacua.




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## Editorial

[General Relativity Still Making Waves \(/prd/edannounce/10.1103/PhysRevD.92.050001\)](https://prd.edannounce/10.1103/PhysRevD.92.050001)

September 24, 2015

Clifford Will discusses the importance of Einstein's general theory of relativity and its relevance for physics research today.

[APS and CERN Sign Open Access Agreement for SCOAP<sup>3</sup> \(https://www.aps.org/publications/apsnews/updates/scoap3.cfm\)](https://www.aps.org/publications/apsnews/updates/scoap3.cfm)

APS and CERN, the host organization of SCOAP<sup>3</sup> (Sponsoring Consortium for Open Access Publishing in Particle Physics), have signed an agreement to make the high-energy physics (HEP) articles published in three leading APS journals open access beginning January 1, 2018. This agreement acts to support the publishing of open access content for wider benefit of the HEP community.

## General Relativity

[2015 - General Relativity's Centennial \(http://journals.aps.org/general-relativity-centennial\)](http://journals.aps.org/general-relativity-centennial)

The editors of the *Physical Review* journals have curated a collection of landmark papers on General Relativity to celebrate its centennial. These papers are currently free to read.

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