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Multiscale Matching of Micro-CT images using Pattern Recognition and Hu moments

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Hu invariant moments have been widely used in pattern recognition applications. In Digital Rock Physics applications several analysis are evaluated in different scales and an integrated approach is needed. In the current work, we present a study of microtomographic images from geological plugs and discuss a simple algorithm to match shapes in different scales. One analyze the efficiency of the matching algorithm comparing patterns obtained in high resolution images with its lower resolution version, and also with noise added.

Keywords: Image Processing; Moment Invariants; Hu moments; Pattern Recognition.

1. INTRODUCTION

The development of scientific and technological instrumentation lead to a huge volume of data to be analyzed and correlated. In particular, for the Geosciences area the instrumentation is applied to the study of rocks and soil using several techniques for observations. In the last years, the computed microtomography (μCT) and image pattern recognition have been widely introduced in different applications inside Geosciences [see, e.g., [1, 2]]. In particular, some of these applications aim the study of petrophysical parameters which are related to rocks and soils. Between these parameters, the porosity and the permeability are the principal targets of many studies [3, 4].

The μCT is a non-destructive imaging method that allows investigating the structure of samples with a high spatial resolution [5]. As pointed out by Kaestner in 2008 [6], the principal advantage of this method is the fact that the sample can be repeatedly scanned under different initial conditions, which allows the process to be spatially and temporally monitored. The principal results obtained by using μCT are strongly dependent of two factors. The first is related to the instrumental resolution (tomographer resolution), while the second one is associated to the image pattern recognition technique used in order to process the acquired data [7]. The aim of these techniques is to classify a pattern or describe objects in an image using a set of features. A typical problem in the field of Pattern Recognition is that the set of objects used to define the pattern should be as representative as possible. Furthermore, the characteristics used to identify the patterns should be uncorrelated.

An approach that has been widely used in many applications related to Patterns Recognition is the multiresolution analysis[1], aimed at the image interpretation invariant to its scale. Geofrey [11] suggests that the multiresolution analysis is composed of two fundamental components: (1) the generation of a multiresolution representation; (2) the information extraction. Regarding the multiresolution analysis, there exist many different computational techniques (or descriptors) which can be used in order to recognize scale-invariant patterns in images. Among these methods or tools we can highlight the Hu Moments [12], Zernick Moments [13], Fourier Transform [14], Pulse Coupled Neural Network [15], among others. In particular, in this work we have chosen to use the Hu Moments (see details in section 2.1) since this technique not only has been often used in patterns recognition, as also it is invariant under translation, rotation and scale (TRS) transformations.

This work reviews the principal pattern recognition techniques used in the processing of images (image enhancement, features extraction, besides others) and methods of invariant features extraction, addresses the strength of the Hu Moments shape descriptors by joining these tools and techniques into a multiresolution background to recognize patterns in μCT images from geological plugs of sedimentary rocks.

The work is organized as follows. In section 2 it is

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[1] Example of multi-resolution techniques: template matching [8], content-based image retrieval (CBIR) [9] and image registration [10].

shown a brief summary on image pattern recognition. Latter, in subsection 2.3 the principal properties about the Hu moments are shown and discussed. Section 3.2 describes the methods and techniques used to process the data, while in subsections 3.2 and 3.4 discuss the principal results obtained in μCT images from geological plugs with these methods. Finally, the conclusions and perspectives are drawn.

2. PATTERN RECOGNITION

There exist many pattern recognition techniques [16] which have been used in several expertise areas. Among them, one in particular, which has received much attention in recent years is the application and characterization of pattern recognition for image processing [14]. This was possible due to the development of computer technology, both in hardware and in software, which has allowed capturing and to storage different types of images. Actually, the image processing is applied in several different fields, for example, medical image, remote sensing, biometric recognition, automatic inspection of industrial components, and so on [17]. Nevertheless, the complexity of the procedures related with image processing, besides the large quantity of data require more efficient techniques and algorithms to processing images.

Many authors have writing relevant articles pointing out the principal definitions and technical details about pattern recognition. In this sense, Bishop [18] defines the pattern recognition as the art of make predictions from data by using probabilistic and statistical tools, computational geometry, machine learning and signal processing. Theodoridis [19] highlights the importance of the machine learning for pattern recognition. Also, he affirms that the machine learning is a central part of many intelligence machine learning systems which are built for decision making. Finally, we can state that pattern recognition is one of the main tools of image processing systems. Its principal objective is to describe the pattern through statistical information extracted from the images pixels.

2.1. Pattern Recognition Systems Steps

As previously mentioned, pattern recognition is the essential part of any image analysis system. Such a system should be able to recognize patterns that are equal under several geometric transformations, like rotation and/or translation. The design of a pattern recognition system in essence involves four main components which are shown in Fig. 1.

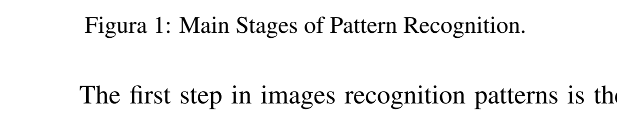

Figura 1: Main Stages of Pattern Recognition.

The first step in images recognition patterns is the acquisition of the image. In this step a sensor can be used for scanning. This sensor can be a scanner, a photograph camera, a tomograph, etc. The nature of the sensor, as also the image obtained is determined by the specific application. In this work, it is used a high-resolution tomograph in order to generate X-ray computed microtomography (μCT) scanning images, to obtain images from geological plugs. The procedure also allows obtaining magnified images of rocks and soils, where it is possible to study some petrophysical properties of the systems, like porosity, permeability, among others [1–3, 6, 7].

In the preprocessing step the data are modified by using several methods in order to improve the quality of the image. Usually, in this stage some imperfections (presence of noisy pixels, contrast and/or inappropriate brightness) which may arise during the image acquisition are corrected. The segmentation process is also performed aiming split the image in its principal units, it means, to separate the principal objects of the image. A robust segmentation procedure can produce excellent solutions regarding the identification of particular objects inside the image. On the contrary, a bad segmentation leads to wrong interpretations of the results. Gonzales [14] point out that the segmentation process is a critical point inside the image processing.

Another method that can be useful in this stage is the Mathematical Morphology. The basic idea of this technique is the comparison of the content of an image with a small image which has a known format. This small image is called structural element and it contains geometric and/or topological features which are related with the relevant information of the original image. Meyer [20] states that this method is an important tool which can be used in different stages during the image processing, pointing out its importance during analyses of microscopic images. The principal operations of the Mathematical Morphology are the erosion and dilation. Both methods are the base for more complex transformations. These forementioned operations depend on the image being processed (i.e. white and black, colors, gray levels). In this particular work, the erosion operator was used to process binary images aiming to eliminate specific pixels which do not correspond to a specific pattern. More information about these operators and its particular details can be found in [14] and [21]. The extraction of image characteristics aims to find some quantitative information in the image which was previously processed by using the image descriptors technique. These descriptors are used in order to describe different properties such as color, texture and shape of its objects. These properties should be represented as a suitable data structure typical of the data recognition algorithm, i.e. the properties representation must be a numeric vector called features vector. It is also important to point out that these properties should contain useful and enough information in order to separate particular objects of the figure, since these features have a critical impact on the results of the next step (decision making). On the other hand, one crucial problem in defining the features vector is that it should be invariant under geometric transformations like rotation, scale and translation. Thus, a robust images pattern recognition system should be enough flexible as to recognize the same pattern transformed under different operations. Details about some particular methods for extracting invariant features will be detailed in section 2.2 of this work. Emphasis would be given to the shape descriptors, in particular, the so-called Hu

Moments.

The decision making step in a pattern recognition process is the recognizing of the object (also can be the shape of the object), or in a more general procedure, specifically, the recognition of some specific fingerprints of the object. According to Jain [17] the four best known approaches for pattern recognition are: pattern matching; statistical classification; syntactic or structural matching and neural networks. The authors claim also that these models are not independent and sometimes the same pattern recognition method can have different interpretations. In this work it is discussed the pattern matching approach aiming to find the similarity between a reference image and the test image through the patterns of invariant features obtained in both images.

2.2. Shape Descriptor

The objects are represented by its features (e.g. color, shape and texture), and many methods are used to compute them. Fig. 2 show some shape descriptors, and a detailed review can be found in [22]. The shape descriptors are classified into two categories: contour-based and region-based. Among of them, some are invariant to rotation and translation, as well as changes in scale, allowing that same objects with different images sizes should be recognized as identical.

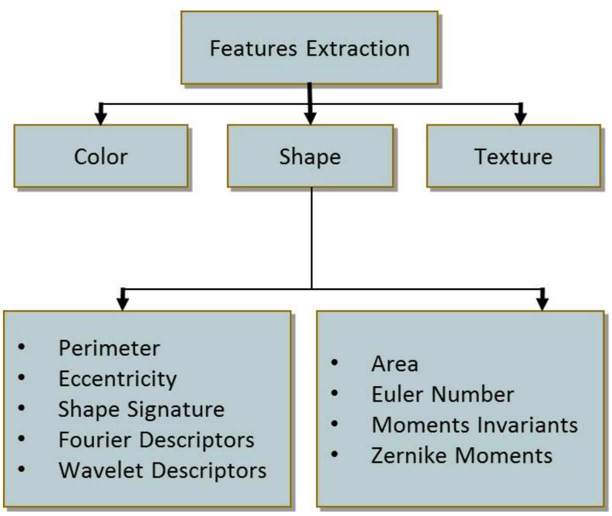

Figura 2: Stages of features extraction and shape description techniques, Adapted from [22].

The Region-Based Shape Descriptors can describe complex objects that consist of multiple disconnected regions, as well as, simple objects with or without holes. Bober [23] points out that the objective of a descriptor are: provide a compact and efficient way to describe properties of multiple disjoint regions, retrieving objects that somehow may be divided into disconnected sub-regions during the segmentation process and robustness to noise. The techniques based on Moments are main representatives of this type of descriptor [24]. There are various techniques mentioned in the literature based on the Moments e.g. Hu Moments [12], Affine Moment Invariant [25], Zernick Moment [13], etc. In this work, we used Hu Moments, which are Region-Based Shape Descriptor, in order to characterize objects in geological microtomography images.

2.3. Hu Moments

In Image Processing, is usual to define moments weightened by the pixel intensity. We expect that moments may characterize the shape of interest and in particular cases that it would be possible to give a resonable interpretation for them. The moment of order $(p+q)$ for 2D objects can be defined as

$$m_{pq} = \int dxdyx^p y^q f(x,y) \quad , \tag{1}$$

where $f(x,y)$ is a weight function, we use in this work as weight function the pixel intensity unless otherwise noted. If one takes $f(x,y) = 1$ inside the shape and $f(x,y) = 0$ outside the moment M_{00} represents the shape area. The centroid is also simply derived

$$\bar{x} = \frac{M_{10}}{M_{00}} \quad , \tag{2a}$$

$$\bar{y} = \frac{M_{01}}{M_{00}} \quad . \tag{2b}$$

From Eq. (1) and (2) it is also possible to define central moments

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\bar{x})^p (y-\bar{y})^q f(x,y)\, dx\, dy \quad . \tag{3}$$

In 1962 Hu [12] introduced a set of o translation, rotation, and scaling (TRS) invariant moments developed in the context of algebraic invariants theory[26–28]. In this forementioned paper were presented a set of 7 moments. The translation invariant moment is defined as

$$\eta_{ij} = \frac{\mu_{ij}}{\mu_{00}^{\left(1+\frac{i+j}{2}\right)}} \quad , \tag{4}$$

and the set of 7 TRS moments are defined as follows

$$I_1 = \eta_{20} + \eta_{02} \quad , \tag{5a}$$

$$I_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \quad , \tag{5b}$$

$$I_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \quad , \tag{5c}$$

$$I_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \quad , \tag{5d}$$

$$I_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \quad , \tag{5e}$$

$$I_6 = (\eta_{20}-\eta_{02})[(\eta_{30}+\eta_{12})^2-(\eta_{21}+\eta_{03})^2]+ 4\eta_{11}(\eta_{30}+\eta_{12})(\eta_{21}+\eta_{03}) \quad , \tag{5f}$$

$$I_7 = (3\eta_{21}-\eta_{03})(\eta_{30}+\eta_{12})[(\eta_{30}+\eta_{12})^2-3(\eta_{21}+\eta_{03})^2]- (\eta_{30}-3\eta_{12})(\eta_{21}+\eta_{03})[3(\eta_{30}+\eta_{12})^2-(\eta_{21}+\eta_{03})^2] \quad . \tag{5g}$$

Note that the first moment is analogous of the z component of the moment of inertia. Numerous works have been devoted to applications of the Hu invariant moments. For example, Flusser and Suk [29] applied moment invariants in template matching and registration of satellite images and Mukundan [30] employed them to estimate the position and the attitude of the object in 3-D space. Wong [31] developed illumination invariants suitable for texture classification.

In the last decade the presented set of 7 classical invariants was proved not independent nor complete [32, 33]. The third moment can be written in terms of other 6. It is also possible to define a third order moment

$$I_8 = \eta_{11}[(\eta_{30}+\eta_{12})^2-(\eta_{03}+\eta_{21})^2]- (\eta_{20}-\eta_{02})(\eta_{30}+\eta_{12})(\eta_{03}+\eta_{21}) \quad , \tag{6}$$

Although those recent developments improve the comprehension on the TRS Hu moments does not diminish the importance nor limit the power of the moments as tools to characterize shapes. In this work we shall concentrate in the 7 classical set.

3. IMAGE MATCHING WITH HU MOMENTS

There are several applications of pattern recognition with Hu invariant moments [34–37]. Since the Hu moments are invariant under translation rotation and scale it turns out it would be a suitable set of features to characterize shapes and match those shapes in two different images. However, these properties are valid in a continuous function. As the images are discrete one must expect and take in consideration some variation after scaling an image. To porsue on the pattern recognition one must also determine if the moments are reasonably dinstinguishable in parameters space whithin the required precision. In this work we concentrate in a specific type of image, μCT scanning images from geological plugs. Those images are taken in slices. The same plug usually is scanned in a low resolution and frequently smaller plugs are produced from the bigger plug. The smaller plugs are usually scanned in higher resolution and sometimes the plug is destroyed to produce samples to be imaged by an scanning electron microscope (SEM). In order to make a more consistent analysis it would be interesting to find which part of the plug is zoomed or imaged by other methods and match those images. For simplicity in our tests we used an uint16 1000×1000 pixels μCT image and processed that image by lowering its scale and adding noise in order to compare with the original image. The test images are matched with the original image by comparing the table of invariant moments extracted in the grains of the test and original sample. In the next section we describe the algorithm used carefully.

3.1. Algorithm

The algorithm used can be seen in the Fig. 3. The original image and the test image are filtered by a Median filter. After that the images are binarized. The Original image used in all tests is shown in Fig. 4, where it can be distinguish 3 phases: a grain phase, a pore phase and not resolved phase, by visually examining the image is clear that the most defined phase is the pore phase which we are used as sources for shapes to match. In order to determine those shapes we used a simple Binarization and labeled considering a 8 connectivity. In Fig. 4 we present the a label image with the 164 objects detected. In the next step the TRS invariant Hu moments are calculated for each shape extracted in the labelization process for both images and two tables are generated, the first containing Hu from the shapes in the original image and the second with the Hu in the test image. Next, the algorithm compares those two tables defining a moment match if

$$I_n^O - 5\% I_n^O \le I_n^T \le I_n^O + 5\% I_n^O \quad , \tag{7}$$

where I_n^O and I_n^T are the n-th Hu moment for the original and test image respectively. In our matching algorithm for each object we search for a match and for false positives separately, this means that the false detection number and the objects recovered number are independent. In the next section we discuss a resonable criteria to determine if we recovered the shape or not.

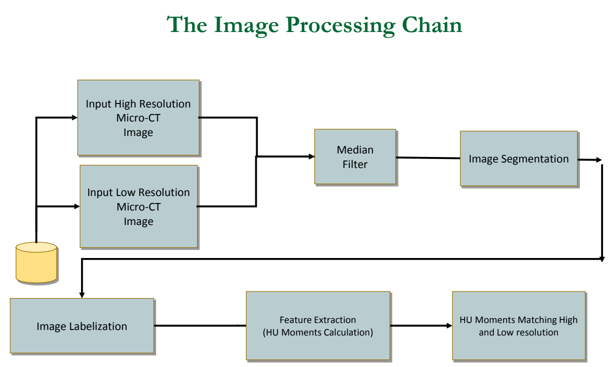

Figura 3: Algorithm Workflow. The Original and test images are inputed pre-processed and binarized, after a labelization process the feature extraction phase calculates the Hu moments in the remaining objects. The table with the Hu moments from the objects in test image are compared to the objects in original images.

3.2. Low Resolution Binary Images

In this first approach we used as test images the high resolution (original) binarized and then resized using the *imresize* function from Matlab Image Processing Toolbox. The main idea of this test is to evaluate the potential of the Hu moments to find the shapes extracted avoiding the segmentation problem. The test images have $1/2$, $1/4$, $1/6$, $1/8$ of the

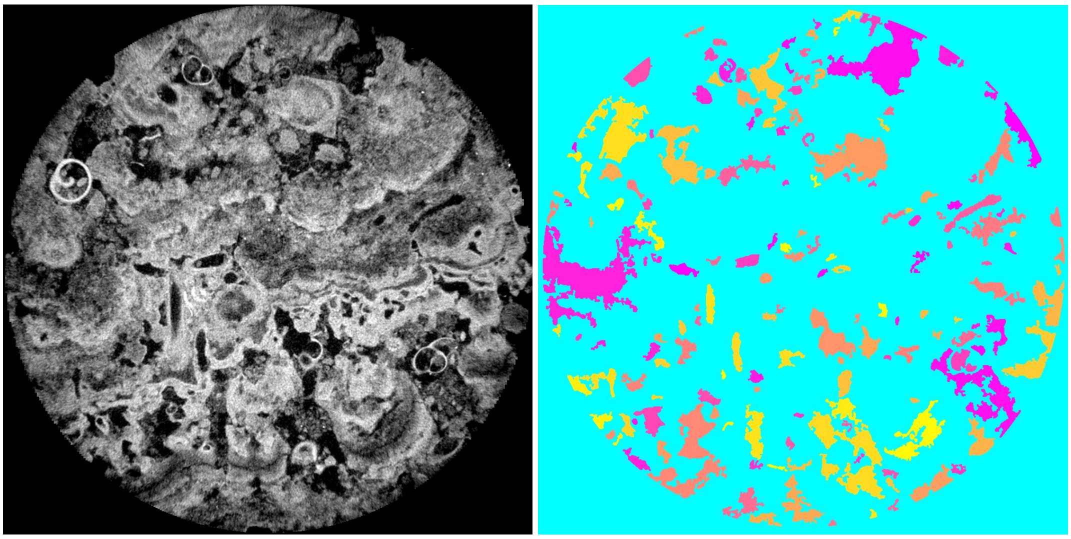

Figura 4: The limestone from Lagoa Salgada outcrop in Rio de Janeiro original μCT image used as input (left) and the label Image with the extracted shapes (right).

original resolution. If ones defines as a sufficient condition to find an object a single Hu match each Hu defines a sample of recovered objects and false positives. By using an $1/2$ binary image with this criteria results in a sample where the fake detections were 163 from 164 shapes, that is all the objects in the image are taken as false positives and the algorithm is not able to distinguish any object. As the Hu moments are scale invariant the main concern is with the number of false detections. It is worth mentioning that our main goal is to match images μCT images, so the completeness of objects recovered is less relevant than the fake detections. Thereafter we define a combination of invariant moments as a criteria to match according to the Table 5.

Hu Moment

Combinations	1	2	3	4	5	6	7
A	X	X					
B	X	X	X				
C	X	X	X	X			
D	X	X	X	X	X		
E	X	X	X	X	X	X	
F	X	X	X	X	X	X	X

Figura 5: Hu moments combinations used as criteria to match objects.

The results for each combination is presented in Fig. 6 for all tested resolutions

The most pure sample is the one with configuration F. The configuration D presents the a balanced results beetween false detections and matches. In Fig. 7 we present a sample of recovered objects for the D configuration.

Even with $1/8$ resolution the algorithm is still capable to detect 2 shapes. For $1/10$ resolution the algorithm could not find any object in any configuration. In the next section we consider the effect of the binarization.

3.3. Different Resolution Images

To take in consideration the effect of the segmentation process we used as test image the original image which we first resize and after this process binarize the image. In this case the algorithm found objects only in the $1/2$ resolution image, the results is presented in Fig.8 and the objects found with D configuration can be seen in Fig. 9

3.4. Images with Noise Added

In this analysis we introduced some noise concerning some possible sistematic problems that may occur during the imaging process. We added poisson noise to the full resolution image and the $1/2$ resolution image. In the Fig. 10 we present our results for the full resolution image and the results for the image with $1/2$ resolution. In both cases the algorithm was able to recover objects in those conditions as presented in Fig. 11 for the configuration D.

4. CONCLUSION

In this work we present an algorithm to match objects extracted from a μ-CT image using TRS Hus moments in diferent resolutions and conditions, although the matching method used was very simple the algorithm was able to recover objects in a 1000×1000 image in $1/8$ resolution image with same binarization. In low resolution images with different binarization case with $1/2$ resolution, the algorithm found more matches than false positives in most of the tested configurations (C, D, E, F) and capable to find objects even after noise addtion. However, the number of false positives is a limitation for this method. The results in this kind of μCT images from plugs of sedimentary rocks suggest that Hus moments may be used as a feature to match the extracted shapes complementary to other features although not alone

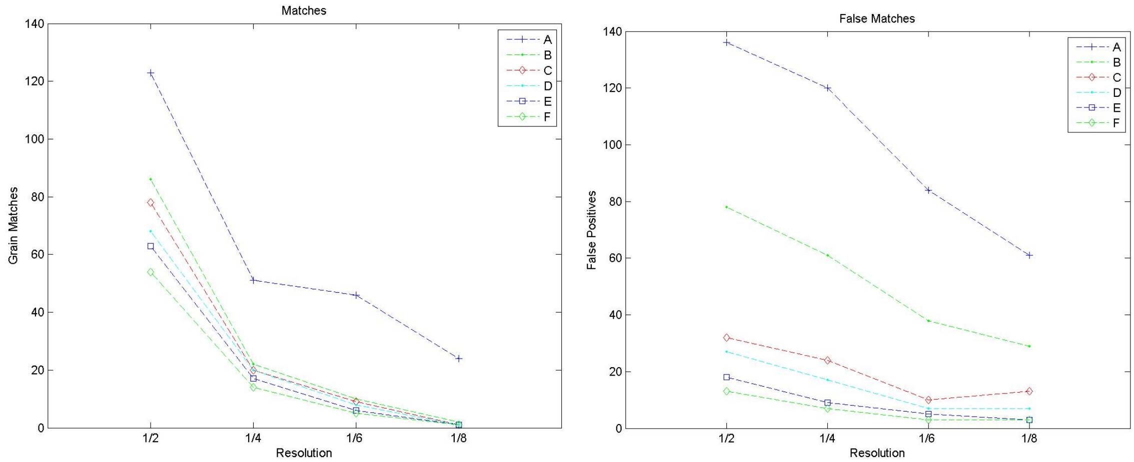

Figura 6: Object Matches for a low resolution Binary image.

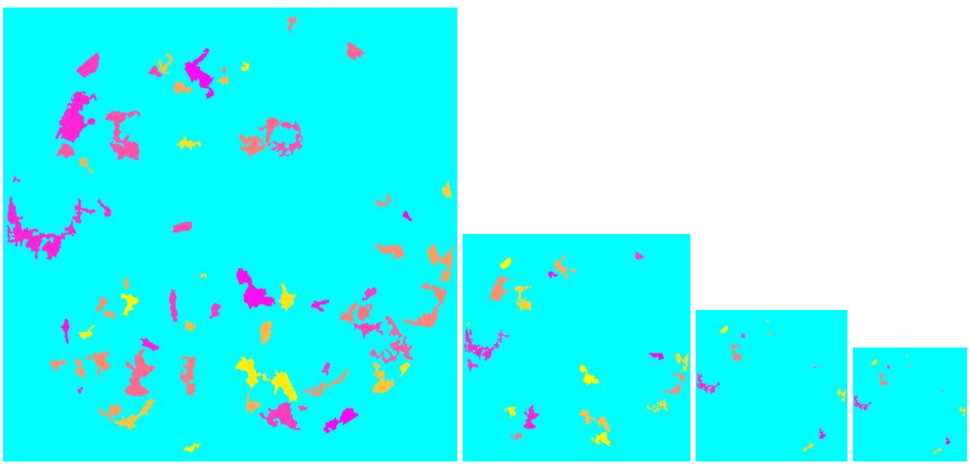

Figura 7: Multiresolution detections after the Hu Matching using configuration D in reduced resolution binary images. From left to right $1/2$, $1/4$, $1/6$ and $1/8$ resolutions.

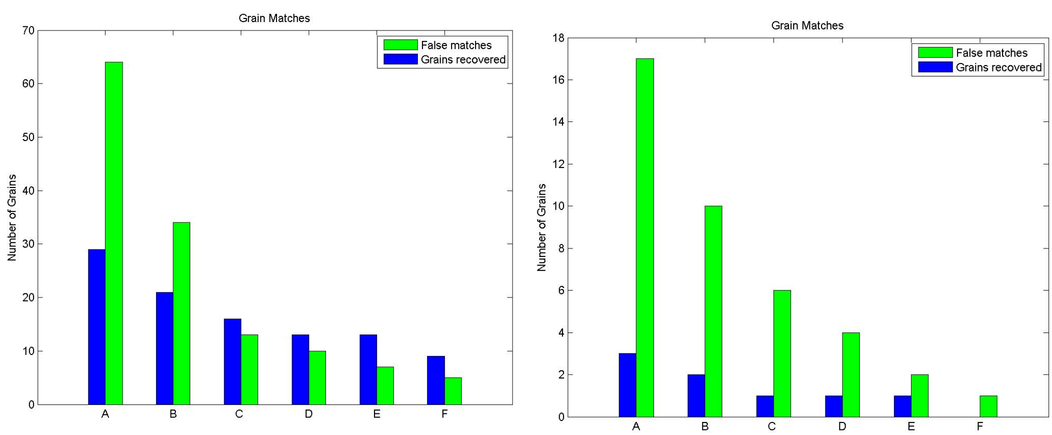

Figura 8: Object Matches for a low resolution image.

using this matching criteria. The choise of a more suitable mathing condition can also increase the results, for example, use Hus diferent distance definitions in parameter space or as input to an unsupervioned Artifical Neural Network. This algorithms are currently under investigation.

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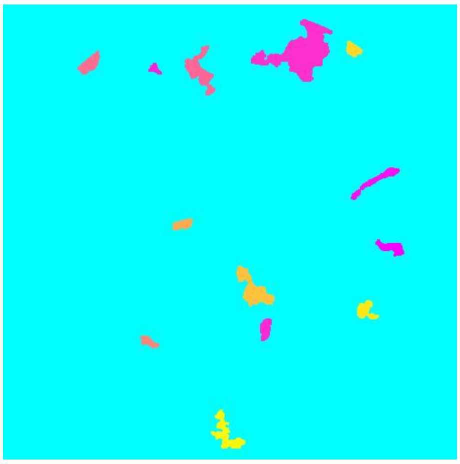

Figura 9: Detections recovered after the Hu Matching for 1/2 resolution

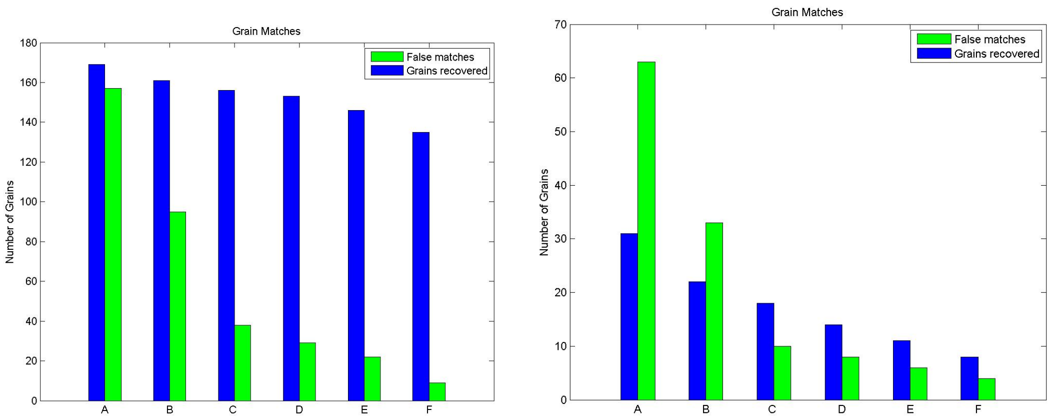

Figura 10: Object Matches for a poisson noise added images for a full resolution image (left) 1/2 resolution image (right).

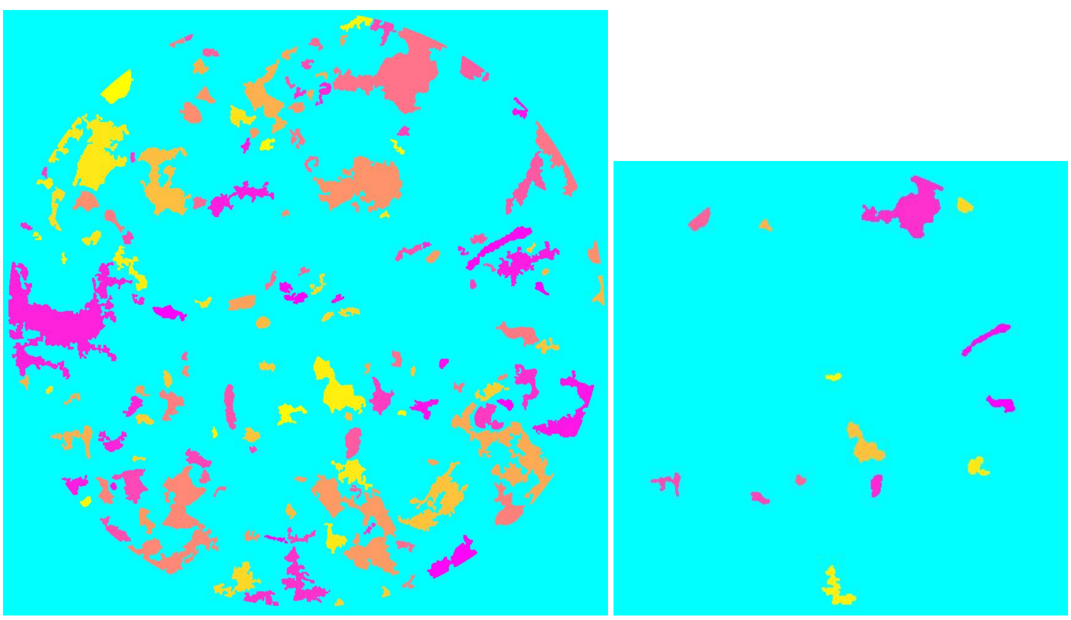

Figura 11: Detections recovered after the Hu Matching for full resolution (left) and (right) 1/2 resolution with poisson noise.

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