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A Screened BFKL Interpretation of F_2 in the Exceedingly Small x Limit

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Abstract

In this letter we show that the behaviour of F_2, at very small x_B, agrees with the behaviour expected from the BFKL evolution equation, when screening corrections are included. We obtain a description which is consistent with the data, however, we require the screening corrections to be relatively large (about a quarter of the total DIS cross section). The relation between the screening corrections and the diffractive DIS cross section is discussed.

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In this letter we discuss the dependence of F_2, the proton structure function, on W, the $\gamma^* p$ c.m. energy, at very small x_B. We are motivated by the recently published data taken at HERA by the ZEUS [1] and H1 [2] collaborations which are shown in Fig.1. Our goal is to extract new information from the experimental data on the deep inelastic scattering (DIS) process, in the region of very small x_B. We will show that the BFKL evolution equation (the BFKL Pomeron) [3], including screening (shadowing) corrections[4], provides a good reproduction of the observed data.

We list first the main qualitative properties of the behaviour of $F_2(W, Q^2)$, as observed by the two experimental groups at HERA and shown in Fig.1.
1) For W values below 130-150 GeV, the measured data points cluster in a narrow linear band. i.e. $F_2(W, Q^2)$ is approximately linear in W and has a weak dependence on Q^2.
2) For higher values of W, $F_2(W, Q^2)$ is dependent on Q^2. The high Q^2 data differs from the lower Q^2 data which seems to reach a local plateau, resulting in a F_2 which is almost constant as a function of W.

The features of the data at low W are compatible with a dominance of the BFKL Pomeron[3] in the small x_B domain of $F_2(x_B, Q^2)$. This is readily seen when we write [5] the BFKL generated structure function

$$F_2^{BFKL}(x_B, Q^2) \;=\; \Sigma_f e_f^2 \cdot \frac{11\pi^2 \alpha_s(Q_0^2)}{32\sqrt{2}} \cdot \frac{G_0}{\sqrt{28 N_c \alpha_s(Q_0^2)\zeta(3)}} \tag{1}$$

$$\cdot \sqrt{\frac{Q^2}{Q_0^2}} \cdot \frac{1}{\sqrt{\ln\frac{1}{x_B}}} \cdot (\frac{1}{x_B})^{\omega_0} \cdot e^{-\frac{\pi(\ln\frac{Q^2}{Q_0^2})^2}{56 N_c \alpha_s(Q_0^2)\zeta(3)\ln\frac{1}{x_B}}}$$

where G_0 denotes the unknown normalization of the gluon distribution at $x_B \sim 1$. The value of ω_0 is given by[3] $\omega_0 = \frac{N_c \alpha_s}{\pi}\, 4\ln 2$. In the following we assume that $\omega_0 = 0.5$, which corresponds to a resonable value of α_s. At small values of x_B, $W^2 = \frac{Q^2}{x_B}$, so we can rewrite Eq.(1) in the form

$$F_2(x_B, Q^2) \;\propto\; \frac{W}{\sqrt{\ln\frac{W}{Q}}} \cdot e^{-\frac{\pi(\ln\frac{Q}{Q_0})^2}{28 N_c \alpha_s(Q_0^2)\zeta(3)\ln\frac{W}{Q}}} \tag{2}$$

The above expression reproduces the qualitative F_2 features listed for the lower W, but fails to reproduce the required high energy characteristics. To improve the "theoretical" behaviour of F_2 at higher values of W, we introduce the shadowing correction[4], illustrated in Fig.2. The DIS structure function can be represented as

$$F_2(x_B, Q^2) \;=\; F_2^{BFKL}(x_B, Q^2) \;+\; \Delta F_2(x_B, Q^2) \tag{3}$$

where ΔF_2 represents the changes in the BFKL structure function which result from screening.

We shall elaborate on the details of the screened diagram calculation, shown in Fig.2, later. Our discussion is based on the main results of Ref.[6] which are summarized as follows: The dominant contribution of interest comes from the exchange of two BFKL ladders (Pomerons), while the upper blob of Fig.2 is suitably given by the GLAP[7] DIS structure function. The integration over x_P (see notations in Fig.2) results in the contribution

$$\Delta F_2(x_B, Q^2) \;=\; -\,\Sigma_f e_f^2 \cdot \frac{11\pi^2\alpha_s(Q_0^2)}{32\sqrt{2}} \cdot \{\frac{Q_0^2}{Q^2}\}^{-\frac{\alpha_s}{2\omega_0}} \tag{4}$$
$$\cdot \frac{G_0}{\sqrt{28N_c\alpha_s(Q_0^2)\zeta(3)}} \cdot \gamma \cdot \frac{\ln\frac{Q^2}{Q_0^2}}{\ln(\frac{1}{x_B})} \cdot (\frac{1}{x_B})^{2\omega_0}$$

where we have absorbed all nonperturbative QCD contributions in the phenomenological triple "ladder" vertex γ. The minus sign in Eq.(4) reflects the shadowing origin of this contribution. Eq.(4) can also be derived from the calculation of the diffraction dissociation cross section using the AGK cutting rules [8], which lead to the relation [9]

$$\Delta F_2 \;=\; -\,F_2^D \tag{5}$$

We note that F_2^D, in the above equation, is related to the total integrated diffractive DIS cross section. Namely, to the two DIS single diffraction channels, as well as the DIS double and central diffraction. It has recently been suggested that these are quite large[10][11]. The restriction implied by Eq.(5) hinders our ability to reconstruct "theoretically", the experimental high energy behaviour of $F_2(W, Q^2)$. The BFKL approach, which qualitatively reproduces the low energy features, fails to do so at high energies. We conclude from this that the necessary corrections, and therefore the diffractive component, must be quite large. As we shall see, when discussing the results of our calculation, we require that $\frac{F_2^D}{F_2} \geq 0.25$. This requirement is not in contradiction to the meagre DIS experimental information presently available, allowing one to check $\frac{F_2^D}{F_2}$. Both ZEUS[12] and H1[13] collaborations find a sizeable diffractive component in their $Q^2 \simeq 0$ photoproduction studies. In DIS, only single diffraction at the γ^* vertex has been measured[14][15], and the ratio of the measured diffraction to the total DIS cross section is about 0.15. Estimates of the non measured diffractive channels are model dependent. Irrespective of our detailed estimate of the unmeasured channels, the overall ratio obtained, is sufficiently large to justify our approach.

As stated, we take $\omega_0 = 0.5$. Seemingly, with value of ω_0 we reach the unitarity limit[4], and expect the diffractive channel to vanish. As s-channel unitarity is not built into this formalism, we can take ω_0 values even larger than 0.5, provided that unitarity corrections, such as screening, are incorporated in the calculation. Absorbing all the unknown factors in a new phenomenological constant $\tilde{G}_0$ we obtain

$$F_2(x_B, Q^2) \;=\; \tilde{G}_0 \cdot \{ \frac{W}{\sqrt{\ln \frac{W}{Q}}} \cdot exp[\, - \frac{\pi (\ln \frac{Q}{Q_0})^2}{28 N_c \alpha_s(Q_0^2) \zeta(3) \ln \frac{W}{Q}} \,] \quad - \; W^2 \cdot \frac{\gamma \; \ln \frac{Q}{Q_0}}{\ln \frac{W}{Q}} \cdot \{ \frac{Q_0^2}{Q^2} \}^{(1 - \frac{1}{4 \ln 2})} \} \; + \; C \tag{6}$$

We have added a (small) constant C to account for the remnant non BFKL contributions.

We now turn to a more detailed discussion of the diagram shown in Fig.2. Our motivation is three fold:

1) We wish to better comprehend the complicated calculation of Bartels, Lotter and Wuesthoff[6] and its consequences.

2) We need to clarify how trustworthy our perturbative QCD calculation is.

3) We need to adjust the results to the relevant kinematic domain at HERA.

We use the expression given for our diagram in Ref.[9]

$$\Delta F_2 = -\gamma \int_0^{\ln \frac{1}{x_B}} d \ln \frac{1}{x_P} \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^4} F_2^{GLAP} (\frac{x_B}{x_P}, \frac{Q^2}{k^2}) \cdot [x_P G_{BFKL}(x_P, k^2)]^2 \tag{7}$$

The integration over k^2 in the above expression leads to an infrared divergency as $k^2 \to 0$. However, the BFKL Pomeron is associated with an anomalous dimension $\gamma(\omega) = \frac{1}{2}$, and thus we get Eq.(1) for which the BFKL gluon distribution is given by

$$x_P G_{BFKL}(x_P, k^2) \;=\; \frac{G_0}{\sqrt{28 N_c \alpha_s(Q_0^2)\zeta(3)}}$$

$$\cdot \sqrt{\frac{k^2}{Q_0^2}} \cdot \frac{1}{\sqrt{\ln\frac{1}{x_P}}} \cdot (\frac{1}{x_P})^{\omega_0} \cdot e^{-\frac{\pi(\ln\frac{k^2}{Q_0^2})^2}{56 N_c \alpha_s(Q_0^2)\zeta(3)\ln\frac{1}{x_P}}} \tag{8}$$

Substituting Eq.(8) in Eq.(7), we rewrite Eq.(7) in a more compact form using new variables $y_P = \ln\frac{1}{x_P}$, $y_B = \ln\frac{1}{x_B}$, $r_Q = \ln\frac{Q^2}{Q_0^2}$ and $r = \ln\frac{k^2}{Q_0^2}$. This yields

$$\Delta F_2 \;=\; -\,\frac{\gamma}{Q_0^2}\int_0^{y_B} dy_P \int_0^{r_Q} dr F_2^{GLAP}(\frac{x_B}{x_P}, \frac{Q^2}{k^2}) \cdot \frac{\pi}{\Delta y_P} \cdot e^{2\omega_0\, y_P - \frac{2r^2}{\Delta y_P}} \tag{9}$$

where $\Delta = 56\zeta(3)\bar{\alpha}_s$ and $\bar{\alpha}_s = \frac{N_c\alpha_s}{\pi}$.

We wish to stress that the infrared divergence of the above integral should be studied in more detail. To this end we consider the situation where r_Q and y_B are sufficiently large so that we can use the solution of the GLAP evolution equation in the region of small x_B to assess F_2^{GLAP} in Eq.(9). We obtain

$$F_2^{GLAP} \;=\; A\, e^{2\sqrt{\bar{\alpha}_s(y_B - y_P)(r_Q - r)}} \tag{10}$$

We fix α_s so as to perform our calculations in a way consistent with the BFKL equation. There is no danger in doing so, as the Q^2 variation in the small x_B HERA kinematic region is negligible, allowing us to use this approach for the analysis of the HERA data. We absorb all irrelevant factors appearing before the exponential in a constant factor A, which appears in front of the expression.

Substituting F_2^{GLAP} in Eq.(9) we reduce the equation to the form

$$\Delta F_2 \;=\; -\,\frac{\gamma A}{Q_0^2}\int_0^{y_B} dy_P \int_0^{r_Q} dr\; e^{2\sqrt{\bar{\alpha}_s(y_B - y_P)(r_Q - r)}} \cdot \frac{\pi}{\Delta y_P} \cdot e^{2\omega_0\, y_P - \frac{2r^2}{\Delta y_P}} \tag{11}$$

It is easy to see that there is no saddle point in the integration with respect to r. Indeed, the equation for the saddle point is

$$\frac{\partial\Psi}{\partial r} \;=\; 0 \tag{12}$$

where

$$\Psi \;=\; 2\sqrt{\bar{\alpha}_s(y_B - y_P)(r_Q - r)} \;+\; 2\omega_0\, y_P - \frac{2r^2}{\Delta y_P} \qquad (13)$$

Eqs.(12,13) give

$$-\sqrt{\frac{\bar{\alpha}_s(y_B - y_P)}{r_Q - r}} \;-\; \frac{4r}{\Delta y_P} \;=\; 0 \qquad (14)$$

The saddle point can only be at negative values of r, but one cannot trust the BFKL equation in this domain, where the virtuality k^2 is less than Q_0^2. At such small values of virtualities there are certainly large corrections, and it does not seem reasonable to expect the BFKL Pomeron description to be valid in this region [4].

We note that the most important region of integration is still $r \;\to\; 0$, or in other words, the dominant value of k^2 remains $k^2 \sim Q_0^2$. This leads us to conclude that the BFKL contribution is questionable and one needs to study the integral of Eq.(11) in more detail, so as to be sure of the domain where it is valid. To this end, we observe that our integral over y_P has a very good saddle point. Indeed, the equation for this saddle point is

$$\frac{\partial \Psi}{\partial y_P} \;=\; 0 \;=\; -\sqrt{\frac{\bar{\alpha}_s(r_Q - r)}{y_B - y_P}} \;+\; 2\omega_0 + \frac{2r^2}{\Delta y_P^2} \qquad (15)$$

Neglecting the last term, we have the saddle point value for y_P

$$y_P^{SP} \;=\; y_B \;-\; \frac{\bar{\alpha}_s(r_Q - r)}{4\omega_0^2} \qquad (16)$$

We now check the reliability of the GLAP approach for the calculation of F_2^{GLAP}. We recall that the typical value of ω, the argument of the anomalous dimension of the GLAP equation[5], is given by

$$\omega \;=\; \sqrt{\frac{\bar{\alpha}_s(r_Q - r)}{y_B - y_P}} \qquad (17)$$

Substituting $y_P \;=\; y_P^{SP}$, we have $\omega \;=\; 2\,\omega_0$.

The BFKL anomalous dimension is given by the series[16]

$$\gamma(\omega) \;=\; \frac{\bar{\alpha}_s}{\omega} \;+\; 2\zeta(3)\,(\frac{\bar{\alpha}_s}{\omega})^4 \;+\; O(\frac{(\bar{\alpha}_s)^5}{\omega^5}) \qquad (18)$$

[4]The position of the saddle point in the region of small virtualities has been studied in all details in Ref.[5].

[5]We denote ω = N - 1, where N is the moment variable.

Substituting $\omega = 2\omega_0$, we see that the BFKL corrections are very small. This does not mean that we do not need the normal GLAP corrections, which are essential (see ref.[17]), but they cannot change the main result of the present problem.

Substituting $y_P = y_P^{SP}$ in Eq.(10), we end up with the following integral over r to be inserted in Eq.(9)

$$\int_0^{r_Q} dr\; e^{2\omega_0 y_B + \frac{\bar{\alpha}_s}{2\omega_0}(r_Q - r) - \frac{2r^2}{\Delta y_B}} \tag{19}$$

for $y_B \gg \frac{\bar{\alpha}_s(r_Q - r)}{4\omega_0^2}$. This is the kinematic region which is most interesting both from the theoretical and experimental points of view. One can see that the integral over r in Eq.(19) is concentrated at small $r \sim \frac{\omega_0}{\bar{\alpha}_s} \propto 0(\alpha_s)$ and at large r_Q and y_B. We face the dilemma of how much trust one can put on the perturbative calculation which estimates the behaviour of the deep inelastic gluon distribution, with virtualities of the order of Q_0^2. Apparently, this calculation is not reliable as the problem reduces to that of the energy behaviour of the typical hadron - hadron interaction at high energy, which is described by the "soft" Pomeron[18][19].

Despite this reservation, the above statement is certainly correct if we consider only small α_s and/or large Q^2, the $\ln(1/x_B)$ parameters in our calculation. Numerically, the situation is more promising. Indeed, as we shall show, the HERA data on F_2 [1][2] confirm the theoretical expectation that the BFKL Pomeron with $\omega_0 = 0.5$ contributes at low x_B. Substituting $\omega_0 = 0.5$ we obtain a typical value of $r \simeq \frac{2\omega_0}{\bar{\alpha}_s} \approx 6$, in the integral of Eq.(19). This value is sufficiently large to justify our using pQCD, to evaluate the diagram of Fig.2. Moreover, $r \approx 6$ is larger than the value of r_Q in HERA kinematic region, so we can estimate the value of the integral in Eq.(19) as r_Q. Collecting all factors together, we obtain Eq.(4), which was used in our description of the HERA data.

Detailed comparisons of our calculations with the data[1][2] are displayed in Fig.3 where we present $F_2(W, Q^2)$ and $F_2(x_B, Q^2)$. We did not attempt a "best fit", nevertheless, our ability to reproduce the gross features of the data is evident. The following comments relate to the data choice and detailed features of our fit:

1) The data considered are bounded by $x_B \leq 10^{-2}$ and $W \geq 50\,GeV$.

2) The following parameters were used in the numerical fit: $\tilde{G}_0 = 0.024$,

$\gamma = 0.015\ GeV^2$, $Q_0 = 1\ GeV$ and C = -0.025. With these parameters we get that $\frac{F_2^D}{F_2} \approx 0.30$ at $Q^2 = 8.5\ GeV^2$ and $W \approx 250\ GeV$. We can reproduce the data with a $\frac{F_2^D}{F_2}$ which is smaller, but, clearly, our requirement for a relatively large DIS diffractive component is essential for this approach.

3) As can be readily seen from Fig.3, we obtain a reasonable description of the data down to values of $Q^2 \approx 3\,GeV^2$. At smaller values of Q^2 we require larger SC to reproduce the data.

Our inability to reproduce the lowest Q^2 data is not surprising. Clearly, the present approach is over simplified, as we fail to take into account the different effects of the SC on the diffractive and the total DIS cross sections. We call attention to the observation[11][19] that in soft hadron interactions $\sigma_{diff}(with\,SC) \approx 0.3\sigma_{diff}(without\,SC)$ whereas $\sigma_t(with\,SC) \approx 0.75\sigma_t(without\,SC)$. As we have shown[20] the strength of the SC in pQCD is determined by a parameter $\kappa = \frac{3\alpha_s\pi}{2BQ^2}[xG(x,Q^2)]$, where B is the elastic slope. At low Q^2, $\kappa \approx 1$, and we doubt the validity of Eq.(5), which is at the core of our calculation. Nevertheless, even when $\kappa \approx 1$, one can still use Eq.(3) with ΔF_2 as defined in Eq.(4), and obtain a better assessment of F_2 at low Q^2 than the one we have presented here. The reason for this is, that the SC to the total DIS cross section turns out to be smaller than the SC to diffraction dissociation. This problem is connected to the broader issue of the transition between the (hard) pQCD and the (soft) non perturbative domain, which we plan to discuss in a forthcoming publication.

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Figure Captions

Fig.1: W dependance of $F_2(W,Q^2)$. Data is taken from ZEUS[1] and H1[2] with $Q^2 \geq 8.5\,GeV^2$.

Fig.2: Diffraction dissociation in perturbative QCD.

Fig.3: Comparison of $F_2(W,Q^2)$ and $F_2(x,Q^2)$ data with our calculations.

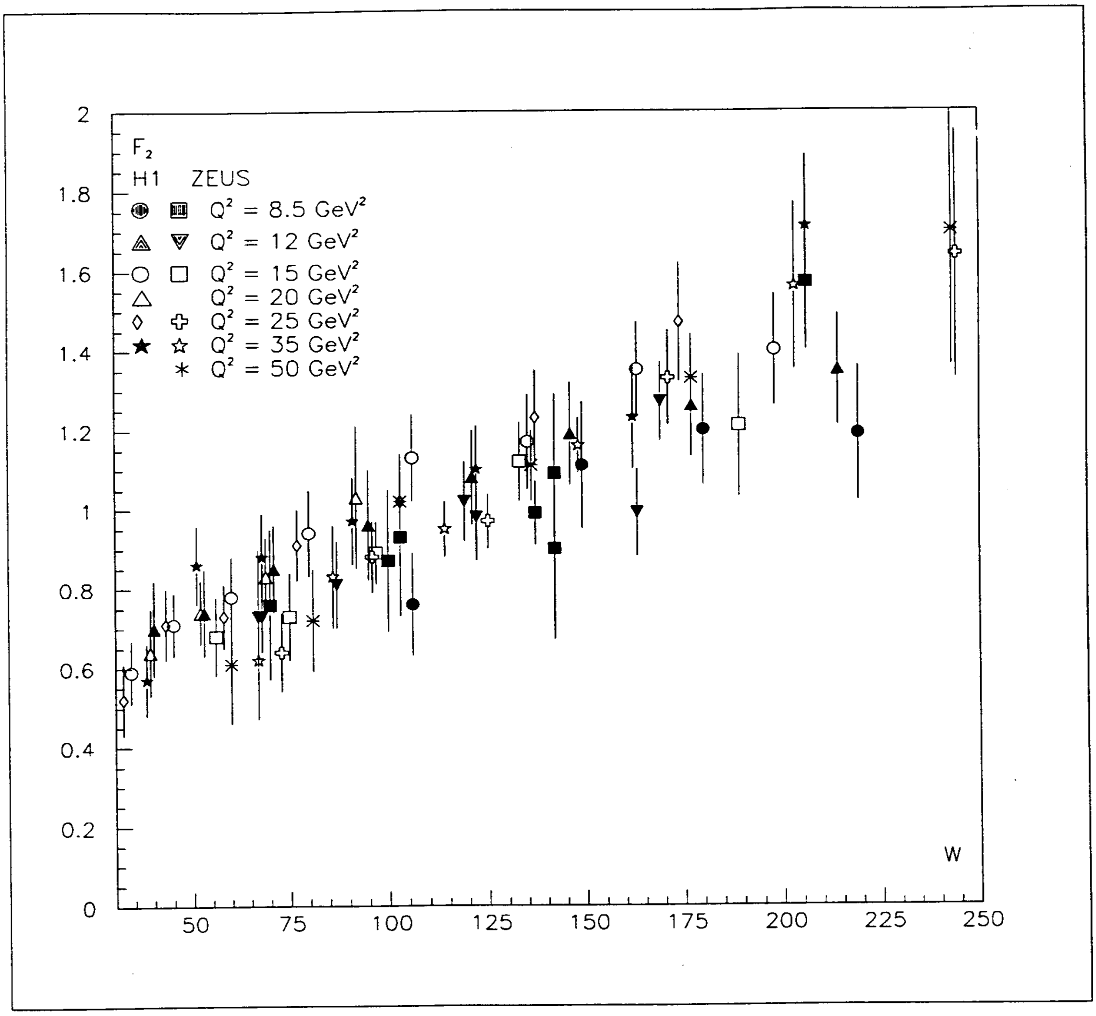


Figure 1: W **dependance of** $F_2(W, Q^2)$**. Data is taken from ZEUS [1] and H1 [2] with** $Q^2 \geq 8.5\,GeV^2$**.**

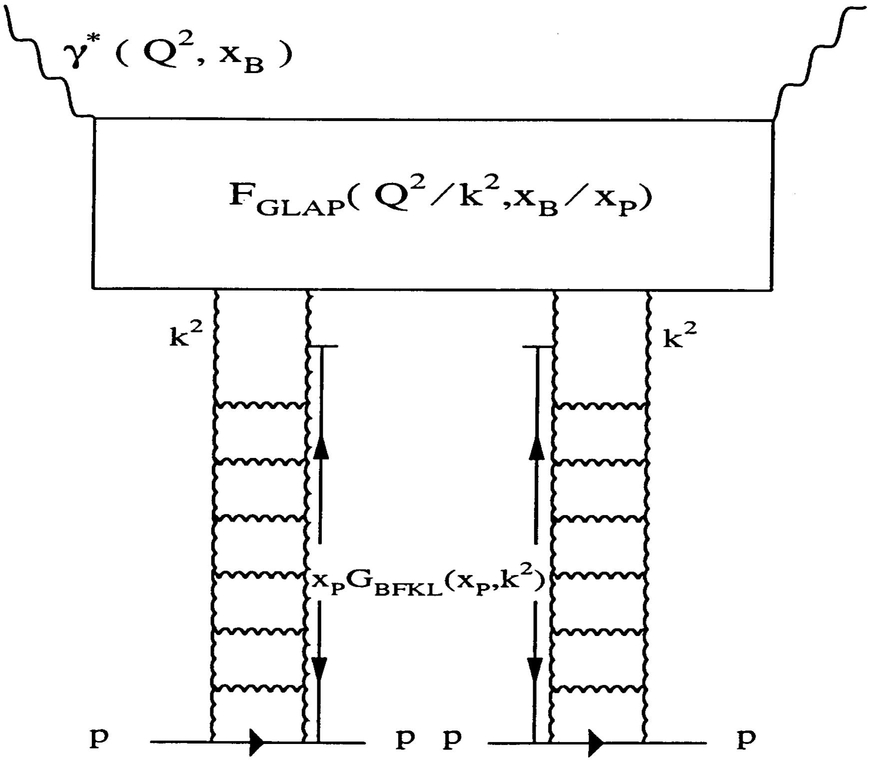
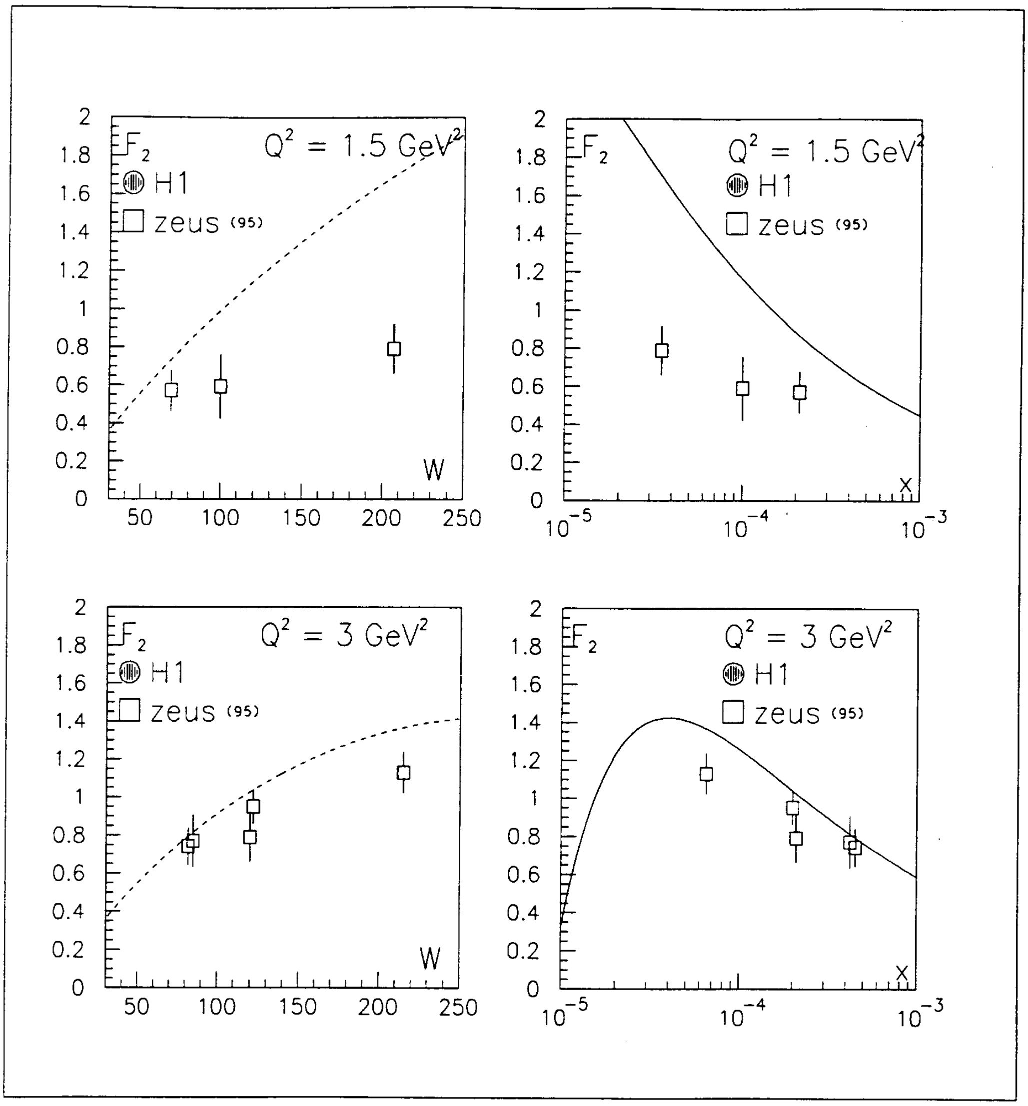


Figure 2:



F_2
$Q^2 = 1.5$ GeV^2
H1
zeus (95)
W
F_2
$Q^2 = 1.5$ GeV^2
H1
zeus (95)
X
F_2
$Q^2 = 3$ GeV^2
H1
zeus (95)
W
F_2
$Q^2 = 3$ GeV^2
H1
zeus (95)
X

Figure 3a.

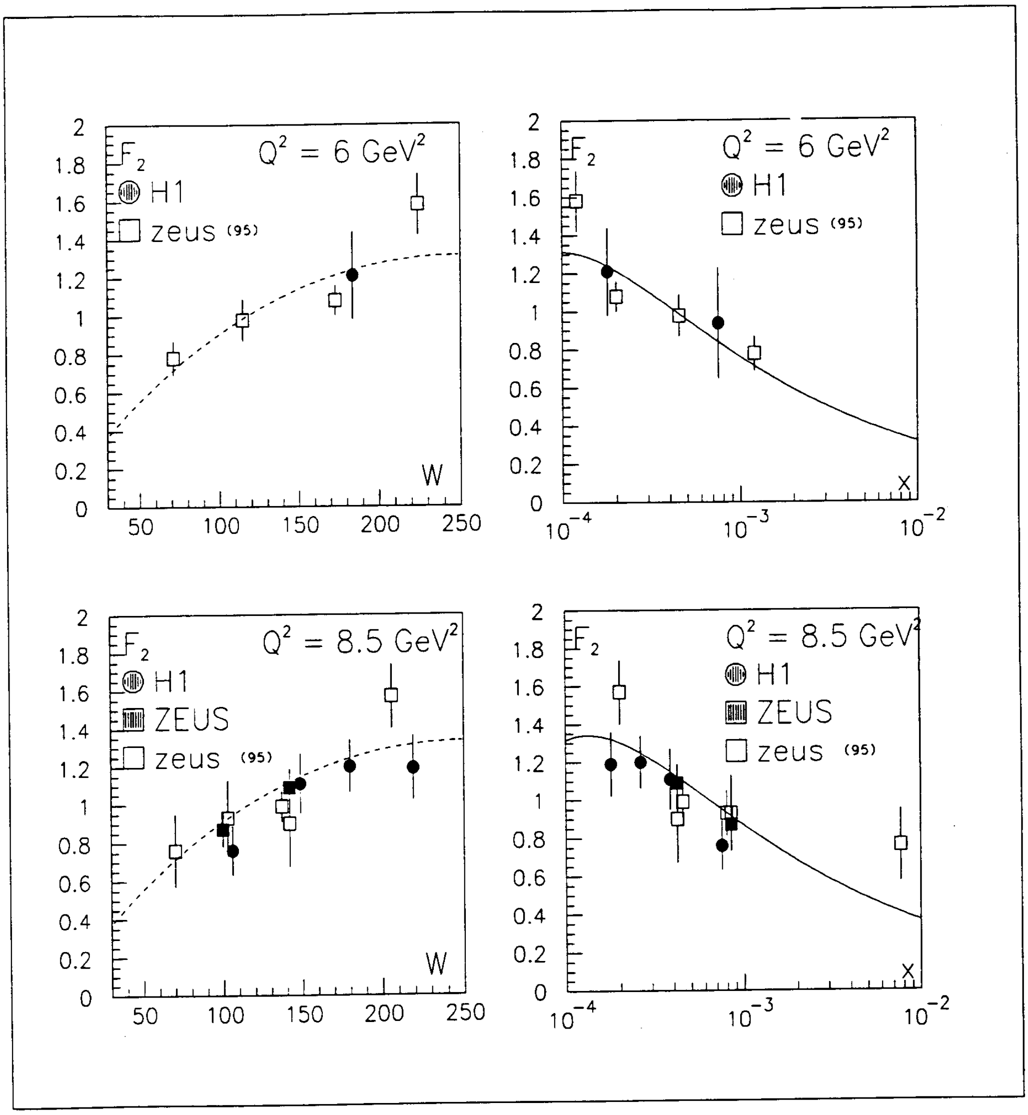

F2
Q² = 6 GeV²
H1
zeus (95)
W
F2
Q² = 6 GeV²
H1
zeus (95)
x
F2
Q² = 8.5 GeV²
H1
ZEUS
zeus (95)
W
F2
Q² = 8.5 GeV²
H1
ZEUS
zeus (95)
x

Figure 3b.

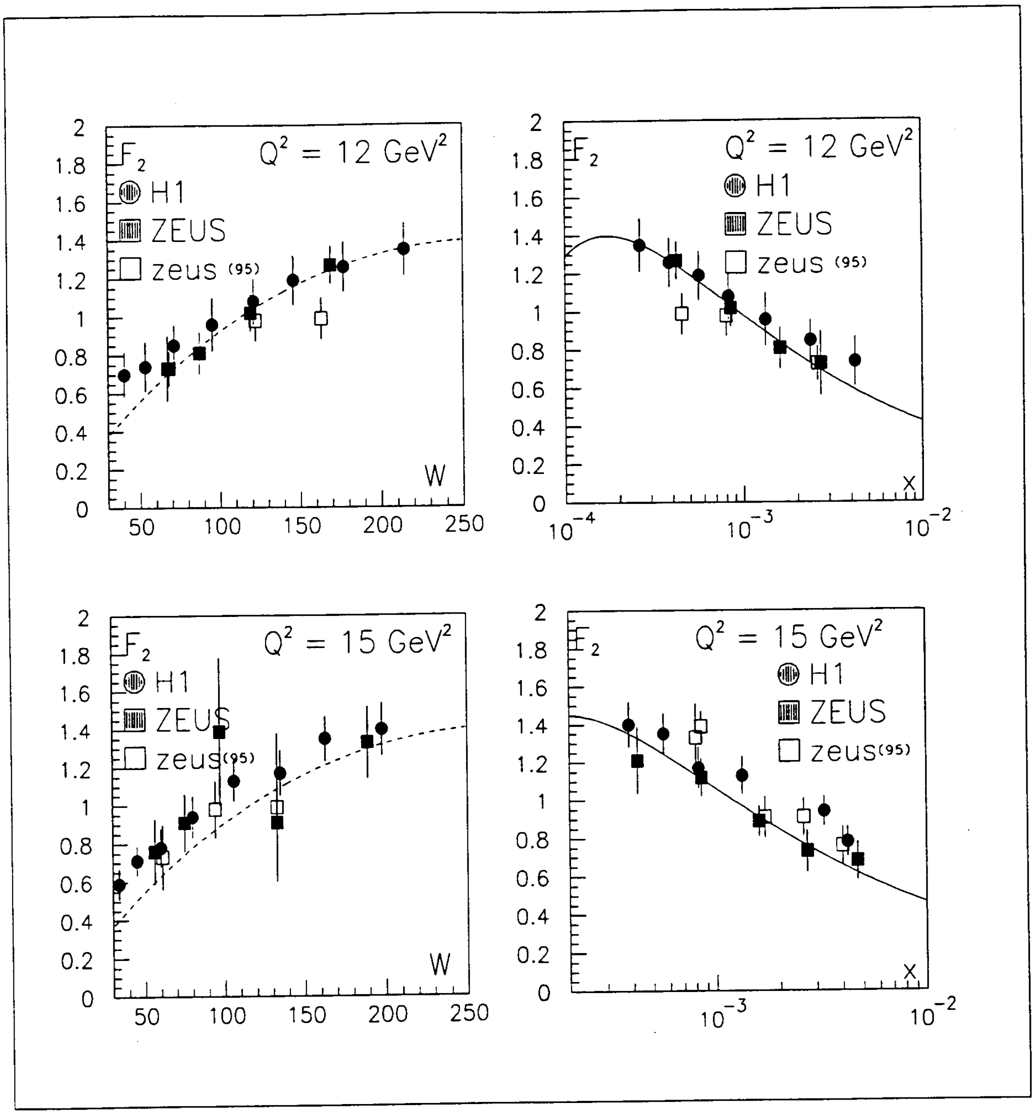


Figure 3c.

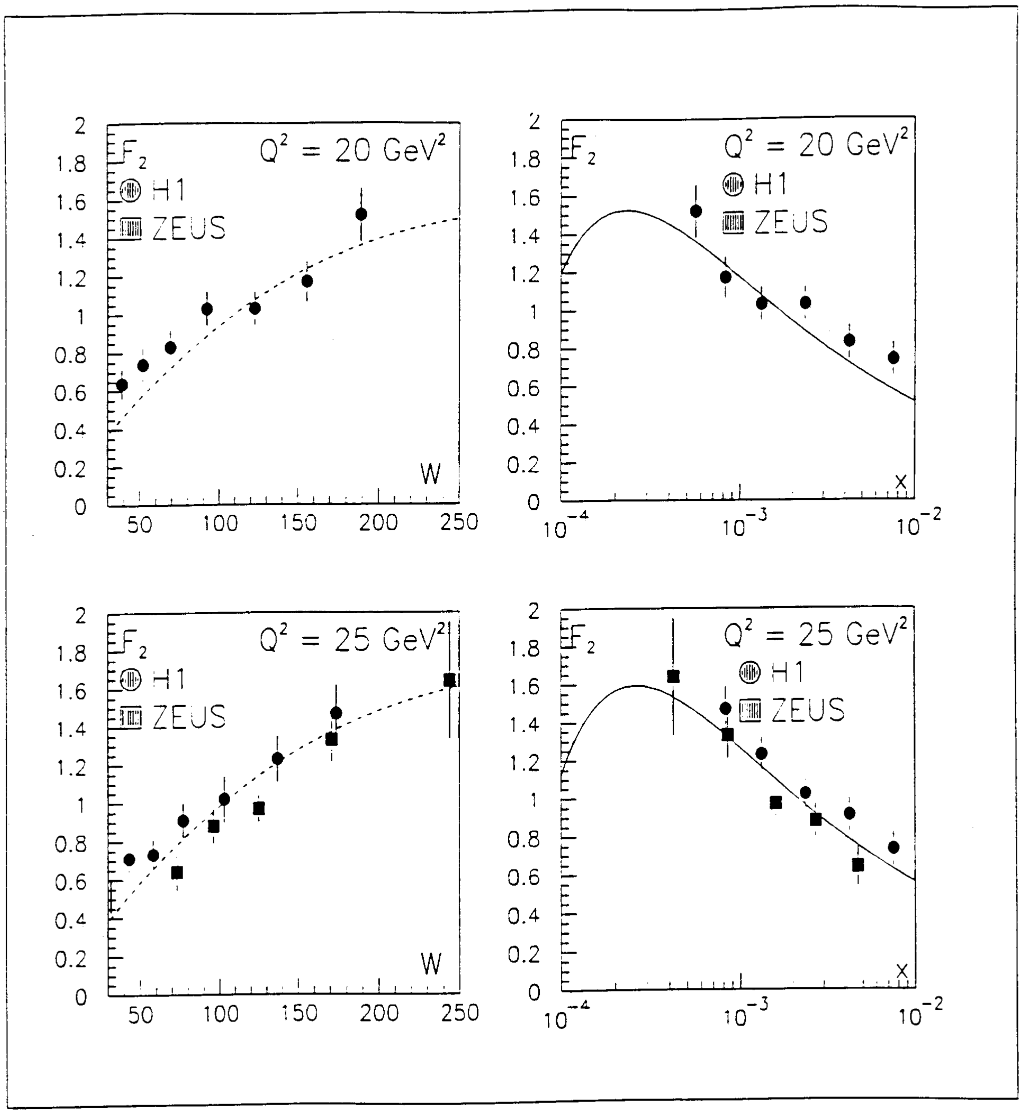


Figure 3d.

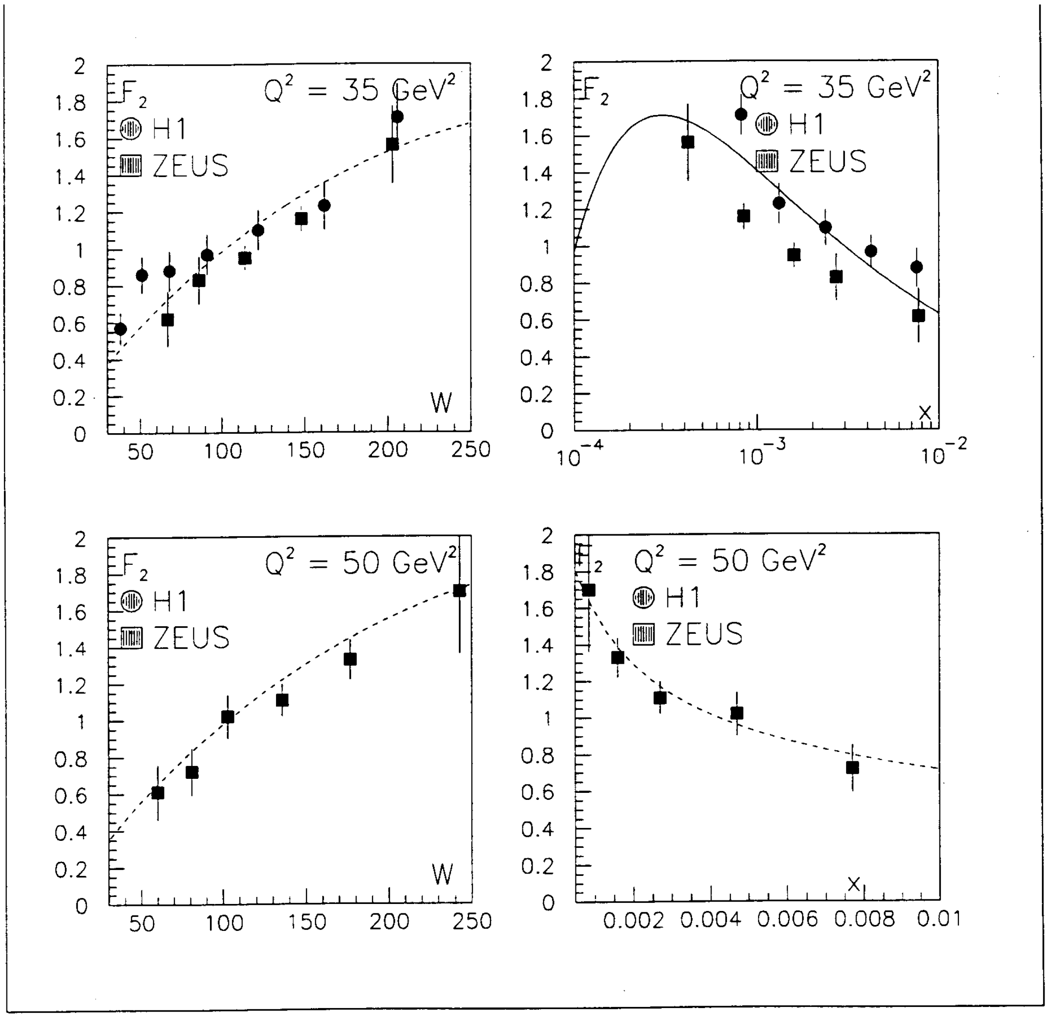


Figure 3e.

Figure 3: Comparison of $F_2(W, Q^2)$ and $F_2(x, Q^2)$ data with our calculations.

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