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What is the Velocity of Gravitational Waves?

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Abstract

We present a new field theory of gravity. It incorporates a great part of General Relativity (GR) and can be interpreted in the standard geometrical way like GR as far as the interaction of matter to gravity is concerned. However, it differs from GR when treating gravity to gravity interaction. The most crucial distinction concerns the velocity of propagation of gravitational waves. Since there is a large expectation that the detection of gravitational waves will occur in the near future the question of which theory describes Nature better will probably be settled soon.

Key-words: Gravity; Gravitational waves; Equivalence principle.

1 INTRODUCTION

1.1 A. Introductory Remarks

There is a general expectation concerning the possibility that we could detect gravitational waves before the next century is born. Such situation is based on the great number of new experimental devices that many laboratories, throughout the world, are constructing. Many scientists are going to become involved into this enterprise. Thus the time is ripe to theoretical re-examination of gravity theory in order to make predictions on the propagation of gravitational disturbances that could be tested in the near future.

Although the great majority of physicists expect that the observation of these waves will confirm the General Relativity prediction[1], it seems worthwhile to remember that such an expectation may just be one of those prejudgements that sometimes spread into the scientific community. On the other hand, we would like to emphasize that if this General Relativity result is proven to be false, it does not destroy the remaining and by far the major part of Einstein's theory that has already been experimentally proved.

The Equivalence Principle - which states that all kinds of matter (including massless particles as photons) interact in an unique and same way with the gravitational field - gave to Einstein the possibility to treat gravitational phenomena as a sort of modification of the spacetime geometry.

Does this universal behavior occur also in the case of gravity-gravity interaction? General Relativity makes an implicit hypothesis by means of which the answer to this question is affirmative. Although there is not even a single observational evidence that this is true, the fact that gravity must carry energy as any other field gave a strong motivation to believe that such an extrapolation concerning gravity-gravity interaction should be valid.

This assumption, that still today remains beyond any real observation, led to the most impressive result of General Relativity, that is, that gravitational processes are nothing but a real universal modification of the spacetime geometry.

However, if we want to keep observation as the true guide of our analysis of Nature, the whole actual situation can be summarized in two statements:

- As far as matter-to-gravity interaction is concerned, the GR scheme of geometrization of gravity seems to be a very good procedure.

[1] The recent spectacular success of the description of pulsar behavior, through loss of gravitational energy, by General Relativity, increased enormously the status of this theory.

- There is not any single direct observational evidence that supports that the self-interaction of gravity can be described in the same way.

Thus, if we limit ourselves to the traditional scientific method of submission of theory to observation, we must say that the universal modification of the geometry proposed by GR is indeed an extrapolation that is not still confirmed by experimental means, as far as the behavior of gravity-gravity coupling is concerned.

We are thus led to take seriously into account the two excludent alternatives concerning such gravity-to-gravity interaction, to wit:

- Gravity couples to gravity as any other form of energy.
- Gravity couples to gravity in a special way distinct from all different forms of energy.

In the first case, the universal modification of the geometry, as proposed in General Relativity, becomes a natural scenario. Neverthless, the complete absence of any experimental evidence that could help us to solve this question tells us that the decision must be dictated by other means, e.g., either by theoretical arguments[2] or by some sort of additional requirements imposed on the interaction mechanism. Einstein's hypothesis that gravitational interaction is the same not only when gravity-matter interaction is concerned but also for gravity-gravity processes, appeared to be the most natural way. In those early times, the complete absence of observations postponed the decision on this for the future. Later, in the fifties, the discovery of a simple way to treat gravity in terms of a standard field theory [1], reducing the gap between GR and all other field theories, made Einstein's hypothesis less questionable.

The purpose of the present work is to show a new way to generate a field theoretical description of gravity.

The fundamental property, that rests on the basis of our theory and that distinguishes the present program of investigation from GR, can be synthesized by the assertion that we will examine the possibility of conciliating the universality of matter to gravity interaction but considering that gravity to gravity coupling is somehow distinct. Since, as we shall see, the most dramatic consequence of the theory we propose here deals with the modification of the velocity of the gravitational waves, we limit our analysis here to the exam of this question.

Just in order to support this statement we note that it is a theoretical prejudice to argue that GR has already settled this question: only observation could do this.

[2] At most times such a decision comes from theoretical prejudices.

The new net result (concerning the propagation of gravitational waves) of our model can be summarized by noting that in this theory gravitational waves propagate in an effective geometry that is not the same as viewed by matter. One can then ask "what is the true geometry of spacetime"? The answer will be: it depends on the instrumentation we use to observe it. All forms of energy, except the gravitational, measure the same universal modification of curved spacetime. However, gravitational waves behave as if they were imbedded in a distinct geometry. What should the origin of this fact be? We postpone this question for latter analysis.

Just to provide a simple example of a theory that displays these ideas we will take as a model a non-linear theory proposed many years ago by Born [4] for spin-one field. We will apply a similar model for the spin-two case. Although in the original proposal of spin-one field such a non-linear theory appeared just as an exotic one, in the case of spin-two that we treat here the non-linearity is mandatory. We shall see in a subsequent section that this situation is just a consequence of the fact that spin-two couples to the energy-momentum tensor and the spin-one field couples to a conserved current. The fact that gravity itself must, for consistency, have energy makes the non-linearity a necessary requirement of any description of gravity.

In the standard geometrical way such self-interaction is described by a non-polinomial Lagrangian. In our re-exam of gravity from the point of view of field theory we will take (just as an example of constructing a coherent field theoretical model) the Born non-polinomial Electrodynamics as a paradigm for the gravitational interaction.

Finally, let us note that although we concentrate here all our presentation of the new theory only to the gravitational waves, we will make a few comments in the final section, anticipating some results that concern the properties of the new theory related to the behavior of the gravitational field in two important situations, to wit:

- The static spherically symmetric field;
- Cosmology.

We leave for a subsequent paper the proof that the standard tests of gravitational processes are satisfied by our present theory.

1.2 B. Synopsis

The presentation of the paper is the following. In Section (2) we present the main definitions and symbols that we use.

In Section (3) we discuss briefly how and why Feynman [1], Deser [2] and others (see [3] for a recent review) were led to describe General Relativity in terms of a field theory only based on the universality of gravitational interaction; we revise the Fierz linear theory and propose a class of non-linear theory of spin-two field. We also study the behaviour of the gravity energy-momentum tensor in the new theory. We present a new derivation of Fierz equation of motion that is worth of generalization in the non-linear case.

In the Section (4) we propose a specific theory and, by analyzing the evolution of the disturbances in this case, we show that the gravitational waves propagate on the null cone of an effective geometry distinct from that one seen by matter.

In Section (5) we present the gravity-matter interaction process and following the standard procedure we show how such an interaction can be described in terms of a modification of the geometry of the spacetime, in the same manner as it occurs in General Relativity.

Finally, in the last Section (6) we make some comments and state future perspectives of our scenario on gravity.

2 DEFINITIONS AND NOTATIONS

The auxiliary metric $\gamma_{\mu\nu}$ of Minkowski geometry[3] is written in an arbitrary system of coordinates in order to exhibit the general covariance of the theory. We define the corresponding covariant derivative by

$$V_{\mu\,;\,\nu} = V_{\mu\,,\,\nu} - \Delta^{\alpha}_{\mu\nu} V_{\alpha} \tag{1}$$

in which

$$\Delta^{\alpha}_{\mu\nu} = \frac{1}{2}\gamma^{\alpha\beta}\,(\gamma_{\beta\mu\,,\nu} + \gamma_{\beta\nu\,,\mu} - \gamma_{\mu\nu\,,\beta}). \tag{2}$$

The associated curvature tensor vanishes identically that is

$$R_{\alpha\beta\mu\nu}(\gamma_{\epsilon\lambda}) = 0. \tag{3}$$

We define a three-index tensor $F_{\alpha\beta\mu}$, which we will call the **gravitational field**, in terms of the symmetric standard variable $\varphi_{\mu\nu}$ (which will be treated as the potential) to describe spin-two fields, by the expression

[3] We shall see in next sections that this metric is not observable neither by matter nor by gravitational field.

$$F_{\alpha\beta\mu} = \varphi_{\mu[\alpha;\beta]} + \varphi_{,[\alpha}\gamma_{\beta]\mu} + \gamma_{\mu[\alpha}\varphi_{\beta]}{}^{\lambda}{}_{;\lambda}. \tag{4}$$

where we are using the anti-symmetrization symbol [ ] like

$$[A, B] \equiv AB - BA. \tag{5}$$

We use an analogous form to the symmetrization symbol ( )

$$(A, B) \equiv AB + BA, \tag{6}$$

From the above definition it follows that this quantity $F_{\alpha\beta\mu}$ is anti-symmetric in the first pair of indices and obeys the cyclic identity, that is

$$F_{\alpha\mu\nu} + F_{\mu\alpha\nu} = 0. \tag{7}$$

$$F_{\alpha\mu\nu} + F_{\mu\nu\alpha} + F_{\nu\alpha\mu} = 0. \tag{8}$$

The trace of the tensor $F_{\alpha\beta\mu}$ is given by

$$F_{\mu} \equiv F_{\mu\alpha\beta}\gamma^{\alpha\beta} = 4(\varphi_{\mu}{}^{\lambda}{}_{;\lambda} - \varphi_{\lambda}). \tag{9}$$

This allow us to re-write the expression of the field in the form

$$F_{\alpha\beta\mu} = \varphi_{\mu[\alpha;\beta]} + \frac{1}{4}F_{[\alpha}\gamma_{\beta]\mu}. \tag{10}$$

The equation of motion of the gravitational field will appear in a more convenient form when written in terms of an associated quantity $M_{\alpha\beta\mu}$ that has the same symmetries as $F_{\alpha\beta\mu}$ and is defined by

$$M_{\alpha\beta\mu} \equiv F_{\alpha\beta\mu} - \frac{1}{2}F_{\alpha}\gamma_{\beta\mu} + \frac{1}{2}F_{\beta}\gamma_{\alpha\mu}. \tag{11}$$

See the Appendix for other properties of these quantities.

The quantity κ represents Einstein's constant, written in terms of Newton's constant G_N and the velocity of light c by the definition

$$\kappa = \frac{8\pi}{G_N c^4}.$$

We set $c = 1$.

3 FROM THE UNIVERSAL COUPLING OF MATTER TO GRAVITY TO THE EINSTEIN GEOMETRIZATION SCHEME

In this section we will briefly review some points that were used in order to implement the hypothesis of universality of gravity interaction by the modification of the metrical properties of the spacetime. We will be interested here not in its historical birth of Einstein's formulation but, instead in its equivalent field theoretical formulation. In other words, we will follow here the path that led from the linear field theory of gravity to its non-linear processes and consequently to the corresponding geometrization scheme[1],[2].

There is no better and simpler manner to describe this than the one set by Feynmann in his lecture notes of 1962. Let us summarize such standard procedure that led from the field theoretical to the geometrical description of gravitational interaction.

The starting point is the linear massless spin-two field theory (Fierz equation), that reads:

$$G^{L}_{\mu\nu} = -kT_{\mu\nu} \tag{12}$$

in which

$$G^{(L)}_{\mu\nu} \equiv 2\phi_{\mu\nu} - \phi^{\alpha}{}_{\mu|\alpha\nu} - \phi^{\alpha}{}_{\nu|\alpha\mu} + \phi^{\alpha}{}_{\alpha|\mu\nu} - \gamma_{\mu\nu}(2\phi^{\alpha}{}_{\alpha} - \phi^{\alpha\beta}{}_{|\alpha\beta}). \tag{13}$$

The quantity $G^{L}_{\mu\nu}$ is divergence-free. This implies that, for compatibility, one must impose the condition that the energy-momentum tensor $T_{\mu\nu}$ of matter should also be divergenceless. Now, since the gravitational field contributes to the balance of the conservation law through its own energy, this imposition faces a difficulty since the matter energy-momentum tensor cannot be separately conserved.

It is precisely at this point that the hypothesis that gravity-gravity process follows the same type of behavior as matter-gravity interaction acts as a guide to the choice of the gravitational contribution to the source of $G^{L}_{\mu\nu}$. This means to add to the energy-momentum tensor of matter, the corresponding energy-momentum tensor for the gravitational field at the right-hand-side of equation (12).

The idea is to proceed step by step. We start by adding to the right-hand-side of equation(13) the tensor $T^{1}{}_{\mu\nu}$, that is the energy momentum tensor of the linear equation for gravity. As a consequence we must add to the original Lagrangian an additional term of higher order that yields, after variation, the term $T^{1}{}_{\mu\nu}$ to be added to the equation of motion. This generates a new compatibility condition, which is solved by adding to the right-hand-side of the equation of motion a new term of higher order, ${T^2}_{\mu\nu}$. This will impose, once more, that a new term must be added to the Lagrangian. This process continues indefinitely, since at each step a new term must be added to the Lagrangian in order to achieve the compatibility lost at the precedent lower order. To accomplish the task and to solve completely the compatibility condition, we must deal with a recurrence procedure such that yields an infinite series to appear [1]. It is precisely the summation of such infinite series that can be described by the equivalent geometrical formulation of General Relativity. Our purpose in the present work is to re-analyse this field theoretical description and to show that the above traditional procedure of searching compatibility is not unique.

3.1 Fierz Linear Theory Revisited

We will now show that the linear theory of spin-2 field can be described using the invariants constructed with the gravitational field $F_{\alpha\mu\nu}$. We will perform this simple exercise here just in order to present the motivation for our further non-linear theory.

There are only two invariants that can be constructed with the field.[4] They are:

$$A \equiv F_{\alpha\mu\nu} F^{\alpha\mu\nu}$$

$$B \equiv F_\mu F^\mu .$$

General covariance imposes that the Lagrangian that one can construct to describe the evolution of the gravitational field must be a functional of these invariants, that is

$$L = L(A, B).$$

The linear theory for the spin-2 field is given by the action

$$S^L = \frac{1}{2} \int \sqrt{-\gamma}\, (-A + \frac{3}{4} B) d^4x. \tag{14}$$

in which γ represents the determinant of $\gamma_{\mu\nu}$.

The proof of this assertion can be made either by a direct inspection on the equation of motion obtained from this Lagrangian or by noting that up to a total divergence we can write

[4] We could use instead of invariant A the one construted with $C_{\alpha\mu\nu}$, the traceless part of $F_{\alpha\mu\nu}$ which employs its irreducible parts. For simplicity of comparison to the traditional Fierz theory we decided here to make the above choice.

$$S^L = \int \sqrt{-\gamma}\varphi^{\mu\nu} G^L_{\mu\nu} d^4x \tag{15}$$

Indeed, we have from eq. (15), up to a total divergence

$$S^L = -\frac{1}{2}\int \sqrt{-\gamma}\varphi^{\mu\nu} M^{\lambda}{}_{(\mu\nu);\lambda} = \frac{1}{2}\int \sqrt{-\gamma}\varphi^{\mu\nu;\lambda} M_{\lambda(\mu\nu)} \tag{16}$$

As a consequence of a direct manipulation of this expression we can show directly that indeed

$$\varphi^{\mu\nu;\lambda} M_{\lambda(\mu\nu)} = -A + \frac{3}{4}B. \tag{17}$$

This demonstrates our assertion. What we have learned from this simple manipulation is that any theory that provides Fierz linear equation of motion in the weak field limit should reduce to the above combination of the invariants A and B. It is tempting then to examine those theories that are functionals only of this combination. We will limit thus all our analysis only to this set of theories. Besides, in the present paper we will consider a specific example of dynamics represented by a Lagrangian that is constructed as a non-polinomial functional of the field variables.

Before going into the exam of such a non-linear theory for the gravitational field, let us make a very short résumé of a typical example of a class of non-linear spin-one theory. We shall see that many properties of this example will have a deep analogy to the spin-two case. This will be useful since it will act as a guide for the analysis of the more complex case of the gravitational field.

3.2 Non-Linear Spin-One Theory

The dynamics is provided by an action[5]

$$S = \int \sqrt{-\gamma} L(F) d^4x \tag{18}$$

where the Lagrangian L that depends non-linearly on the invariant F constructed with the field $F_{\mu\nu}$ by the product[6]

$$F \equiv F_{\mu\nu} F^{\mu\nu}$$

The corresponding equation of motion that follows is[7]

[5] The attentive reader should notice that in this section the quantity $F_{\mu\nu}$ represents the Electromagnetic field.

[6] We consider here, just for simplicity the particular form of the theory which does not contain the invariant constructed with the dual of the field.

[7] We remind the reader that although we deal here with Minkowski background metric, we are using covariant derivatives just in order to exhibit the general covariance of physics.

$$\{L_F F^{\mu\nu}\}_{;\nu} = \frac{1}{4} J^{\mu} \tag{19}$$

where L_F represents the functional derivative of the Lagrangian with respect to the invariant. Maxwell theory is the case in which this derivative is the constant $-\frac{1}{4}$.

We will limit our analysis here to the theory that represents a non-linear electrodynamics, which was suggested by Born and developed by Infeld many years ago. The Lagrangian is given by

$$L_B = -\frac{1}{4} \left\{ \sqrt{b^4 + 2b^2 F} - b^2 \right\}, \tag{20}$$

in which the constant b has the meaning of the maximum possible value of the field. There are two important properties of such theory that interest us, to wit:

- The theory is non-linear.
- The propagation of the electromagnetic waves can be described as if the metrical properties of the spacetime were changed by the presence of the non-linear electromagnetic field.

These two qualities of this type of theory will be explored in the sequence in order to construct a non-linear spin-two field theory.

Using the general form of expressing the energy-momentum tensor $T_{\mu\nu}$, presented in a previous section, we obtain from the Lagrangian eq (20) the following:

$$T_{\mu\nu} = -L\gamma_{\mu\nu} - 4L_F F_{\mu\alpha} F^{\alpha}{}_{\nu} \tag{21}$$

We note that this quantity has basically all algebraic properties and symmetries that appear in the linear Maxwell case. For our purposes here the interesting property in the non-linear case is the fact that the interaction of the field with external currents remains the same. This can be seen most easily by a direct evaluation of the exchange of field's energy with its sources. Since we are dealing in the present paper with a most complex situation for the case of spin-two field, let us spend some time here and show this simple result in detail in the simpler case of spin-one field, just in order to get some insight of what should be expected for the spin-two case.

Thus, our task now is to evaluate the ratio of exchange of energy of the field, that is, the divergence of the symmetric energy-momentum tensor $T_{\mu\nu}$ of the class of non-linear spin-one field,. From the equation of motion (19) contracted with $F_{\alpha\mu}$ we obtain

$$\{L_F F_{\alpha\mu} F^{\mu\nu}\}_{;\nu} - L_F F_{\alpha\mu;\nu} F^{\mu\nu} = \frac{1}{4} F_{\alpha\mu} J^{\mu}$$

Using the expression of the tensor $T_{\mu\nu}$ we re-write this under the form

$$T^{\alpha\mu}{}_{;\mu} + L_F F_{,\alpha} + 4 L_F F^{\mu\nu} F_{\alpha\mu;\nu} = -F_{\alpha\mu} J^{\mu}$$

and thus finally

$$T^{\alpha\mu}{}_{;\mu} = -F_{\alpha\mu} J^{\mu} \tag{22}$$

The remarkable fact that follows from this expression is the well-known result that the balance of forces trough the exchange of energy of the field and the currents is independent of the form of the dependence of the Lagrangian on the invariant F. This is the lesson we learn from this simple analysis. Let us pass now to the gravitational field.

3.3 A Class of Non-linear Spin-Two Theory

As we saw in the previous section, when passing from the linear theory of gravity to the general case the standard procedure is to add to the energy-momentum tensor of matter, the corresponding energy tensor for the gravitational field. Proceeding step by step, the first non-linear term contains $T^{(1)}_{\mu\nu}$, which is the energy-momentum tensor obtained from the linear part. This procedure is based on the implicit hypothesis that one should treat the gravitational energy in the same foot as any other form of energy. In other words, the gravitational field generated by the gravitational energy is not distinct from the field generated by any other form of energy. This is a further extrapolation of the Equivalence Principle, applied to gravitational energy. At this point we take a path which is different from the one followed by Feynman [1], Deser [2], and others. Instead of adding to the source of the field the successive energy-momentum tensors of the gravitational field for each order of non-linearity, we make the hypothesis that these terms that represent gravity-to-gravity interaction must be constructed as a functional of the two invariants A and B. We thus set our action for the free gravitational field to be given by

$$S = \int \sqrt{-\gamma} L(A, B) d^4x \tag{23}$$

Variation of the potential $\varphi_{\mu\nu}$ yields

$$\delta S = \int \sqrt{-\gamma}\, \Theta^{\lambda}{}_{\mu\nu;\lambda}\, \delta\varphi^{\mu\nu} d^4x \tag{24}$$

giving the equation of motion

$$\Theta^{\lambda}{}_{\mu\nu;\lambda} = 0 \tag{25}$$

and $\Theta^{\lambda}{}_{\mu\nu}$ is

$$\Theta_{\lambda\mu\nu} \equiv 2L_A \left\{F_{\lambda(\mu\nu)} - F_\nu\gamma_{\lambda\mu} + 2F_\lambda\gamma_{\mu\nu}\right\} - 4L_B \left\{F_\mu\gamma_{\nu\lambda} + F_\nu\gamma_{\mu\lambda} - 2F_\lambda\gamma_{\mu\nu}\right\} \tag{26}$$

in which we have used the definitions $L_A \equiv \delta L/\delta A$. In the special linear case in which $L_A = -1/2$ and $L_B = 3/8$ this expression reduces to the Fierz case:

$$\Theta^L_{\lambda\mu\nu} = -M_{\lambda\mu\nu}. \tag{27}$$

in which the upperscript L stands for the linear case. Using the property above (see Section 2) it follows that the equation of motion reduces to

$$M^{\lambda}{}_{(\mu\nu);\lambda} = -2G^{L}{}_{\mu\nu} = 0. \tag{28}$$

and $G^{L}{}_{\mu\nu}$ is the Fierz linear operator (see eq. (12)).

Since we would like to impose that our theory should provide the good weak field limit, that is, Fierz linear equation, we will restrict our analysis in this paper to those Lagrangians whose dependence on the invariants obeys the relationship:

$$L_B = -\frac{3}{4} L_A. \tag{29}$$

Under this hypothesis the equation of motion for the free gravitational field within our scheme, equation (25) takes the form

$$\left\{L_A M^{\lambda}{}_{(\mu\nu)}\right\}_{;\lambda} = 0. \tag{30}$$

Using the properties of $M^{\lambda}{}_{(\mu\nu)}$ we can re-write this expression in a more convenient form:

$$G^{L}{}_{\mu\nu} = \frac{1}{2} L_A{}^{-1} \left\{L_{A;\lambda} M^{\lambda}{}_{(\mu\nu)}\right\} \tag{31}$$

Note that this is an exact equation, that is, it does not contain any sort of approximation term. We have just isolated the linear Fierz operator on the left-hand side and set all non-linearity terms to the right-hand side. Besides, under this form one can see directly that the source of the non-linearity, contrary to the case of GR, is not expressed in terms of the energy-momentum tensor of the gravitational field. We will make this point clearer when we treat the case of interaction with matter.

3.4 The Gravitational Energy-Momentum Tensor

The standard definition of the energy-momentum tensor for any field is provided by the variation of the Lagrangian with respect to the underlying metric, through the expression

$$T^{g}{}_{\mu\nu} = \frac{2}{\sqrt{-\gamma}} \frac{\delta L \sqrt{-\gamma}}{\delta \gamma^{\mu\nu}} \tag{32}$$

For the Lagrangians that we examine here (which obey the condition set up trough equation (29)) we have

$$T^{g}{}_{\mu\nu} = -L\gamma_{\mu\nu} + 2L_A \left\{ \frac{\delta A}{\gamma^{\mu\nu}} - \frac{3}{4} \frac{\delta B}{\gamma^{\mu\nu}} \right\} \tag{33}$$

After a rather long although direct calculation we obtain the form:

$$T^{g}{}_{\mu\nu} = -L\gamma_{\mu\nu} + L_A \left\{ 4F_{\mu\alpha\beta} F_{\nu}{}^{\alpha\beta} + 2F_{\alpha\beta\mu} F^{\alpha\beta}{}_{\nu} - 3F^{\alpha} F_{\alpha(\mu\nu)} - \frac{5}{2} F_{\mu} F_{\nu} + F^{\epsilon} F_{\epsilon} \gamma_{\mu\nu} \right\} \tag{34}$$

in which the symbol g stands for gravity.

This is the form of the gravitational energy-momentum tensor of the gravitational field, obtained from the Lagrangian L.

Take the trace of the above form to arrive at

$$T^{g} = -4L + 6L_A \left\{ A - \frac{3}{4} B \right\} \tag{35}$$

It is a direct exercise, left to the reader, to show that in the particular case of the linear action, (equation (14)), this expression reduces to Gupta energy-momentum tensor.

The tensor $T^{g}{}_{\mu\nu}$ differs from the one obtained from Noether's theorem by a total divergence. For future references it is useful to write the Noether energy-momentum tensor:

$$N_{\mu\nu} = -L\gamma_{\mu\nu} - 2\varphi_{\alpha\beta;\nu}\, L_A \left\{ F_{\mu(\alpha\beta)} + \frac{1}{2} F^{(\alpha}\gamma^{\beta)\mu} - F^{\mu}\gamma^{\alpha\beta} \right\} \tag{36}$$

or, equivalently

$$N_{\mu\nu} = -L\gamma_{\mu\nu} - \varphi_{\alpha\beta;\nu}\, L_A M_{\mu\alpha\beta} \tag{37}$$

Under this form the examination of the balance of energy between the gravitational field and its sources assumes a very simple expression

$$N^{\mu}{}_{\nu;\mu} = 2T_{\alpha\beta}\,\varphi^{\alpha\beta}{}_{;\nu} \tag{38}$$

This can be shown either using an analogous trick as in the spin-one case as described above or just by direct calculation. Let us remark that, like in the previous case of spin-one, the balance of energy between the gravitational field and its sources is independent of the form of the Lagrangian one takes to represent the gravitational field.

In the next section we will consider a specific simple case of gravitational theory by a choice of the Lagrangian in this scheme.

4 A SUGGESTIVE MODEL TO GRAVITATION

We have dealt in the precedent sections with the general scenario for our construction of a theory of gravity. The aim of this section is to produce a specific characterization of the gravitational equations of motion by searching a Lagrangian that satisfies the requirements set up in the precedent sections. This means that we will undertake now the task to produce a specific example of a field theory for the gravitational field that fulfills both conditions:

- obeys the requirements set up in the precedents section (including that it satisfies the equation 30).
- agrees with the observed tests of the gravitational field.

Just to simplify our explanation of the properties of our proposal we will take as a model of our scenario the non-linear theory of electromagnetic field proposed by Born and Infeld that we presented above. We do not intend to solve completely our task of searching a field theory for the gravitational field with such a naive model, but only to present our general scheme to deal with this question.

Taking this theory as an example, let us assume as a toy model for the self-interacting gravitational field the Lagrangian

$$L^{g} = \frac{1}{\kappa}\left\{\sqrt{b^4 + b^2(-A + \frac{3}{4}B)} - b^2\right\} \tag{39}$$

The quantity b has the dimension of $(lenght)^{-1}$. At this level of the theory it is a free parameter that can be choose either by some speculative consideration (e.g.by setting it equal to Planck's lenght, for instance) or by future observational requirements. By the time being we will left it free. Let us remark that, for the class of Lagrangians (eq. (39)), this quantity does not have the same meaning as in the original Born proposal of the maximum value possible for the field.

In the present presentation we will concentrate our interest on the consequences of such theory in the propagation of the gravitational waves. We leave the exam of the consequences of this model to a subsequent paper[8].

4.1 GRAVITATIONAL WAVES

The main purpose of this section is to examine the behavior of the gravitational waves in this theory. In order to do this we will analyse the evolution of discontinuities of the equation of motion through a characteristic surface Σ. This analysis give us the velocity of the gravitational wave that will be the key point to distinguish this kind of theory from General Relativity. For pedagogical reasons we start by examining the non-linear spin-one case.

4.1.1 Propagation of the Discontinuities: The Non-Linear Spin-1 Case

Let Σ be the surface of discontinuity. We set (using Haddamard's condition)

$$[F_{\mu\nu}]_\Sigma = 0, \tag{40}$$

and

$$[F_{\mu\nu,\lambda}]_\Sigma = f_{\mu\nu} \mathsf{k}_\lambda. \tag{41}$$

in which the symbol $[J]_\Sigma$ represents the discontinuity of the function J through the surface Σ.

Applying this into the equation of motion (19) we obtain

$$L_F f^{\mu\nu} \mathsf{k}_\nu + 2 L_{FF} \eta F^{\mu\nu} \mathsf{k}_\nu = 0, \tag{42}$$

where η is

[8] See Conclusions for other remarks related to the properties of this theory.

$$F^{\alpha\beta} f_{\alpha\beta} \equiv \eta.$$

After some algebraic manipulations the equation of propagation of the disturbances is provided by

$$\left\{ \gamma_{\mu\nu} (\frac{L_F^2}{L_{FF}} + L) + T_{\mu\nu} \right\} k^\mu k^\nu = 0 \tag{43}$$

In the case of the Born theory this expression reduces to

$$\left\{ \gamma_{\mu\nu} + \frac{4}{b^2} T_{\mu\nu} \right\} k^\mu k^\nu = 0 \tag{44}$$

In this particular non-linear non-polynomial theory we see that the disturbances propagate in the modified geometry, changing the background geometry $\gamma_{\mu\nu}$ into an effective one $g_{\mu\nu}$, which depends on the energy distribution of the field. We shall see later that the same structure occurs for spin-2.

4.1.2 Propagation of the Discontinuities: The Non-Linear Spin-2 Case

In an analogous way we set for the spin-2 field the discontinuity conditions

$$[F_{\mu\nu\alpha}]_\Sigma = 0 \tag{45}$$

and

$$[F_{\mu\nu\alpha;\lambda}]_\Sigma = f_{\mu\nu\alpha} \mathsf{k}_\lambda \tag{46}$$

that imply

$$[M_{\mu\nu\alpha;\lambda}]_\Sigma = m_{\mu\nu\alpha} \mathsf{k}_\lambda \tag{47}$$

which is the compact form that appears in the equation of motion (30). Taking the discontinuity of this equation we obtain

$$m_{\mu(\alpha\beta)} \mathsf{k}_\mu + \frac{4}{b^2} L_A^2 (\eta - \frac{3}{4}\Lambda) M_{\mu(\alpha\beta)} \mathsf{k}_\mu = 0 \tag{48}$$

in which the quantities η and Λ are defined by

$$\eta \equiv F_{\alpha\beta\mu} f^{\alpha\beta\mu}$$

$$\Lambda \equiv M_\mu m_\mu$$

After some algebraic manipulations (see the Appendix for some details) the equation of propagation of the disturbances is written as

$$\left\{ \gamma_{\mu\nu} (\frac{b^2}{2L_A} + L) + T^g{}_{\mu\nu} \right\} k^\mu k^\nu = 0 \tag{49}$$

In the case of the Born-like Lagrangian (see equation (39)) this expression reduces to

$$\left\{ \gamma_{\mu\nu} - \frac{1}{b^2} T^g{}_{\mu\nu} \right\} k^\mu k^\nu = 0 \tag{50}$$

in which $T^g{}_{\mu\nu}$ is the energy-momentum tensor of the gravitational field defined previously.

We note that in our theory we face a very similar behavior as in the previous spin-1 case. Indeed, the disturbances propagate in a modified geometry, changing the background geometry $\gamma_{\mu\nu}$, into an effective one $\tilde{g}_{\mu\nu}$ which depends on the energy distribution of the field $F_{\alpha\beta\mu}$. This fact shows that such a property stems from the structural form of the Lagrangian.

From the above calculations we conclude that, differently from General Relativity, in the present theory the caracteristic surfaces of the gravitational waves propagate on the null cone of an effective geometry distinct of that observed by all other forms of energy and matter. This result gives a possibility to choose between these two theories just by observations of the gravitational waves. This is a challenge that is expected to be solved in the near future.

5 THE GRAVITY-MATTER INTERACTION

As we pointed out in Section (1) there are strong evidences that matter couples universally with gravitation in such a way that its net effect can be described as if matter is imbedded in a Riemannian geometry produced by the gravitational field. Many authors [2] [3] have shown that this geometry can be written in terms of an unobservable background geometry $\gamma^{\mu\nu}$ and the gravitational field through the potential $\varphi^{\mu\nu}$ as[9]

$$g^{\mu\nu} \equiv \gamma^{\mu\nu} + \varphi^{\mu\nu} \tag{51}$$

[9]For some practical simplifications some authors do not deal with such a definition, but instead with pseudo-tensors obtained by multiplication of these tensors by the square-root of the corresponding determinants. See, for instance, [3].

This means that in order to know how matter couples to gravity one must just make the substitution of the background geometry $\gamma^{\mu\nu}$ by an effective geometry $g^{\mu\nu}$ and change accordingly the derivatives by the covariant derivatives constructed with $g^{\mu\nu}$. Since such procedure is already a standard one and there are very good reviews on this in the scientific literature we will not enter here in more details on this field theoretical equivalence of description of General Relativity. The reader that is not familiar with this should consult the review done in [3].

Let us just take here as a very simple example, the case of a scalar field Ψ. The free field equation of motion is provided by the action

$$S = \int \sqrt{-\gamma}\Psi_{,\mu}\Psi_{,\nu}\gamma^{\mu\nu} d^4x \tag{52}$$

In order to couple the scalar matter field with gravity we make the substitution of the metric $\gamma^{\mu\nu}$ into the effective one defined as above. The action takes then the form

$$S = \int \sqrt{-g}\Psi_{,\mu}\Psi_{,\nu}(\gamma^{\mu\nu} + \varphi^{\mu\nu}) d^4x \tag{53}$$

that is

$$S = \int \sqrt{-g}\Psi_{,\mu}\Psi_{,\nu}g^{\mu\nu} d^4x \tag{54}$$

It seems worth to remark that in this expression we have set the determinant of the inverse

$$g \equiv det\, g_{\mu\nu} = det(\gamma^{\mu\nu} + \varphi^{\mu\nu})^{-1}.$$

Note that the quantity $g_{\mu\nu}$ is the inverse of $g^{\mu\nu}$ and constitutes an infinite series in terms of the tensors $\gamma^{\mu\nu}$ and $\varphi^{\mu\nu}$. Then the equation of motion of the scalar field becomes

$$2\Psi \equiv \frac{1}{\sqrt{-g}} \left\{\sqrt{-g}\Psi_{,\mu}g^{\mu\nu}\right\}_{,\nu} = 0. \tag{55}$$

We can thus obtain the modification of the equation of motion of the gravitational field in presence of matter. In the above hypothesis (borrowed from General Relativity) that the matter feels the gravitational field only by the combination $g^{\mu\nu} \equiv \gamma^{\mu\nu} + \varphi^{\mu\nu}$ it follows that

$$\frac{\delta L}{\delta \gamma^{\mu\nu}} = \frac{\delta L}{\delta g^{\mu\nu}}$$

Thus in the theory developed here the general equation of motion of the gravitational field containing source terms takes the form

$$\left\{L_A M^{\lambda}{}_{(\mu\nu)}\right\}_{;\lambda} = -T_{\mu\nu}, \tag{56}$$

or the equivalent one

$$G^{L}{}_{\mu\nu} = \frac{1}{2} L_A{}^{-1} \left\{L_{A;\lambda} M^{\lambda}{}_{(\mu\nu)} + T_{\mu\nu}\right\} \tag{57}$$

in which $T_{\mu\nu}$ represents the matter energy-momentum tensor of matter evaluated by the equation (32).

Finally, we would like to emphasize that, as we have pointed out before, we can thus conclude that, in what concerns the matter to gravity interaction the theory proposed here is indistinguishable of the General Relativity.

6 CONCLUSION

Einstein's theory of General Relativity is one of the most beautiful, comprehensive and deeply important achievements of classical field theory. Not only it contains and exhibits such a simplicity and internal coherence but, more than this, it provides a sound step towards the understanding of gravitational processes which has had no rival since the early times of Newton hypothesis concerning the existence of the universal gravitational atraction. Thus, any theory that dares to propose even a small modification on its scenario faces not only the challenge to present a sound new argument -but besides, it has to deal with the enormous difficulty of intending the substitution of a successful paradigm.

Thanks to the work of many physicists as Feynmann, Deser and others, we have learned that Einstein's geometric vision of General Relativity admits an equivalent field theoretical presentation. In such a formulation, the phenomenon of gravity becomes more similar to the rest of Physics and, besides (and by far the more important) we obtain a deeper comprehension on the mechanism of self-interaction of the gravitational field. It is precisely the analysis of this mechanism that we think is worth to re-examine. We can thus realize that there are two reasons for this:

- The theoretical challenge of providing another coherent manner to describe gravitational self-interaction process.
- The perspective of test alternative theories by the observation, in the near future, of gravitational waves.

In the present paper we describe a new program of analysis of gravitational interaction that has a very similar geometrical interpretation as General Relativity as far as matter to gravity processes are concerned. However, it becomes distinct from GR in the description of gravity to gravity interaction. We have exhibited a specific example of a theory in order to show how these ideas can be implemented. We have evaluated the propagation of gravitational waves within such a theory and shown that the velocity of the waves does not coincide with the GR prediction. Once there is a large expectative that these waves will be detected in the next few years, we could test this aspect of these theories soon.

Finally, we should mention that we have found two particular solutions of the theory we present here: the spherically symmetric static case and a cosmology. We can anticipate that both solutions are in good agreement to actual observations. Both will be published soon.

7 APPENDIX: SOME USEFUL FORMULAE

7.1 The Traceless Part of the Gravitational Field

The traceless tensor $C_{\alpha\mu\nu}$ is defined by

$$C_{\alpha\mu\nu} \equiv F_{\alpha\mu\nu} - \frac{1}{3} F_\alpha \gamma_{\mu\nu} + \frac{1}{3} F_\mu \gamma_{\alpha\nu}$$

The invariant constructed with this quantity C is given by

$$C \equiv C_{\alpha\mu\nu} C^{\alpha\mu\nu}$$

It then follows that between this quantity and the invariants dealt with in the paper (A and B)there is the following relation

$$C = A - \frac{2}{3} B.$$

We could then use the irreducible parts of the gravitational field to describe its dynamics. In this case the generic form of the Lagrangian should be

$$L[B, C].$$

.

7.2 Variational Formulae

The evaluation of the energy-momentum of the gravitational field needs some technicalities concerning the dependence of $F^{\alpha\mu\nu}$ on the background metric $\gamma_{\mu\nu}$. There are some expressions that simplify this manipulation. Let us present here some of these.

We have

$$\delta A = \left\{ 2F_{\mu\alpha\beta}F_{\nu}{}^{\alpha\beta} + F_{\alpha\beta\mu}F^{\alpha\beta}{}_{\nu} + \frac{3}{2}F^{\alpha}F_{\alpha(\mu\nu)} + F_{\mu}F_{\nu} - F^{\epsilon}F_{\epsilon} \right\} \delta\gamma^{\mu\nu}$$

To show the validity of this expression we must use the relations

$$F^{\alpha\beta\mu}\delta F_{\alpha\beta\mu} = \frac{1}{2}F^{\lambda}\gamma^{\beta\mu}\delta F_{\lambda\beta\mu}$$

and

$$F^{\lambda}\gamma^{\beta\mu}\delta F_{\lambda\beta\mu} = \left\{ \frac{3}{2}F^{\alpha}F_{\alpha(\mu\nu)} + F_{\mu}F_{\nu} - F^{\epsilon}F_{\epsilon}\gamma_{\mu\nu} \right\} \delta\gamma^{\mu\nu}$$

For the invariant B we obtain

$$\delta B = \left\{ 4F^{\alpha}F_{\alpha(\mu\nu)} + 3F_{\mu}F_{\nu} - 2F^{\epsilon}F_{\epsilon}\gamma_{\mu\nu} \right\} \delta\gamma^{\mu\nu}$$

7.3 Algebraic Formuli

We list some useful relations that are worth obtaining in order to deal with the quantities presented in the text.

A simple inspection on the equations of motion of the dynamics of the present theory shows that it depends only on the symmetric part of the two last indices of the gravitational field. This is a consequence of the symmetry of the potential $\varphi_{\mu\nu}$. Note however that even the invariants of the theory have already this property. Indeed, we can write

$$A \equiv F_{\alpha\mu\nu}F^{\alpha\mu\nu} = \frac{1}{3}F_{\alpha(\mu\nu)}F^{\alpha(\mu\nu)}.$$

Another consequence of this symmetry is explicitated through the expression

$$F_{\alpha(\mu\nu)} + F_{\mu(\alpha\nu)} + F_{\nu(\alpha\mu)} = 0.$$

Consequently, the same formula is valid for the associated quantity $M_{\alpha\mu\nu}$, that is,

$$M_{\alpha(\mu\nu)} + M_{\mu(\alpha\nu)} + M_{\nu(\alpha\mu)} = 0.$$

These expressions allow us to re-write the gravitational energy-momentum tensor ${T^g}_{\mu\nu}$ under the equivalent form

$$ {T^g}_{\mu\nu} = -L\gamma_{\mu\nu} + \frac{1}{3}L_A \left\{ 2M_{\mu(\alpha\beta)}{M_\nu}^{(\alpha\beta)} + 4M_{\alpha(\beta\mu)}{M^\alpha}_{(\beta\nu)} + 3M^\alpha M_{\alpha(\mu\nu)} - \frac{3}{2}M_\mu M_\nu \right\} $$

There is a cyclic identity[10] concerning the first derivative of the field which is given by

$$ {F_{\lambda\alpha}}^\nu{}_{;\beta} + {F_{\alpha\beta}}^\nu{}_{;\lambda} + {F_{\beta\lambda}}^\nu{}_{;\alpha} = \frac{1}{2}\left\{ \delta^\nu_\alpha A_{[\beta\lambda]} + \delta^\nu_\beta A_{[\lambda\alpha]} + \delta^\nu_\lambda A_{[\alpha\beta]} \right\} $$

in which we have defined

$$ A_{\mu\nu} \equiv -{F^\alpha}_{\mu\nu;\alpha} - F_{\mu;\nu} $$

In terms of the quantity $M^{\alpha\beta\nu}$ this expression then takes the form

$$ {M_{\lambda\alpha}}^\nu{}_{;\beta} + {M_{\alpha\beta}}^\nu{}_{;\lambda} + {M_{\beta\lambda}}^\nu{}_{;\alpha} = $$
$$ \frac{1}{2}\delta^\nu_\alpha \left\{ {M_{\beta\lambda}}^\epsilon;\epsilon - \frac{1}{2}M_{[\beta\lambda]} \right\} + \frac{1}{2}\delta^\nu_\beta \left\{ {M_{\lambda\alpha}}^\epsilon;\epsilon - \frac{1}{2}M_{[\lambda\alpha]} \right\} + \frac{1}{2}\delta^\nu_\lambda \left\{ {M_{\alpha\beta}}^\epsilon;\epsilon - \frac{1}{2}M_{[\alpha\beta]} \right\} $$

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[2] S.Deser, J. Gen. Rel. Grav. 1, 9, (1970)

[3] L.P.Grischuck, A.N.Petrov and A.D.Popova, Commun. Math. Phys. 94, 379 (1984)

[4] M.Born and L.Infeld, Nature 133, 63 (1934)

[5] S.L.S. Duque, Ph.D Thesis (1994), unpublished.[11]

[10] See, for instance [5]

[11] A copy in Portuguese may be available. Write to Novello@lafexsu2.lafex.cbpf.br.