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Light-front Quantized Scalar Field Theory and Phase Transition

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Abstract:

The self-consistent Hamiltonian formulation of the scalar field theory on the light-front, which contains also a constraint equation, is constructed and compared with the equal-time formulation. In two dimensions the mass renormalization condition and the renormalized constraint equation contain all the information to describe the phase transition in the ϕ^4 theory, which is found to be of the second order.

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1- Introduction:

Dirac [1] suggested the possibility of building dynamical theory of a physical system on the three dimensional hypersurface in space-time formed by a plane wave front advancing with the velocity of light. The initial conditions on the dynamical variables are here specified on the hyperplane (*light-front*), say, $\tau = (x^0 + x^3)/\sqrt{2} = 0$, which has a light like normal- the *front form* dynamics. The conventional formulation uses instead the $x^0 \equiv t = 0$ hyperplane- the *instant form* dynamics. In both the cases the separation between two points is space like in general which may become light like in particular cases. Latter the *front form* dynamics was rediscovered by Weinberg [2] in the infinite momentum frame rules in the quantized field theory. The rules were clarified by Kogut and Soper [3] to correspond to the theory quantized on the light-front. Even earlier [4] the $p \to \infty$ technique played an important role in the derivation of the current algebra sum rules and it was also observed that it amounted to using appropriate light-front current commutators. The recent revival of interest [5,6] in the light-front theory has been motivated by the difficulties faced in the nonperturbative QCD in the conventional formulation. We have the problem of reconciling the standard constituent (valence) quark model and QCD, where the vacuum state itself contains an infinite sea of constituent quarks and gluons (partons), to describe the hadrons. The problem of describing relativistic bound-state of light quarks in the presence of the complicated vacuum in the *instant form* seems difficult but it was found that the Light-front Tamm-Dancoff method [5] may be feasible. The *front form* dynamics may serve as a complementary tool where we have a simple vacuum while the complexity of the problem is now transferred to the light-front Hamiltonian. This was illustrated recently also in the light-front quantized scalar field theory where it is found [7] to be nonlocal, in contrast to the polynomial form of the equal-time formulation, due to the presence of a *constraint equation* in the theory which allows also to describe the spontaneous symmetry breaking. It was shown [7] that the physical outcome in the case of continuous symmetry is the same in the two forms of dynamics, however, it is achieved through different mechanisms. In fact, we will show that in the case below many of the external ingredients, which we usually *add* to the scalar theory treated in the *instant form* upon invoking the physical considerations, are already contained in the *front form* through a set of self-consistency constraints.

The simplicity of the light-front vacuum, which may often coincide with the free theory one, derives in particular from the observation that the longitudinal momentum $k^+ = (k^0+k^3)/\sqrt{2}$ of a free massive particle is necessarily positive. Another general feature of the *front form* theory is that it describes a constrained dynamical system. The Dirac procedure [8] to construct a self-consistent Hamiltonian formulation is convenient to use. The theory may be quantized via the correspondence of the Dirac brackets with the commutators of the corresponding operators and these brackets, available in the Hamiltonian framework, are again useful for unifying [1] also the principle of (special) relativity in the theory. We also note that if we assume the *microcausality* principle both the equal-time and equal-τ commutators of two, say, scalar observables may take nonvanishing values only on the light-cone.

We discuss in the paper the stability of the vacuum in the ϕ^4 theory when the coupling constant is increased from vanishingly small values to larger values. The light-front framework seems very attractive to study this problem. On renormalizing the theory we have now, in addition to the usual equations like the mass renormalization conditions, also the renormalized constraint equations to deal with. In our context we remind that there are rigorous proofs [9] on the triviality of ϕ^4 theory in the continuum for more than four space-time dimensions and on its interactive nature for dimensions less than four. In the important case of four dimensions the situation is still unclear [9] and light-front dynamics may throw some light on it. In view of the complexity of the renormalization problem in this case we will illustrate our points by considering only the two dimensional theory, which is of importance in the condensed matter physics. For example, from the well established results on the generalized Ising models, Simon and Griffiths [10] conjectured some time ago that the two dimensional ϕ^4 theory should show the *second order* phase transition. We do find it to be so by quantizing the theory on the light-front. The variational methods based on the *instant form* like the Hartree approximation or Gaussian effective potential [11], or the one based on a scheme of canonical transformations [12], or using the *front form* theory but ignoring the constraint [13], all seem to give a first order phase transition contradicting the conjecture. The *instant form* post-Gaussian approximation [14] and the non-Gaussian variational method [15] give a second order transition for a particular value of the coupling constant. Our result shows second order transition for any coupling above a critical value. In view of the remarks made at the end of the first paragraph *front form* theory should be able to throw light on this quite old problem. The procedure used in the paper is the well established Dyson-Wick expansion [16] of perturbation theory and may be improved systematically computing still higher order corrections which is difficult to do in the variational methods. A brief sketch of the front form Hamiltonian formulation is given in Sec. 2, the renormalization and the phase transition are discussed in Sec. 3, and the conclusions summarized in Sec. 4.

2- Light-front Hamiltonian formulation:

The Lagrangian density in the *front form* is given by $[\dot{\phi}\phi' - V(\phi)]$, where an overdot and a prime indicate the partial derivatives with respect to the light-front *time* $\tau \equiv x^+ = (x^0 + x^1)/\sqrt{2}$ and the longitudinal coordinate $x \equiv x^- = (x^0 - x^1)/\sqrt{2}$ respectively. The eq. of motion is $2\dot{\phi}' = -V'(\phi)$, where a prime on V indicates the variational derivative with respect to ϕ. It shows that the classical solutions, $\phi = const.$ are allowed and given by solving $V'(\phi) = 0$. We write [7] $\phi(x,\tau) = \omega(\tau) + \varphi(x,\tau)$ where the variable ω corresponds to the bosonic condensate and φ describes the fluctuations above the former. Our interest being in the ground state we assume (in view of the translational invariance of the vacuum) ω to be independent of τ. The Lagrangian then reads as

$$\int_{-L/2}^{L/2} dx\, [\dot{\varphi}\,\varphi' - V(\phi)], \tag{2.1}$$

where $L \to \infty$ and $V(\phi) = (1/2){m_0}^2\phi^2 + (\lambda/4)\phi^4 + const.$, $\lambda > 0$, with the *correct sign* for the *bare* mass term. The Lagrangian being singular we follow the Dirac method [8] to construct the Hamiltonian which describes the evolution of the system in τ. Indicating by $\pi(x,\tau)$ the momenta conjugate to $\varphi(x,\tau)$, the primary constraint is found to be $\Phi \equiv \pi - \varphi' \approx 0$ while the canonical Hamiltonian density is $\mathcal{H}_c = V(\phi)$, with the symbol $\approx$ standing for the weak equality. We postulate the standard Poisson brackets with the nonvanishing one give by $\{\pi(x,\tau), \varphi(y,\tau)\} = -\delta(x-y)$ and adopt for the preliminary Hamiltonian

$$H'(\tau) = H_c(\tau) + \int dy\; u(\tau,y)\Phi(\tau,y), \tag{2.2}$$

where u is a Lagrange multiplier function. We find

$$\dot{\Phi} = \{\Phi, H'\} \approx -V'(\phi) - 2u'. \tag{2.3}$$

The persistency requirement $\dot{\Phi} \approx 0$ results in a consistency condition for determining u and does not generate a new constraint. The constraint $\Phi(x) \approx 0$ is second class since, $\{\Phi(x), \Phi(y)\} = -2\partial_x \delta(x-y) \equiv C(x,y) = -C(y,x) \neq 0$. The unique inverse with the correct symmetry property is $C^{-1}(x,y) = -C^{-1}(y,x) = -\epsilon(x-y)/4$ so that the the equal-τ *Dirac bracket* $\{,\}_D$ may be defined by

$$\{f(x), g(y)\}_D = \{f(x), g(y)\} + \frac{1}{4}\int\int du dv \{f(x), \Phi(u)\}\epsilon(u-v)\{\Phi(v), g(y)\}. \quad (2.4)$$

Since $\{f, \Phi\}_D = 0$ for any arbitrary functional $f(\varphi, \pi)$ we may set now $\pi = \varphi'$ even inside the Dirac bracket and (2.2) reduces to the following light-front Hamiltonian

$$H^{l.f.} = \int_{-L/2}^{L/2} dx \left[\omega(\lambda\omega^2 + m_0{}^2)\varphi + \frac{1}{2}(3\lambda\omega^2 + m_0{}^2)\varphi^2 + \lambda\omega\varphi^3 + \frac{\lambda}{4}\varphi^4 + const.\right]. \quad (2.5)$$

From (2.4) we derive also $\{\varphi(x,\tau), \varphi(y,\tau)\}_D = -(1/4)\epsilon(x-y)$.

The quantization is done by associating to each dynamical variable a field operator and we make the correspondence $i\{f,g\}_D \rightarrow [f,g]$ where $[f,g]$ indicates a commutator (or anticommutator) between the operators. The well known light-front commutator

$$[\varphi(x,\tau), \varphi(y,\tau)] = -(i/4)\epsilon(x-y). \quad (2.6)$$

is then reobtained. Its right hand side is not a delta function and does not vanish for distinct points with $x^- \neq y^-$ on the light-front $x^+ = \tau$ in contrast to the case of equal-time commutator. The antisymmetry property of the commutator requires that we define $\epsilon(0) = 0$. The Hamilton's eq. is found to be

$$\begin{aligned} \dot{\varphi}(x,\tau) &= \{\varphi(x,\tau), H^{l.f.}(\tau)\}_D \\ &= -\frac{1}{4}\int dy\, \epsilon(x-y)\, V'(\phi(y,\tau)) \end{aligned} \quad (2.7)$$

and we recover the Euler-Lagrange eq.

$$\dot{\varphi}'(x,\tau) = -\frac{1}{2}V'(\phi(x,\tau)) \quad (2.8)$$

assuring us of the self-consistency [8]. If we substitute the value of $V'(\phi)$ obtained from (2.8) into (2.7) we find on an integration by parts

$$\dot{\varphi}(x,\tau)=\dot{\varphi}(x,\tau)-\frac{1}{2}\Big[\dot{\varphi}(\infty,\tau)\epsilon(\infty-x)-\dot{\varphi}(-\infty,\tau)\epsilon(-\infty-x)\Big]. \tag{2.9}$$

For finite x we obtain $\dot{\varphi}(\infty,\tau)+\dot{\varphi}(-\infty,\tau)=0$. On the other hand the commutator (2.6) may be realized in the momentum space through the expansion

$$\varphi(x,\tau)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dk\,\frac{\theta(k)}{\sqrt{2k}}\,[a(k,\tau)e^{-ikx}+a^{\dagger}(k,\tau)e^{ikx}] \tag{2.10}$$

where $a(k,\tau)$ and $a^{\dagger}(k,\tau)$ satisfy $[a(k,\tau),a(k',\tau)^{\dagger}]=\delta(k-k')$ etc. Now if we integrate the momentum space expansion of $\varphi'(x,\tau)$ over x we show that $\varphi(\infty,\tau)-\varphi(-\infty,\tau)=0$. Combining this with the condition obtained above we are led to $\partial_{\tau}\varphi(\pm\infty,\tau)=0$ as another consistency condition. This is similar to $\partial_t\varphi(x^1=\pm\infty,t)=0$ which in contrast is *added* to the equal-time theory upon invoking physical considerations.

The following nonlocal *constraint equation* in the *front form* theory then follows if we integrate the Lagrange eq. of motion (2.8) over the coordinate x and use the constraint just obtained

$$\beta(\tau)\equiv\omega(\lambda\omega^2+m_0{}^2)+lim_{L\to\infty}\frac{1}{L}\int_{-L/2}^{L/2}dx\Big[(3\lambda\omega^2+m_0{}^2)\varphi+\lambda(3\omega\varphi^2+\varphi^3)\Big]=0. \tag{2.11}$$

Eliminating ω using (2.11) would result in a nonlocal and involved Hamiltonian in the place of the local and polynomial type adopted in the conventional approach. At the tree level since the φ is an ordinary bounded function, the second term in the expression of β in (2.11) drops out and we obtain the conventional result $V'(\omega)=0$. The extension [7] to $3+1$ dimensions and the continuous symmetry is straightforward. In fact, general arguments which include the *microcausality* postulate may be given on the light-front to show the possibility of the appearance of nonlocality in the longitudinal coordinate, while the theory remains local in the transverse ones. We do find in ref. 1 some examples where constraints on the potential arise due to the necessity of incorporating special relativity in the theory.

3- Renormalization. Phase transition in two dimensions:

The theory based on (2.5), (2.6), and (2.11) may be renormalized. We do not solve (2.11) but instead obtain the renormalized constraint equation. We set

$M_0^2(\omega) = (m_0^2 + 3\lambda\omega^2)$ and choose $\mathcal{H}_0 = {M_0}^2\varphi^2/2$ so that $\mathcal{H}_{int} = \lambda\omega\varphi^3 + \lambda\varphi^4/4$. In view of the superrenormalizability of the two dimensional theory we need to do only the mass renormalization. We assume that the bare mass is nonvanishing so that [17] $\int dx\varphi(x,\tau) = \sqrt{2\pi}\,\tilde{\varphi}(k=0,\tau) = 0$, e.g., $k \equiv k^+ > 0$, and the corresponding term in (2.5) and (2.11) drops out. We could follow as is usually done [18] the old fashioned perturbation theory, but the Dyson-Wick expansion based on the Wick theorem [16]

$$T[e^{i\int d^2x\, j(x)\varphi(x)}] = e^{-\frac{1}{2}\int\int d^2x d^2y\, j(x)G_0(x-y)j(y)} : [e^{i\int d^2x\, j(x)\varphi(x)}] :$$

is convenient. Here T indicates the ordering in τ and G_0 is the free scalar field propagator.

The self-energy correction to the *one loop order* is

$$\begin{aligned} -i\Sigma(p) &= -i\Sigma_1 - i\Sigma_2(p) \\ &= (-i6\lambda)\frac{1}{2}D_1({M_0}^2) + (-i6\lambda\omega)^2\frac{1}{2}(-i)D_2(p^2,{M_0}^2), \end{aligned} \tag{3.1}$$

where the divergent contribution D_1 refers to the one-loop tadpole while D_2 to the one-loop finite contribution coming from the φ^3 vertex. The latter carries the sign opposite to that of the first and it will be argued below to be of the same order in λ as the first one, because of the presence of ω in it. We have shown explicitly the symmetry and other factors in (3.1). The one particle reducible graphs coming from the cubic vertex are ignored and also $\langle\varphi(x)\rangle = 0$. On using the dimensional regularization [19] with the minimal subtraction prescription.

$$\begin{aligned} D_1(M_0) &= \frac{1}{(2\pi)^n}\int\frac{d^nk}{(k^2+{M_0}^2)} = \mu^{(n-2)}\frac{1}{4\pi}\left(\frac{{M_0}^2}{4\pi\mu^2}\right)^{(\frac{n}{2}-1)}\Gamma(1-\frac{n}{2}) \\ &\to \frac{\mu^{(n-2)}}{4\pi}\Big[\frac{2}{(2-n)} - \gamma - ln(\frac{M_0^2}{4\pi\mu^2})\Big] \end{aligned} \tag{3.2}$$

where the limit $n \to 2$ is to be taken at the end and we suppress the terms which vanish in this limit. Also

$$D_2(p^2,{M_0}^2) = \int\frac{d^2k}{(2\pi)^2}\frac{1}{(k^2+M_0^2)[(p-k)^2+{M_0}^2]}, \qquad D_2(p^2,M_0^2)|_{p^2=-M_0^2} = \frac{\sqrt{3}}{18M_0^2}. \tag{3.3}$$

The physical mass $M(\omega)$ is defined [16,19] by

$$M_0{}^2(\omega) + \Sigma(p)|_{p^2=-M^2(\omega)} = M^2(\omega) \tag{3.4}$$

where p^μ is the Euclidean space 4-vector and $M(\omega)$ determines the pole of the renormalized propagator. We obtain from (3.1-4)

$$M_0{}^2(\omega) = M^2(\omega) + \frac{3\lambda}{4\pi}\left[\gamma + ln(\frac{M^2(\omega)}{4\pi\mu^2})\right] + 18\lambda^2\omega^2 D_2(p, M^2)|_{p^2=-M^2} + \frac{3\lambda}{2\pi}\frac{1}{(n-2)}. \tag{3.5}$$

Here we have taken into account that in view of the tree level result $\omega(\lambda\omega^2 + m_0{}^2) = \omega[M_0{}^2(\omega) - 2\lambda\omega^2] = 0$ the correction term $\lambda^2\omega^2$ (when $\omega \neq 0$) is, really of the first order in λ. We ignore terms of order λ^2 and higher and remind that M_0 depends on ω which in its turn is involved in the constraint equation (2.11). To maintain consistency we replace M_0 by M in the terms that are already multiplied by λ.

From (3.5) we obtain the *mass renormalization condition*

$$M^2 - m^2 = 3\lambda\omega^2 + \frac{3\lambda}{4\pi}ln(\frac{m^2}{M^2}) - \lambda^2\omega^2\frac{\sqrt{3}}{M^2} \tag{3.6}$$

where $M(\omega) \equiv M$ and $M(\omega = 0) \equiv m$ indicate the phsyical masses in the asymmetric and symmetric phases respectively. The eq. (3.6) expresses the invariance of the bare mass and for $\omega = 0$ or $\lambda = 0$ it implies $M^2 = m^2$.

We deal next with the constraint equation (2.11). To the lowest order [16] we find for the relevant vacuum expectation values

$$\begin{aligned} 3\lambda\omega\langle\varphi(0)^2\rangle &\simeq 3\lambda\omega.iG_0(x,x) = 3\lambda\omega.D_1(M), \\ \lambda\langle\varphi(0)^3\rangle &\simeq \lambda(-i\lambda\omega).6.\int dx\langle T(\varphi(0)^3\varphi(x)^3)\rangle_c^0, \\ &= -6\lambda^2\omega D_3(M) = -6\lambda^2\omega\frac{b}{(4\pi)^2M^2}, \end{aligned} \tag{3.7}$$

where c indicates *connected* diagram and [16] D_3 is a finite integral like D_2 with three denominators and a numerical computation gives $b \simeq 7/3$. From (2.11) on making use of (3.5-7) we find that the divergent term cancels giving rise to the *renormalized constraint equation*

$$\beta(\omega) \equiv \omega\Big[M^2 - 2\lambda\omega^2 + \lambda^2\omega^2\frac{\sqrt{3}}{M^2} - \frac{6\lambda^2}{(4\pi)^2}\frac{b}{M^2}\Big] = 0. \tag{3.8}$$

We will verify below that β coincides with the total derivative with respect to ω, in the equal-time formulation, of the (finite) difference $F(\omega)$ (see Sec. 3 below) of the renormalized vacuum energy densities in the *asymmetric* ($\omega \neq 0$) and *symmetric* ($\omega = 0$) phases in the theory. The last term in β corresponds to a correction $\simeq \lambda(\lambda\omega^2)$ in this energy difference and thus may not be ignored just like in the case of the self-energy discussed above. In the equal-time case (3.8) would be required to be *added* to the theory upon physical considerations. It will ensure that the sum of the tadpole diagrams, to the approximation concerned, for the transition $\varphi \rightarrow$ *vacuum* vanishes [19]. The physical outcome would then be the same in the two forms of treating the theory here discussed. The variational methods write only the first two ($\approx$ tree level) terms in the expression for β and thus ignore the terms coming from the finite corrections. A similar remark can be made about the last term in (3.6). Both of the eqs. (3.6) and (3.8) and the difference of energy densities above are also found to be independent of the arbitrary mass scale introduced in the dimensional regularization and contain only the finite physical parameters of the theory.

Consider first the *symmetric phase* with $\omega \approx 0$, which is allowed from (3.8). From (3.6) we compute $\partial M^2/\partial\omega = 2\lambda\omega(3-\sqrt{3}\lambda/M^2)/[1+3\lambda/(4\pi M^2)-\sqrt{3}\lambda^2\omega^2/M^4]$ which is needed to find $\beta' \equiv d\beta/d\omega = d^2F/d\omega^2$, the second derivative of the above mentioned energy difference. Its sign will determine the nature of the stability of the vacuum. We find $\beta'(\omega = 0) = M^2[1 - 0.0886(\lambda/M^2)^2]$, where by the same arguments as made above in the case of β we may not ignore the λ^2 term. The β' changes the sign from a positive value for vanishingly weak couplings to a negative one when the coupling increases. In other words the system starts out in a stable symmetric phase for very small coupling but passes over into an unstable symmetric phase for values greater than $g_s \equiv \lambda_s/(2\pi m^2) \simeq 0.5346$.

Consider next the case of *the spontaneously broken symmetry phase* ($\omega \neq 0$). From (3.8) the values of ω are now given by

$$M^2 - 2\lambda\omega^2 + \frac{\sqrt{3}\lambda}{2} = 0, \qquad (\omega \neq 0), \tag{3.9}$$

where we have made use of the tree level approximation $2\lambda\omega^2 \simeq M^2$ when $\omega \neq 0$. The mass renormalization condition becomes

$$M^2 - m^2 = 3\lambda\omega^2 + \frac{3\lambda}{4\pi} ln(\frac{m^2}{M^2}) - \lambda\frac{\sqrt{3}}{2}. \tag{3.10}$$

On eliminating ω from (3.9) and (3.10) we obtain the *modified duality relation*

$$\frac{1}{2}M^2 + m^2 + \frac{3\lambda}{4\pi} ln(\frac{m^2}{M^2}) + \frac{\sqrt{3}}{4}\lambda = 0. \tag{3.11}$$

which can also be rewritten as $[\lambda\omega^2 + m^2 + (3\lambda/(4\pi))ln(m^2/M^2)] = 0$ and it shows that the real solutions exist only for $M^2 > m^2$. The finite corrections found here are again not considered in the references cited in Sec. 1, for example, they assume (or find) the tree level expression $M^2 - 2\lambda\omega^2 = 0$. In terms of the dimensionless coupling constants $g = \lambda/(2\pi m^2) \geq 0$ and $G = \lambda/(2\pi M^2) \geq 0$ we have $G < g$. The new self-duality eq. (3.11) differs from the old one [11,13] and shifts the critical coupling to a higher value. We find that: *i)* for $g < g_c = 6.1897$ (*critical coupling*) there is no real solution for G, *ii)* for a fixed $g > g_c$ we have two solutions for G one with the point lying on the upper branch ($G > 1/3$) and the other with that on the lower branch ($G < 1/3$), of the curve describing G as a function of g and which starts at the point ($g = g_c = 6.1897, G = 1/3$), *iii)* the lower branch with $G < 1/3$, approaches to a vanishing value for G as $g \to \infty$, in contrast to the upper one for which $1/3 < G < g$ and G continues to increase. From (3.10) and $\beta = \omega[M^2 - 2\lambda\omega^2 + \sqrt{3}\lambda/2]$ we determine $\beta' \approx (1 + 0.9405G)$ which is always positive and thus indicates a minimum of the difference of the vacuum energy densities for the nonzero values of ω.

The energetically favored broken symmetry phases become available only after the coupling grows to the critical coupling $g_c = 6.18969$ and beyond this the asymmetric phases would be preferred against the unstable symmetric phase in which the system finds itself when $g > g_s \simeq 0.5346$. The phase transition is thus of the *second order* confirming the conjecture of Simon-Griffiths. If we ignore the additional finite renormalization corrections we obtain complete agreement with the earlier results, e.g., the symmetric phase always remains stable but for $g > 1.4397$ the energetically favored asymmetric phases also do appear, indicating a first order transition.

Vacuum energy density:

The expression for the vacuum energy density in the equal-time formulation is given by

$$\mathcal{E}(\omega) = I_1(M_0) + \frac{1}{2}{m_0}^2\omega^2 + \frac{\lambda}{4}\omega^4 + \frac{\lambda}{4}.3.D_1(M_0)^2 + (-i6\lambda\omega)^2.\frac{1}{2!}.\frac{1}{6}.D_3(M_0). \quad (3.12)$$

Here the first term is the vacuum energy density with respect to the free propagator with mass ${M_0}^2$ and is given by [11]

$$I_1(M_0) = \frac{1}{(2\pi)^{(n-1)}} \int d^{(n-1)}k \; \frac{1}{2} \sqrt{\vec{k}^2 + {M_0}^2} = \frac{{M_0}^n}{(4\pi)^{\frac{n}{2}}} \; \frac{1}{n} \; \Gamma(1-\frac{n}{2})$$
$$\rightarrow \mu^{(n-2)} \, \frac{M_0^2}{4\pi} \, \frac{1}{2} \left[\frac{2}{(2-n)} + 1 - \gamma - ln(\frac{{M_0}^2}{4\pi\mu^2}) \right] \quad (3.13)$$

The ${D_1}^2$ term represents the two-loop correction of the order λ and so does the last one in view of the discussion above except for that it is finite and carries an opposite relative sign. We remark that the last term is non-vanishing even in the light-front computation where we find in the integrand $\theta(k)\theta(k')\theta(k'')\delta(k+k'+k'')$ multiplied by another distribution. This product, however, may not be considered vanishing. The last term of β in (3.8) corresponds to the derivative with respect to ω of the last term in (3.12).

From (3.5) we have $M_0^2 \approx M^2\Big[1 + (3\lambda/(2\pi M^2))\{A + 1/(n-2)\}\Big]$, where $A = (4\pi)\Big[(1/8\pi)ln(M^2/\bar{\mu}^2) + 3\lambda\omega^2 D_2\Big]$, $\bar{\mu}^2 = 4\pi\mu^2 exp(-\gamma)$, and $\gamma \simeq 0.5772$. We rewrite (3.12) as

$$\Big[\frac{M^n}{(4\pi)^{\frac{n}{2}}} \, \frac{1}{n}\Gamma(1-\frac{n}{2}) \; - \; \mu^{(n-2)}\frac{M^2}{4\pi}\frac{1}{(2-n)}\Big]$$

$$+\frac{3\lambda}{(4\pi)^2}\Big[\frac{M^{(n-2)}}{(4\pi)^{\frac{n}{2}-1}} \, \Gamma(1-\frac{n}{2}) \; - \mu^{(n-2)}\frac{2}{(2-n)}\Big] A$$

$$+\frac{3\lambda}{(4\pi)^2}\mu^{2(n-2)}\Big[\frac{M^{2(\frac{n}{2}-1)}}{(4\pi\mu^2)^{(\frac{n}{2}-1)}} \, \frac{1}{2}\Gamma(1-\frac{n}{2}) - \frac{1}{(2-n)}\Big]^2$$

$$+\frac{1}{2}M^2\omega^2 + \frac{3\lambda}{4\pi}\omega^2 A - \frac{3}{2}\lambda\omega^4 + \frac{\lambda}{4}\omega^4$$

$$+(-i6\lambda\omega)^2.\frac{1}{6}.D_3(M) \; + \mu^{(n-2)}\frac{3\lambda}{(4\pi)^2}\frac{1}{(2-n)^2} + \frac{m_0^2}{4\pi}\frac{1}{(2-n)}. \quad (3.14)$$

Except for the last two terms containing the poles the expression involves only the finite terms. Taking the limit $n \to 2$ we obtain the following finite expression for the difference of the vacuum energy densities in the broken and the symmetric phases

$$\begin{aligned} F(\omega) &= \mathcal{E}(\omega) - \mathcal{E}(\omega = 0) \\ &= \frac{(M^2 - m^2)}{8\pi} + \frac{1}{8\pi}(m^2 + 3\lambda\omega^2)\, ln(\frac{m^2}{M^2}) + \frac{3\lambda}{4}\left[\frac{1}{4\pi} ln(\frac{m^2}{M^2})\right]^2 \\ &\quad + \frac{1}{2}m^2\omega^2 + \frac{\lambda}{4}\omega^4 + \frac{1}{2!}.(-i6\lambda\omega)^2.\frac{1}{6}.D_3(M), \end{aligned} \tag{3.15}$$

which is also found to be independent of the arbitrary mass μ on using (3.6).

We verify that $(dF/d\omega) = \beta$ and $d^2F/d\omega^2 = \beta'$ and except for the last term in (3.15) it coincides with the result in the earlier works. From numerical computation we verify that at the minima corresponding to the nonvanishing value of ω the value of F is negative and that for a fixed g it is more negative for the point on the lower branch ($G < 1/3$) than for that on the upper branch ($G > 1/3$). To illustrate we find: for $g = 6.366$ and $G = 0.263$ we get $|\omega| = 0.736$, $F = -0.097\lambda$ while for the same g but $G = 0.431$ we find $|\omega| = 0.617$, $F = -0.082\lambda$. For $g = 11.141$ and $G = 0.129$ we get $|\omega| = 1.050$, $F = -0.174\lambda$ while for the same g but $G = 1.331$ we find $|\omega| = 0.493$, $F = -0.111\lambda$. The symbolic manipulation was found very handy in treating the coupled eqs. (3.6) and (3.8).

4- Conclusion:

The present work and the earlier one on the mechanism of spontaneous continuous symmetry breaking [7] add to the previous experience [2-6] that the *front form* dynamics is a useful complementary method and needs to be studied systematically in the context of QCD and other problems. The physical results following from one or the other form of the theory should come out to be the same though the mechanisms to arrive at them may be different. In the equal-time case we are required to introduce external considerations in order to constrain the theory (e.g., in the variational methods) while the analogous conditions in the light-front formulation seem to be already contained in it through the self-consistency equations. The phase transition of the second order in the two dimensional ϕ^4 theory follows if we include also the finite renormalization corrections and without them our results agree with those obtained in the variational methods.

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We note that from (2.10) we find $iG_0(x;0) \equiv \langle 0|T(\varphi(x,\tau)\varphi(0,0))|0\rangle = \int (dk/4\pi k)\,\theta(k)\,[\theta(\tau)\,e^{-i(kx+\epsilon_k\tau)} + \theta(-\tau)\,e^{i(kx+\epsilon_k\tau)}]$ where $2k\,\epsilon_k = M_0^2$. Using the well known integral representation of $\theta(\tau)$ and the identity $[\theta(k) + \theta(-k)] = 1$, true in the sense of distribution theory (*cf.* D. Jones, *Theory of Generalized Functions*, Cambridge University Press, 1982.), it may be rewritten as $\int\int (dk^+ dk^-/(2\pi)^2)\,[i(2k^+k^- - M_0^2 + i\epsilon)^{-1}]\,e^{-i(k^+x^- + k^-x^+)}$ where $k^\pm$ are now dummy variables taking values from $-\infty$ to ∞ and the integration over k^- is understood to be performed first like in the equal-time case where the k^0 integral is understood to be done first, (e.g., see J.M. Jauch and F. Rohrlich, *Theory of Photons and Electrons*, Addison-Wesley, Massachusetts, 1954).

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