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ON THE WAY TO A LARGE UNIFICATION THEORY

by

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ABSTRACT

An alternative model for restoring broken symmetries is proposed, leading to the prediction of a massive spin-two particle with $m = 1.33\ \mathrm{TeV}/c^2$.

Key-words: Broken symmetries; Lanczos potential; Gravitation theory.

The large class of the so-called gauge theories has given physics at least one great success: the experimental observation of the vector boson which confirmed the electromagnetic and weak forces unification as proposed by Salam[1] and Weinberg[2] some years ago.

The general idea behind these gauge theories is the restoration of a dynamics symmetry which was broken by a space-time dependent map. The usual procedure is the introduction of an extended covariant derivative in which one can absorb the non-invariant part of the Lagrangian.

Regardless of the general acceptance upon looking at the electromagnetic field $f_{\mu\nu}$ as <u>the</u> compensatory field which restores the dynamics symmetry of a scalar (or spinorial) field $\phi(x)$ under a space-time dependent phase transformation

$$\phi(x) \rightarrow \tilde{\phi}(x) = e^{i\Lambda(x)} \phi(x) \qquad , \tag{1}$$

we shall take the risk of claiming that it is possible to construct a model in which invariance under transformation (1) is obtained without making appeal to the interaction with vector fields. The Lagrangian

$$L = \partial_\mu \phi^* \partial_\nu \phi \, g^{\mu\nu} - V(\phi^* \phi) \tag{2}$$

can be written in an equivalent but rather unusual form, $L = \frac{1}{6} s^*_{\mu\nu\lambda} s^{\mu\nu\lambda} - V(\phi^*, \phi)$, provided we define the "current" $s_{\mu\nu\lambda} = \partial_\mu \phi g_{\nu\lambda} - \partial_\nu \phi g_{\mu\lambda}$, where $s_{\mu\nu\lambda} = -s_{\nu\mu\lambda}$ and $s_{\mu\nu\lambda} + s_{\nu\lambda\mu} + s_{\lambda\mu\nu} = 0$. In fact it is straightforward to show that

$$L = \frac{1}{6}\, s^*_{\mu\nu\lambda} s^{\mu\nu\lambda} - V(\phi^*\phi) \equiv \frac{1}{6}\left[\partial_\mu\phi^* g_{\nu\lambda} - \partial_\nu\phi^* g_{\mu\lambda}\right]\left[\partial^\mu\phi g^{\nu\lambda} - \partial^\nu\phi g^{\mu\lambda}\right] - V(\phi^*\phi)$$

$$= \partial_\mu\phi^*\partial_\nu\phi\, g^{\mu\nu} - V(\phi^*\phi) \qquad (3)$$

According to a formal analogy with gauge theories, we introduce the interaction of ϕ with a new tensorial field $A_{\mu\nu\lambda}$ through

$$L_{\phi int} = \frac{1}{6}\left[\partial_\mu\phi^* g_{\nu\lambda} - \partial_\nu\phi^* g_{\mu\lambda} + i g_{new}\, \phi^* A_{\mu\nu\lambda}\right]\left[\partial^\mu\phi g^{\nu\lambda} + \right.$$

$$\left. - \partial^\nu\phi g^{\mu\lambda} - i g_{new}\, \phi\, A^{\mu\nu\lambda}\right] , \qquad (4)$$

where g_{new} is a new coupling constant.

Doubtless, the third order tensor $A_{\mu\nu\lambda}$ shall have the same symmetries as $s_{\mu\nu\lambda}$. Besides, one should not be surprised that the invariance of Lagrangian (4) under map (1) requires $A_{\mu\nu\lambda}$ to transform as

$$A_{\mu\nu\lambda} \rightarrow \tilde{A}_{\mu\nu\lambda} = A_{\mu\nu\lambda} + \frac{1}{g_{new}}\left[\partial_\mu\Lambda g_{\nu\lambda} - \partial_\nu\Lambda g_{\mu\lambda}\right] . \qquad (5)$$

At this point we are faced with two possibilities. Either we look for some kind of hithertho unknown interaction — we shall leave to others this task — or else we try to identify $A_{\mu\nu\lambda}$ with some field already discussed in the literature. Surprisingly enough, we have found a realization of this second possibility in a 1962 article by Lanczos, which does not seem to have deserved the attention of physicists during a long time, in spite of its potentiality[3,4,5].

Indeed it would have been beyond one's forces to resist the temptation of identifying the tensor $A_{\mu\nu\lambda}$ above introduced with the Lanczo's potential. This potential has as its major role providing us the means of writing the Weyl conformal tensor $C_{\alpha\beta\mu\nu}$ as first order derivatives, that is

$$
\begin{aligned}
- C_{\alpha\beta\mu\nu} = A_{\alpha\beta\mu;\nu} - A_{\alpha\beta\nu;\mu} + A_{\mu\nu\alpha;\beta} + \\
- A_{\mu\nu\beta;\alpha} + \frac{1}{2} (A_{\nu\alpha} + A_{\alpha\nu}) g_{\mu\beta} + \\
+ \frac{1}{2} (A_{\mu\beta} + A_{\beta\mu}) g_{\alpha\nu} - \frac{1}{2} (A_{\alpha\mu} + A_{\mu\alpha}) g_{\beta\nu} + \\
- \frac{1}{2} (A_{\beta\nu} + A_{\nu\beta}) g_{\alpha\mu} + \frac{2}{3} A^{\sigma\lambda}{}_{\sigma;\lambda} (g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu})
\end{aligned}
\tag{6}
$$

in which $A_{\alpha\mu} \equiv A_{\alpha}{}^{\sigma}{}_{\mu;\sigma} - A_{\alpha}{}^{\sigma}{}_{\sigma;\mu}$ and ; means covariant derivative. Lanczos' tensor $A_{\alpha\beta\mu}$ has twenty independent components, for it obeys relations $A_{\alpha\beta\mu} = - A_{\beta\alpha\mu}$ and $A_{\alpha\beta\mu} + A_{\beta\mu\alpha} + A_{\mu\alpha\beta} = 0$. Therefore, there is not a unique relation between $C_{\alpha\beta\mu\nu}$ and $A_{\alpha\beta\mu}$, because equation (6) is invariant under the gauge $A_{\mu\nu\lambda} \to A_{\mu\nu\lambda} + W_{\mu} g_{\nu\lambda} - W_{\nu} g_{\mu\lambda}$ for an arbitrary vector W_{μ}, of which relation (5) is a special case.

From dimensional analysis we can not avoid to perceive the first advantage of our model. As $[\dim A^{\alpha\beta\mu}] = (\text{length})^{-1}$, the coupling constant g_{new} can be eradicated. One could argue that g_{new} being dimensionless does not helplessly implies that $g_{new} = 1$. We would certainly not disagree, but in this paper only this possibility will be considered.

There can be no doubt that the introduction of the coupling of the scalar field with $A_{\mu\nu\lambda}$ restores the symmetry broken by transformation (1), provided $A_{\mu\nu\lambda}$ transforms according to (5). This means that the interaction with vector fields is nothing but one of the possible ways of getting over the symmetry breaking of Lagrangian (2). Neverthless, we shall not regret taking it into account. As a matter of fact, the introduction of terms that couple the scalar field to the electro-weak vector fields W^μ will reveal a far more interesting result.

If the dynamics is described by the Lagrangian

$$L_N = \frac{1}{6}\,[(\partial_\mu\phi^* + ie\phi^* W_\mu)g_{\nu\lambda} - (\partial_\nu\phi^* + ie\phi^* W_\nu)\,g_{\mu\lambda} + $$

$$+ i\phi^* A_{\mu\nu\lambda}][(\partial^\mu\phi - ie\phi W^\mu)g^{\nu\lambda} - (\partial^\nu\phi - ie\phi W^\nu)\,g^{\mu\lambda} + $$

$$- i\phi A^{\mu\nu\lambda}] + \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}\,f^{\mu\nu}f_{\mu\nu} + $$

$$+ \frac{1}{8}\,C^{\alpha\beta\mu\nu}\,C_{\alpha\beta\mu\nu} + L_{rest} \quad , \qquad (7)$$

the Higgs mechanism which yields a non-zero value for the mass m_W of the Salam-Weinberg vector boson will also be responsible for the appearance of the mass m_{new} of the tensorial field $A_{\mu\nu\lambda}$. The identification of W^μ with the vector boson in Salam-Weinberg unified model allows us to predict that $m_{new} = 1.33\ \text{TeV}/c^2$, since $m_W = e\sigma$ and $m_{new} = \sqrt{2}\,\sigma$, where σ is the non-zero vacuum expectation value of our scalar field ϕ.

The complete structure SU(2) ⊗ U(1) of Salam-Weinberg model is obviously not made explicit in (7). We have chosen so for the sake of simplicity since the only terms necessary to reach our purpose are those written out in Lagrangian (7).

At this point, an extremely important remarks has to be made. The potential $A_{\alpha\beta\mu}$ describes a kind of short-range gravity and in order to have a complete theory we should have a doublet of Lanczos' potentials where one of them is a massles spin-two field which generates the usual long range gravity.

The free $C^{\alpha\beta\mu\nu}$ fields term in Lagrangian (7) originates Einstein's equations in Jordan's formulation[3]:

$$C^{\alpha\beta\mu\nu}{}_{;\nu} = J^{\alpha\beta\mu} \tag{8}$$

where $J_{\alpha\beta\mu}$ is a current to be constructed with matter terms.

Contrary to a largely spread belief[6], general covariance is not destroyed if a massive term is added, either through spontaneous symmetry breaking or any other way else, as is transparent from the equation below.

$$C^{\alpha\beta\mu\nu}{}_{;\nu} + m^2_{new} A^{\alpha\beta\mu} = J^{\alpha\beta\mu} \quad . \tag{9}$$

For weak gravitational fields, that is, $g_{\mu\nu} \approx \eta_{\mu\nu} + \varepsilon\, \psi_{\mu\nu}$ with $\varepsilon^2 << \varepsilon$, Lanczos potential can be written as

$$A_{\alpha\beta\mu} = \frac{1}{4}\left[\psi_{\alpha\mu,\beta} - \psi_{\mu\beta,\alpha} + \frac{1}{6}\psi_{,\alpha}\eta_{\mu\beta} - \frac{1}{6}\psi_{,\beta}\eta_{\mu\alpha}\right]$$

in which $\psi = \psi_{\mu\nu}\eta^{\mu\nu}$, and equation (9) has the property of reducing to the good limit of massive Fierz-Pauli spin-two theory[3].

Nothing else is left but to conclude the existence of a new massive spin-two particle with $m_{new} = 1.33$ TeV/c^2. Since this energy is within the reach of modern accelerators it would be desirable to experimentaly probe our model.

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