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PSEUDOSCALAR MESONS AND SCALAR DIQUARKS DECAY CONSTANTS

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Summary

The decay constants of pseudoscalar mesons and scalar diquarks are calculated in an approximated way within a quark model developped previously.

Key-words: Diquark; Decay constant; Pseudoscalar; Λ_c^+ charmed lambda.

1. Introduction

The quark-diquark picture of baryons has been recently shown to be a viable and appropriate description of light and heavy hadron spectroscopy within a relativistic scheme([1]). This picture is being presently extended to take into account also the decay widths([2]). Among other things, this extension could be important to shed some light on the possible existence of two charmed baryons Λ_c^+ which are predicted to exist within the quark-diquark picture([3]) (some estimates of the Λ_c^+ decay exist([4]) based on a non relativistic approximation of the wave function of three quarks at the origin). To this aim, the preliminary task is to provide an estimate of the diquarks decay constants.

In this paper, we begin by analyzing the decay constants of pseudoscalar mesons and scalar diquarks using a non-relativistic limit of the equations developped in ref. 1. To avoid difficulties and ambiguities, we take *ratios* of decay constants so as to refer all of them to a basic one (that of the pion). The formalism of ref. 1. does, in principle, allow one to evaluate also the decay constants of vector mesons and diquarks as well as of baryons. This will be done in subsequent work.

2. The decay constants and the wave function at the origin

The fundamental ingredient to evaluate the decay constants in the nonrelativistic limit is the celebrated formula of Van

Royen and Weisskopff which is written, in the pseudoscalar case(5)

(2.1) $$f_p = 2 \frac{G_A'}{G_V'} \frac{|\psi_p(0)|}{\sqrt{M_p}}$$

where G_V' and G_A' are the renormalized interaction constants of quarks for vector and axial couplings; $\psi_p(0)$ is the wave function at the origin and M_p is the (pseudoscalar) meson mass.

Recently(1,6), a relativistic, spin dependent equation has been proposed for dealing with the spectroscopy of quarks of current (or bare) mass m_1, m_2

(2.2) $$[-\nabla^2 + 2\mu V_s - (E_R - V)^2 + (m_R + S)^2]\,\psi = 0$$

where, as discussed in ref. 1.,

$$E_R = \frac{W^2 - (m_1^2 + m_2^2)}{2W} .$$

and

$$m_R = \frac{m_1 m_2}{W}$$

are the relativistic generalizations of the effective energy and reduced mass(7), respectively (W being the total energy in the c.m.). We shall see later on the importance of using m_R in our calculation. V(r) is the (Coulomb-like) part of the potential which simulates one-gluon exchange and transforms like the time component of a vector

$$(2.3) \qquad V(r) \simeq -\left(\frac{6\pi}{27}\,\frac{1-\lambda r}{r\ell n\lambda r} + V_o\right) \vec{F}_1 \cdot \vec{F}_2$$

(λ being related to the QCD scale parameter Λ by $\lambda = \Lambda\,\exp(\gamma)$ where γ is the Euler Mascheroni constant). $S(r)$ is the confining part of the potential which transforms like a scalar,

$$(2.4) \qquad S(r) = -\left[\frac{6\pi}{27}\lambda\frac{(\lambda r-1)}{\ell n\lambda r}\right] \vec{F}_1 \cdot \vec{F}_2$$

In (2.3,4), $\vec{F}_1$ and $\vec{F}_2$ are the color SU(3) F spins of the two interacting quarks. For a quark-antiquark system in a singlet $\vec{F}_1 \cdot \vec{F}_2 = -4/3$ whereas $\vec{F}_1 \cdot \vec{F}_2 = -2/3$ for two quarks to form a diquark in a $\bar{3}$ representation.

In (2.2) μ is defined as

$$(2.5) \qquad \mu = \frac{(m_1+S(r)/2)\,(m_2+S(r)/2)}{m_1+m_2+S(r)}$$

and $\langle m_i+S(r)/2\rangle$ are the effective (constituent) quark masses(1). Finally,

$$(2.6) \qquad V_S = -\frac{g_1 g_2}{2\mu}\,\nabla^2 \ell n\left(1-\frac{V(r)}{3(m_1+m_2+S(r))}\right) \vec{S}_1 \cdot \vec{S}_2$$

is the spin-dependent potential where $\vec{S}_1$, $\vec{S}_2$ are the spins of the two particles and g_1 and g_2 are the g factors which arise for the color magnetic moments of the two particles.

In the above mentioned formalism, the evaluation of $\psi(0)$ is technically complicated by the fact that eq. (2.2) has a singularity at the origin which is only logarithmically less singu lar than $1/r^2$. If, in fact, the bound state Coulomb problem is solved with a relativistic wave equation, the wave function is singular at the origin(8). In the case of (2.2-6), asymptotic freedom reduces the degree of singularity at r=0 but only by means of logarithms. This implies that although $\psi(0)$ is finite in our case extreme care must be paid on how to proceed in its evaluation(2). An alternative way to overcome the above difficulty is the one that we propose to follow in this paper. Our recipe is: i) to take the non-relativistic limit of (2.2) and, ii) to evaluate only ratios of wave functions at the origin. The determination of all decay constants will therefore be made in terms of a basic one (and only one) inserted from the outside and to which all the others will be referred. This procedure, while greatly reducing the above mentioned technical difficulties should also bypass the ambiguities connected with the evaluation of the wave function at the origin.

Going now back to equation (2.2), we take its nonrelativistic limit $\vec{p}^2 = E_R^2 - m_R^2 \simeq 0$ (implying $E_R \simeq m_R$) in the spinless case ($V_S=0$). Neglecting V^2 and S^2 we get

$$[-\nabla^2 + 2m_R U(r)]\,\psi = 0 \tag{2.7}$$

where

$$U(r) = V(r) + S(r) = -\vec{F}_1 \cdot \vec{F}_2 \left(\frac{6\pi}{27} \frac{(1-\lambda r)^2}{r \ell n \lambda r} + V_0 \right) \tag{2.8}$$

Around $r \simeq 0$ the dominant contribution to $U(r)$ is of the

Coulomb form 1/r which leads to

$$(2.9) \qquad |\psi(0)| \simeq K \sqrt{m_R C}$$

where K is an undetermined constant and

$$(2.10) \qquad C = -\vec{F}_1 \cdot \vec{F}_2 \, \frac{12\pi}{27}$$

(so that $C_{pseudoscalar} = \frac{16\pi}{27}$ and $C_{diquark} = \frac{8\pi}{27}$). We checked numerically that in the region where $\frac{1}{r \ell n \lambda r}$ differs significantly from 1/r the wave functions is already significantly smaller than at $r = 0$.

We should also mention that to obtain the value of K we should have to solve the equation (2.7) up to $r \to \infty$, thus in the region where our approximation fails badly.

3. The ratios of decay constants (pseudoscalar mesons)

In eq. (2.9) we have obtained the wave function at the origin as the result of some approximations and few comments are in order. First of all, we notice that the reduced mass m_R which appears in (2.9) was defined previously in terms of the bare (current) quark masses m_i and of the invariant mass of the system W which was evaluated in ref. 1. for the various configurations. The result (2.9) has been obtained in the non--relativistic approximation and doubts may be raised on the validity of such an approximation in the light meson sector.

If, however, instead of f_p (eq.(2.1)) we demand only *ratios* of decay constants such as

(3.1) $$\frac{f_p}{f_{p'}} = \frac{|\psi_p(0)|}{|\psi_{p'}(0)|}\frac{\sqrt{M_{p'}}}{\sqrt{M_p}} \simeq \frac{\sqrt{m_R^p}}{\sqrt{m_R^{p'}}}\frac{\sqrt{M_{p'}}}{\sqrt{M_p}}$$

the approximations made may become much more acceptable and these ratios are expressed entirely in terms of current quark masses and of their bound states. Therefore if the meson P(P') is made of a quark-antiquark pair i, $\bar{j}$ (i', $\bar{j}$'), using the definition of m_R in (3.1) we get

(3.2) $$\frac{f_p}{f_{p'}} = \frac{M_{p'}}{Mp}\sqrt{\frac{m_i m_j}{m_{i'} m_{j'}}}$$

Notice that $f_p/f_{p'}$ is expressed intirely in terms of ratios of current quark masses and of their bound states which are less model dependent than the masses themselves([10]). Equation (3.2) is our main result which we now apply to evaluate all possible ratios of decay constants for all pseudoscalars ($\pi^{\pm}$, $K^{\pm,0}$, $D_c^{\pm,0}$, $F^{\pm}$). A few examples follow

(3.3) $$\begin{cases} \dfrac{f_{k^\pm}}{f_{\pi^\pm}} = \dfrac{M_{\pi^\pm}}{M_{k^\pm}}\sqrt{\dfrac{m_s}{m_d}} \; ; \; \dfrac{f_{k^0}}{f_{\pi^\pm}} = \dfrac{M_{\pi^\pm}}{M_{k^0}}\sqrt{\dfrac{m_s}{m_u}} \\ \dfrac{f_{D_c^\pm}}{f_{\pi^\pm}} = \dfrac{M_{\pi^\pm}}{M_{D_c^\pm}}\sqrt{\dfrac{m_c}{m_u}} \; ; \; \dfrac{f_{D_c^0}}{f_{\pi^\pm}} = \dfrac{M_{\pi^\pm}}{M_{D_c^0}}\sqrt{\dfrac{m_c}{m_d}} \; ; \; \dfrac{f_{F^\pm}}{f_{\pi^\pm}} = \dfrac{M_{\pi^\pm}}{M_{F^\pm}}\sqrt{\dfrac{m_s m_c}{m_u m_d}} \end{cases}$$

but other ratios could be considered such as

(3.4) $$\frac{f_{k^\pm}}{f_{k^0}} = \frac{M_{k^0}}{M_{k^\pm}}\sqrt{\frac{m_u}{m_d}} \quad ; \quad \frac{f_{D_c^\pm}}{f_{k^0}} = \frac{M_{k^0}}{M_{D_c^\pm}}\sqrt{\frac{m_c}{m_s}} \quad ; \ldots$$

4. Numerical results (case of pseudoscalar mesons)

In the above formulae (3.3,4) we take the physical values(9) of the pseudoscalar meson masses whereas for the quark masses we take(10)

(4.1) $$\begin{cases} m_u = 5.1 \pm 1.5 \text{ Mev} & m_s = 175 \pm 55 \text{ Mev} \\ m_d = 8.9 \pm 2.6 \text{ Mev} & m_c = 1.27 \pm 0.05 \text{ Gev} \end{cases}$$

This leads to the following ratios

(4.2) $$\begin{cases} \frac{f_{k^\pm}}{f_{\pi^\pm}} = 1.25 \; ; \; \frac{f_{k^0}}{f_{\pi^\pm}} = 1.64 \; ; \; \frac{f_{D_c^0}}{f_{\pi^\pm}} = 0.89 \; ; \; \frac{f_{D_c^\pm}}{f_{\pi^\pm}} = 1.18 \\ \frac{f_{F^\pm}}{f_{\pi^\pm}} = 4.83 \; ; \; \frac{f_{k^\pm}}{f_{k^0}} = 0.76 \; ; \; \frac{f_{D_c^\pm}}{f_{k^0}} = 0.72 \; ; \; \frac{f_{D_c^0}}{f_{k^0}} = 0.54 \\ \frac{f_{D_c^\pm}}{f_{k^\pm}} = 0.94 \; ; \; \frac{f_{D_c^0}}{f_{k^\pm}} = 0.71 \; ; \; \frac{f_{F^\pm}}{f_{k^\pm}} = 3.85 \; ; \; \frac{f_{D_c^\pm}}{f_{D_c^0}} = 1.32 \ldots \end{cases}$$

If we now inject, as a further information the well determined experimental value(11)

(4.3) $$f_{\pi^\pm}^{exp} \simeq 131.75 \pm 0.13 \text{ Mev}$$

we get the absolute predictions

$$(4.4)\qquad \begin{cases} f_{k^\pm} = 164.69 \pm 0.16 \ \text{Mev} \\ f_{k^0} = 216.07 \pm 0.21 \ \text{"} \\ f_{D_c^\pm} = 155.47 \pm 0.15 \ \text{"} \\ f_{D_c^0} = 117.26 \pm 0.12 \ \text{"} \\ f_{F^\pm} = 636.35 \pm 0.63 \ \text{"} \end{cases}$$

Using the same experimental value (4.3) for $f_{\pi^\pm}$ we can now determine the constant k in eq. (2.9). With some trivial algebra and taking into account the color factor ($\sqrt{3}$) missing in (2.1) we have

$$(4.5)\qquad k = \frac{3}{8\sqrt{\pi}}\, f^{exp}_{\pi^\pm}\, \frac{M_{\pi^\pm}}{\sqrt{m_u m_d}}$$

which gives

$$(4.6)\qquad k \simeq 0.58 \ \text{Gev}$$

5. Diquark decay constants

The above results enable us now to go back to the diquark model([1]). Indeed we can extend our predictions to the diquark decay constants. To this aim we apply again eq. (2.9) where, as already mentioned, $C_{diquark}/C_{pseudoscalar} = 1/2$. Furthermore, in the case of diquarks, we shall take the diquark masses as calculated in ref. 1. We also denote by $M_{M(i,\bar{j})}$ and $M_{D(i,j)}$ the masses of pseudoscalar mesons and diquarks with the same quark content. We then have

$$(5.1) \qquad f_{D(i,j)} = \frac{1}{\sqrt{2}} f_{M(i,\bar{j})} \frac{M_{M(i,\bar{j})}}{M_{D(i,j)}}$$

For example, using[1] $M_{D(cu)} \simeq m_{D(cd)} \simeq 1895$ Mev, $M_{D(ud)} \simeq 531.$Mev, $M_{D(us)} \simeq M_{D(ds)} \simeq 807.$Mev, and using our previous results, we get

$$f_{D(cd)} \simeq 108.04 \pm 0.11 \quad \text{Mev}$$

$$f_{D(cu)} \simeq 82.08 \pm 0.08 \quad "$$

$$f_{D(ud)} \simeq 24.49 \pm 0.02 \quad "$$

$$f_{D(us)} \simeq 71.45 \pm 0.07 \quad "$$

$$f_{D(ds)} \simeq 94.38 \pm 0.09 \quad "$$

for which, however, it was assumed $m_u = m_d$, which should not be very serious shortcoming.

6. Concluding remarks

In this paper we have presented an approach for evaluating hadronic decay constants within the general relativistic scheme of ref. 1. This has been done using a non-relativistic limit and referring all calculations to the fundamental input constant f_π circumventing difficulties and ambiguities present in the general case, in the hope that the general results be maintained in a future relativistic calculation. Very important was, in the present case, the use of the reduced relativistic mass instead of its nonrelativistic limit $\mu = m_1 m_2/(m_1+m_2)$.

Other calculations[12] within potential models have already been proposed, without, however, being careful to take all the precautions we have discussed here. Besides we have only one undetermined constant. Thus the value $f_{k^\pm} = 1.25\, f_{\pi^\pm}$ (see 4.2) predicted in our scheme is in complete agreement with experimental results.

The inputs used have been, the pion decay constant f_π, the current quark masses, the experimental pseudoscalar masses and the calculated diquark masses[1].

Notice that different values were obtained for the decay constants of neutral and charged mesons in agreement with expectation in current algebra calculations[13].

It is also quite interesting that our results (3.3) are compatible, and in fact, in agreement, with results obtained within QCD sum rules calculations[14]. In particular, the first of our equations (3.3) coincides with eq. (6.25) of ref. 14.

$$\frac{m_s}{m_d} = \frac{M_k^2}{M_\pi^2}\,\frac{f_k^2}{f_\pi^2}$$

(see also eq. (6.33) of ref. 14. as compared with the third of eq. (3.3) above).

In conclusion, we have developped a scheme of approximations within the approach of ref. 1. to derive decay constants. This has been applied so far to pseudoscalar mesons and scalar diquarks. We plan to extend the calculation to take into account also vector mesons and baryons.

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