THE ROLE OF UNITARITY IN HARD DIFFRACTION [1]

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Abstract: The role of s-channel unitarity in DIS hard diffraction is discussed with special emphasise on the following issues: 1) Froissart-Martin and s-channel unitarity bounds and their implied bound for $xG(x, Q^2)$. 2) The Q^2 dependence of the hard Pomeron intercept $\alpha_P(0)$. 3) The t-dependence of diffractive electroproduction of vector mesons: shrinkage and diffraction dips. 4) The determination of $\Psi^V(0, z)$, the vector meson wave function at the origin, from vector meson electroproduction.

This talk is based on research done with E.M. Levin and E. Gotsman [1] [2].

Total hadronic cross sections cannot exceed the Froissart-Martin bound $\sigma_{tot} \leq \frac{\pi}{\mu^2} ln^2 \frac{s}{s_0}$ [3], where μ is the pion mass which is the lightest particle which can be exchanged in the crossed channel. The bound is a consequence of s-channel unitarity combined with analyticity and crossing symmetry. Even though there is some ambiguity concerning the choice of μ, it is generally agreed that this bound is still considerably higher than presently available hadronic total cross section data. Over the last few years it became evident [4] that s-channel unitarity, on its own, provides a very effective, even though less general, bound which is relevant to present day hadronic cross sections. This is analyzed best in b-space, where s-channel unitarity translates into a black disc limit, $|a(s, b)| \leq 1$. $a(s, b)$ is the b-space scattering amplitude and b is the impact parameter. As shown [4], if we take the Donnachie-Landshoff parameterization [5] as a good reproduction of the $\bar{p}p$ or pp total cross section data, the black disc limit will be attained, for small b, just above the Tevatron range. As long as this is a small b effect, its implications for σ_{tot} are rather small. The CDF measurement [6] of $a(\sqrt{s} = 1800, b = 0) = 0.984 \pm 0.016$ verifies our theoretical expectations. The search for unitarity effects is, thus, confined practically to small b phenomena. Such is the process of diffractive scattering which has a much smaller unitarity saturation scale than elastic scattering. This is manifested by screening corrections (SC) setting on at relatively low energies. This theoretical observation [7] translates into different

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energy dependences of σ_{el} and σ_{diff}, which are observed experimentally [8]: whereas $\frac{\sigma_{el}}{\sigma_{tot}}$ grows over the ISR-Tevatron energy range, the ratio $\frac{\sigma_{diff}}{\sigma_{tot}}$ is decreasing significantly over the same energies. A well known observation [9], originating from the same mechanism, is the diffractive $\frac{d\sigma}{dt}$ dip at $|t| \approx 1.4\,GeV^2$ in pp and $\bar{p}p$ scattering.

These considerations do not apply in a trivial manner to photoproduction and deep inelastic scattering (DIS), where the E.M. photon coupling, the absence of an elastic channel and the photon mass (as an additional kinematic variable) complicate our analysis. A significant simplification of the above has been suggested by Gribov [10]. Gribov's main observation was that at high energies (small $x = \frac{Q^2}{s}$), the photon fluctuates into an hadronic system ($\bar{q}q$ to the lowest order) with a coherence length ($l_c = \frac{1}{mx}$, m being the target mass) which is much bigger than the target radius. This enables us to describe DIS as a two step process: i) The photon transforms into a $\bar{q}q$ system before the interaction with the target. ii) The hadronic system, just produced, interacts with the target. This simple picture enables us to adopt ideas and procedures, taken from hadron physics, and use them in the analysis of photoproduction and DIS. Accordingly, we can describe, to the lowest order, the hard Pomeron by the well defined, and relatively easy to calculate, pQCD dipole two gluon exchange model [11].

Gribov has made, also, two technical assumptions: i) A dispersion relation, without subtractions, can be written for the produced hadronic mass M^2 at the photon vertex. This connects M^2 to the mass spectra produced in e^+e^- annihilation. ii) The hadronic interaction is a black disc interaction. Hence the incoming and outgoing $q\bar{q}$ masses are equal. With these assumptions we obtain:

$$\sigma(\gamma^* N) = \frac{\alpha_{em}}{3\pi} \int \frac{R(M^2)M^2 dM^2}{(Q^2+M^2)^2}\sigma_{M^2N}(s), \tag{1}$$

where

$$R(M^2) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}. \tag{2}$$

Note that the cross section of Eq.(1) has two contributions. The first comes from σ_{M^2N} which is a pure hadronic cross section. The second comes from the M^2 integration which diverges logarithmically with s, leading to a bound [12]

$$\sigma(\gamma^* N) \leq \frac{\alpha_{em}}{3\pi} R_\infty ln^2 \frac{s}{s_0} ln \frac{1}{x}, \tag{3}$$

where R_∞ denotes the high energy limit of $R(M^2)$.

The result just quoted is somewhat disturbing being less stringent than the Froissart-Martin hadronic bound. We note that the extra logarithm in Eq.(3) is a direct consequence of the M^2 integration. Moreover, an arbitrary $M^2 \leq 0.05s$ cut, which is commonly used, does not eliminate this problem. Trying to improve on this result, we have recently repeated this calculation [2], giving up the black disc assumption for large M^2. Our physics reason for doing so is that the quark-antiquark pair with a large mass has a very small transverse size, $r_\perp \propto \frac{1}{M}$, and, hence, the black disc approximation is not appropriate. Moreover, the interaction at high M^2 takes place at short distances. This enables us to replace the arbitrary M^2 cutoff with a pQCD calculation which provides a natural cutoff at very high M^2. In our calculations we adopted a very simple approach where we assume an M_0^2 cutoff. We assume that the production of $M^2 < M_0^2$ is soft, whereas the production of $M^2 \geq M_0^2$ is hard. Accordingly,

$$\sigma(\gamma^* N) = \sigma^{soft} + \sigma_{hard}, \tag{4}$$

where σ^{soft} is calculated from the e^+e^- annihilation data assuming Gribov's black disc assumption. σ^{hard} is calculated from pQCD in the leading $lnQ^2 ln\frac{1}{x}$ approximation. M_0^2 is a free parameter.

Here, I wish to report just on the general result we have obtained, where Eq.(3) is replaced by

$$\sigma(\gamma^* N) \leq C ln^2 \frac{s}{s_0} ln \frac{Q^2 + M^2(x)}{Q^2 + M_0^2}, \tag{5}$$

where C contains all the constants we have specified thus far and $M^2(x)$ is the solution of

$$\frac{4\pi\alpha_s}{3R_N^2 M^2(x)} x G^{DGLAP}(x, M^2(x)) = 1. \tag{6}$$

The details of our calculation depend, thus, on the gluon density input and R_N^2, the gluon correlation radius which has been estimated [13] from the HERA diffraction dissociation data. Our calculations show a severe hard pQCD background at $Q^2 = 0$, as well as an important soft component persisting up to relatively high $Q^2 = 50\,GeV^2$. The high energy limit of Eq.(5) gives us a better bound

$$\sigma(\gamma^* N) \leq ln^2 \frac{s}{s_0} ln^{\frac{1}{2}}(\frac{1}{x}). \tag{7}$$

This is a preliminary result which we hope to improve. An obvious deficiency of our calculation is that diffraction dissociation at the nucleon vertex is not included. Since this is a strong interaction vertex , we have tacitly assumed that it is bound by the Froissart-Martin bound [3] as well.

In general, a bound on $\sigma(\gamma^* N)$ implies a bound on $xG(x, Q^2)$. Recent model dependent calculations [14] [15] have suggested a more stringent, Froissart-Martin like, bound where $xG(x, Q^2)$ behaves as $ln(\frac{1}{x})$. The question if one can obtain such a bound based on general grounds is still opened. Clearly, some relevant experimental input, such as Ref. [16], is of much importance.

In the continuation I shall concentrate on hard diffractive leptoproduction of vector mesons (DLVM). DLVM is a hard, short distance, DIS process where Q^2 is the measure of the hard scale. In real photoproduction of J/Ψ, $4m_\Psi^2$ replaces Q^2. As such, we can calculate these cross sections in pQCD [17] [18] and check for signatures which are typical phenomena associated with SC and are significantly different [1] from the non screened pQCD prediction [17]. To this end we wish to study the forward DLVM differential cross section which is commonly presented as

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{t=0} e^{-B|t|}, \tag{8}$$

where B is the $t=0$ differential slope. Expressing the hard Pomeron by the Regge notation, we have

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t = 1 + \epsilon + \alpha'_P t. \tag{9}$$

The leading energy dependence of the forward cross section is determined by ϵ, whereas the slope $B = B_0 + 2\alpha'_P \, ln(\frac{s}{s_0})$ depends on α'_P, where $\alpha'_P > 0$ implies a shrinkage of the forward differential cross section. In our context, we are interested in the Q^2 dependence of the above two parameters in DLVM. We note that the DGLAP equations, which are adequate in the HERA kinematic domain, produce a hard Pomeron with $\alpha'_P = 0$ and ϵ which is growing fast with Q^2 attaining a high, BFKL like, $\epsilon \approx 0.5$ at $Q^2 = 50\,GeV^2$.

A general approach to calculate DLVM in DIS, with SC, has been developed in our earlier paper [18], following Refs. [11] [19]. The general expression for the screened DLVM amplitude is

$$A(Q,x;b) = C \int \frac{d^2 r_\perp}{\pi} \int_0^1 dz\, \Psi^{\gamma^*}(Q, r_\perp, z)\, 2\left(1 - e^{-\kappa(r_\perp, x; b)}\right) [\Psi^V(r_\perp = 0, x)]^*, \tag{10}$$

where Ψ^{γ^*},Ψ^V denote the wave functions of the virtual photon and vector meson respectively. $r_\perp$ denotes the transverse distance between the quark and the antiquark. z is the fraction of the photon energy carried by the quark or antiquark. W is the energy and b the impact parameter of the reaction. $x = \frac{Q^2 + m_V^2}{W^2}$, where m_V is the vector meson mass. C was calculated in Ref. [18].

In a one radius model for the target structure we have [18]

$$\kappa(r_\perp, x; b) = \frac{2\pi\alpha_s r_\perp^2}{3}\, \Gamma(b)\, xG(x, \frac{4}{r_\perp^2}) \tag{11}$$

which is a measure of the degree of SC applied in our calculation. The relation between the profile $\Gamma(b)$ and the two gluon form factor is

$$\Gamma(b) = \frac{1}{\pi} \int e^{-i(b\cdot q)}\, F(t)\, d^2 q \tag{12}$$

with $t = -q^2$. To simplify the numerics we use a Gaussian approximation

$$\Gamma(b) = \frac{1}{R^2}\, e^{-\frac{b^2}{R^2}}. \tag{13}$$

Following Ref. [18], we obtain after the $r_\perp$ integration

$$A(Q,x;b) = C \int_0^1 dz\, 2\,[1 - Y(\kappa(x,b))]\; \Psi^V(0,z), \tag{14}$$

where

$$Y(\kappa) = \frac{1}{\kappa}\, e^{\frac{1}{\kappa}}\, E_1(\frac{1}{\kappa}), \tag{15}$$

$$\kappa(x,b) = \frac{2\pi\alpha_s}{3a^2R^2}\, e^{-\frac{b^2}{R^2}}\, xG(x, a^2). \tag{16}$$

$a^2 = Q^2 z(1-z)+m_Q^2$, m_Q being the quark mass. $A(Q,x;b)$ provides us with the full information on the differential forward cross section where we have

$$B = \frac{1}{2}\frac{\int b^2\, A(Q,x;b)\, d^2b}{\int\, A(Q,x;b)\, d^2b}, \tag{17}$$

$$\frac{\frac{d\sigma}{dt}}{(\frac{d\sigma}{dt})_{t=0}} = \left| \frac{\int J_0(b\sqrt{|t|}\, A(Q,x;b)\, d^2b}{\int A(Q,x;b)\, d^2b} \right|^2. \tag{18}$$

Recent HERA data [20] show a remarkable difference between the elastic and inelastic slopes of J/Ψ in both photo and lepto production. $B_{el}^{\Psi} = 4\, GeV^2$ whereas $B_{in}^{\Psi} = 1.7\, GeV^2$, even though the integrated cross sections are approximately equal. This observation may be explained by a two radii description of the proton. We have extended our model to accommodate such a description and have obtained reasonable results which indicate again some general features which are particular to models with screening corrections. After a relatively simple algebra we end with the following [1]

$$A_{el}(Q,x;b) = c\int_0^1 dz\,(1 - Y(\kappa_1) - \frac{\kappa_2^2}{\kappa_1(\kappa_1 - \kappa_2)}[\frac{1}{\kappa_1}(1 - Y(\kappa_1)) - Y(\kappa_1)] \tag{19}$$

$$- \frac{\kappa_2^2}{(\kappa_1 - \kappa_2)^2}[Y(\kappa_2) - Y(\kappa_1)])\frac{\Psi^V(0,z)}{a^2}$$

and

$$A_{in}(Q,x;b) = C\int_0^1 dz\, \frac{\kappa_2}{\kappa_1 - \kappa_2}[Y(\kappa_1) - Y(\kappa_2)]\frac{\Psi^V(0,z)}{a^2}. \tag{20}$$

$$\kappa_1(x,b) = \frac{2\pi\alpha_s}{3a^2R_1^2}\, e^{-\frac{b^2}{R_1^2}}\, xG(x,a^2), \tag{21}$$

$$\kappa_2(x,b) = \frac{2\pi\alpha_s}{3a^2R_2^2}\,\frac{R_1}{R_2}\, e^{-\frac{b^2}{R_2^2}}\, xG(x,a^2). \tag{22}$$

Our results, compared with the available relevant data [20] [21] [22],are shown in Figs.1-4. The numerics is determined by our choice for κ_i, which depend on the gluons profile function $\Gamma(b)$ and density $xG(x,Q^2)$ [2], and on R_i^2. As can be seen the J/Ψ data is well reproduced with $R_1^2 = 6$ and $R_2^2 = 2\, GeV^{-2}$ which comply with the experimental values of $B_{el}^{\Psi} = 4.0 \pm 0.3$ and $B_{in}^{\Psi} = 1.6 \pm 0.3\, GeV^{-2}$. Since pQCD is well suited for the calculation of J/Ψ photo and electro production, we consider these parameters to be rather reliable. The situation is more complicated for ρ DIS electroproduction, where we have a non neglgible soft background. Indeed, we fit the higher values of B_{el}^{ρ} and B_{in}^{ρ} with $R_1^2 = 10$ and $R_2^2 = 3\, GeV^{-2}$. We note that at higher Q^2 values the decreasing ρ slopes approach the J/Ψ high Q^2 slope values, indicating

[2]Note that our SC include only the screening due to the propagation of the $\bar{q}q$ through the target. The SC in the gluon sector are effectively included in the GRV parametrization [23] that we use for $xG(x,Q^2)$.

a diminishing role for the soft contribution.

Regardless of the fine details of the model that we chose to describe hard diffraction, I wish to comment on a few signatures that are particular to the fact that we have included SC in our DGLAP pQCD diffractive calculation. An experimental validation of these signatures will be, thus, very supportive to the picture I am trying to promote in which unitarity screening corrections are meaningful at present day HERA and Tevatron energies. Specifically:
1) The DGLAP (as well as the BFKL) Pomeron is flat, i.e. $\alpha'_P > 0$, and we expect no shrinkage of $\frac{d\sigma}{dt}$. SC induce a shrinkage to which we can assign an effective α'_P. In our calculations for J/Ψ production we get $\alpha'_P = 0.08\,GeV^{-2}$ at $Q^2 = 0$, changing slowly to 0.05 at $Q^2 = 50\,GeV^2$. The values of α'_P obtained for ρ DIS electroproduction are moderately higher.
2) DGLAP without SC induces a fast increase of $\epsilon(Q^2)$. For J/Ψ production $\epsilon(Q^2 = 0) = 0.32$ growing to $\epsilon(Q^2 = 50) = 0.52$. When we include SC, these values are reduced to 0.21 and 0.46 respectively. A similar suppression is also observed for ρ DIS electroproduction.
3) SC induce a diffraction dip in $\frac{d\sigma}{dt}$. Its exact location depends on the choice of R_i^2. This behavior creates a positive curvature in $\frac{d\sigma}{dt}$. As a result we recommend that the t-slope be determined by a relatively narrow binning.
4) The above analysis implies that we cannot simply estimate $\Psi^V(0, z)$, the vector meson wave function at the origin, as might be concluded from the non screened pQCD calculations [17]. Our calculations, either in the one radius [18] or in the two radii [1] approximations, clearly indicate that the ambiguity induced by the SC makes any such estimate non reliable!
5) Our results strongly suggest the existence of a sizeable hard pQCD component in $Q^2 = 0$, commonly supposed to be dominated by soft physics. Also, an important soft background accompanies the hard diffractive phenomena up to Q^2 as high as $50\,GeV^2$. We, thus, conclude that the next stage in the analysis of DIS diffraction will require some meshing of soft and hard processes.

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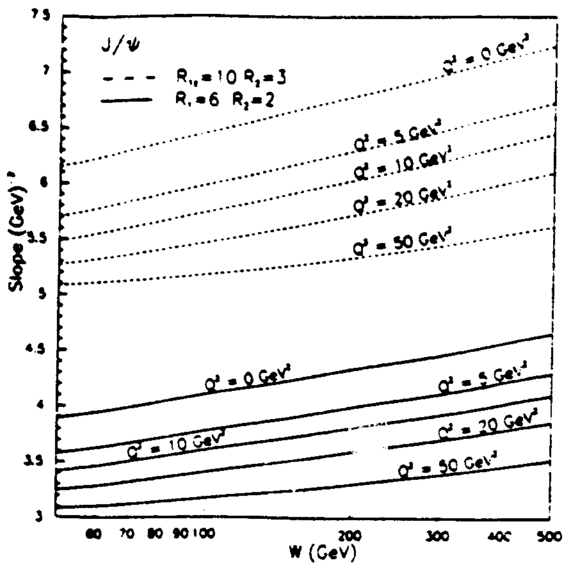

Fig.1: W and Q^2 dependences of the J/Ψ elastic $t = 0$ slopes.

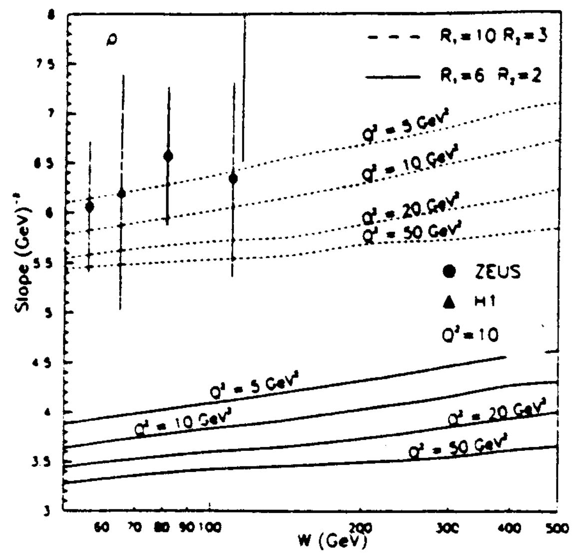

Fig.2: W and Q^2 dependences of the ρ elastic $t = 0$ slopes.

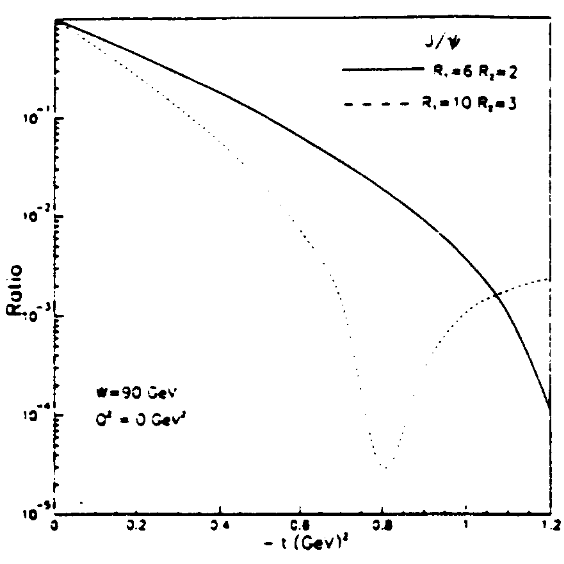

Fig.3: $\frac{d\sigma}{dt}$ for elastic J/Ψ elastic production.

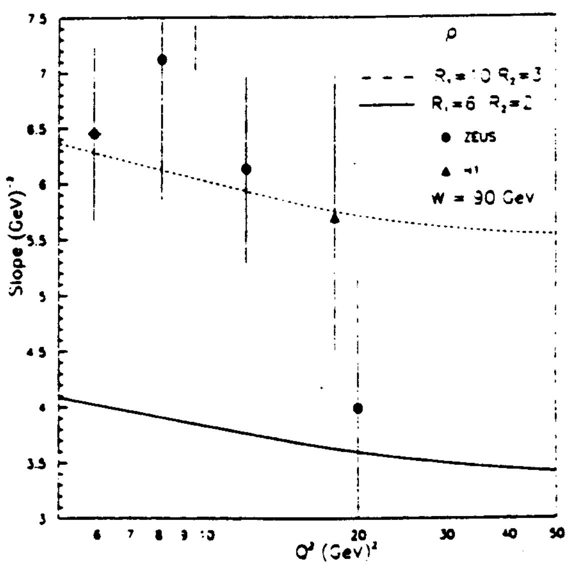

Fig.4: Q^2 dependence of the ρ elastic $t = 0$ slope.