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LOW x PROBLEMS: ON THE WAY TO ANALYTIC SOLUTION

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Abstract

This talk is the attempt to give the review of main ideas and results in theory of deep inelastic scattering at low x to the more general audience than the expert's meeting. At low x we deal with a dense relativistic system of gluons in a nonequiliblium state in which the coupling constant of QCD is still small. This limit of QCD opens new window to nonperturbative properties of QCD and has many features in common with early stages of formation of the quark - gluon plasma in ion - ion collissions and with the high energy behaviour of the total cross section of hadron - hadron scattering.

1 Introduction

The title of this talk I stole from David Gross who in his Cornell Summary gave ten predictions for the year 2008, the second one of which was: *" Analytic treatments in QCD will be developed to describe small x_B physics, Regge behaviour and hadronic fragmentation functions".*

I am going to give you the outline of our hopes,results and problems that we have understood during more that one dacade of efforts in this direction. Let me start with long citation from the report of the DPF Long Range Planning Group on QCD [1]: *" Small x physics is still in its infancy. Its relation to heavy ion physics, mathematical physics and soft hadron physics along with a rich variety of possible experimental signatures make it central for QCD studies over the next decade".*

The main goal of this talk to prepare you for this next decade sharing with you the main ideas and approaches that have been developed during the previous decade. I hope to convince you that we are on the right road but still only in the beginning.

The main process which I am going to discuss is the deep inelastic scattering (DIS) at low x_B. As you know the deeply inelastic process is the big microscope which allows us to resolve the structure of a hadron at short disdances $\Delta r \sim \frac{1}{Q}$ at energy $s = \frac{Q^2}{x_B}$, where Q is the momentum transferred from incoming lepton to recoil one.

Let me try to answer the first natural question:

2 Why is it interesting to study DIS at high energy?

2.1 Past (< 1974) = Reggeon Calculus.

To answer this question let me remind you that in the past before the year 1974 the study high energy asymptotic was high priority job, because we believed that

$$\text{" Analyticity } + \text{ Asymptotic } = \text{ Theory of Everything".}$$

Roughly speaking we needed asyptotic at high energy to specify how many substructions we have to make in the dispersion relation.

Now situation is quite different, we have good microscopic theory (QCD) and certainly we have a lot of problems in QCD which have to be solved. High energy asymptotic is only one of many. I think it is time to ask yourselve why we spend our time and brain trying nevertheless to find the high energy asymptotic in DIS.

Let me before I'll try to answer this question to share with you the lessons that me and all experts that I know learned from the first attempt to built the effective theory at high energy, so called Reggeon Calculus.

1. The longitudional and transverse degrees of freedoms look differently at high energy and can be treated separately and in different ways.

2.The effective theory at high energy can be reduced to two dimensional theory and we need to find the energy spectrum of this two dimensional theory to specify the high energy asymptotic.

3.The major problem of any affective theory at high energy is to find correct degree of freedom or in other words the correct effective particle which is responsible for high energy asymptotic. The concrete realization, namely Reggeon Calculus failed, because the Pomeron, effective particle of that time, turnes out to be not the correct one to construct theory at high energy (see key papers of ref. [2] on this subject).

4. The only way to invent correct effective particle is to develop the consistent approach starting from microscopic theory but not from phenomenology.

At that time we had no microscopic theory and in the best tradition of high energy physics experts left the field. Now we have such a theory and it seems extremely interesting to go back to old problems to see how far away was our guess from sistematic approach. It is rather private but strong motivation for many experts including me.

2.2 The map of QCD.

Before the full answer to the question in the title of this section I want to summarize what we have learned about QCD. To illustrate the situation with QCD as we have understood by now we present the map of QCD in Fig.1. Shown are three separate regions, distinguished by the size of the distances that can be resolved in the process and by the value of parton densities that can be reached in the process.

1. *The region of small parton density at small distances (low density (pQCD) region).*

This is the region where we can apply the powerful methods of perturbative QCD since the value of running coupling constant $\alpha_s(1/r^2)$ is small ($\alpha_s(1/r^2) \ll 1$). During two decades remarkable theoretical progress has been achieved here (GLAP evolution equation [3], gluon bremshtrahlung for jet decay [4] , factorization theorem (J.Collins,D.Soper and G.Sterman (1983) [5]) and the main property of "hard" processes has been experimentally confirmed at LEP and at the Tevatron.

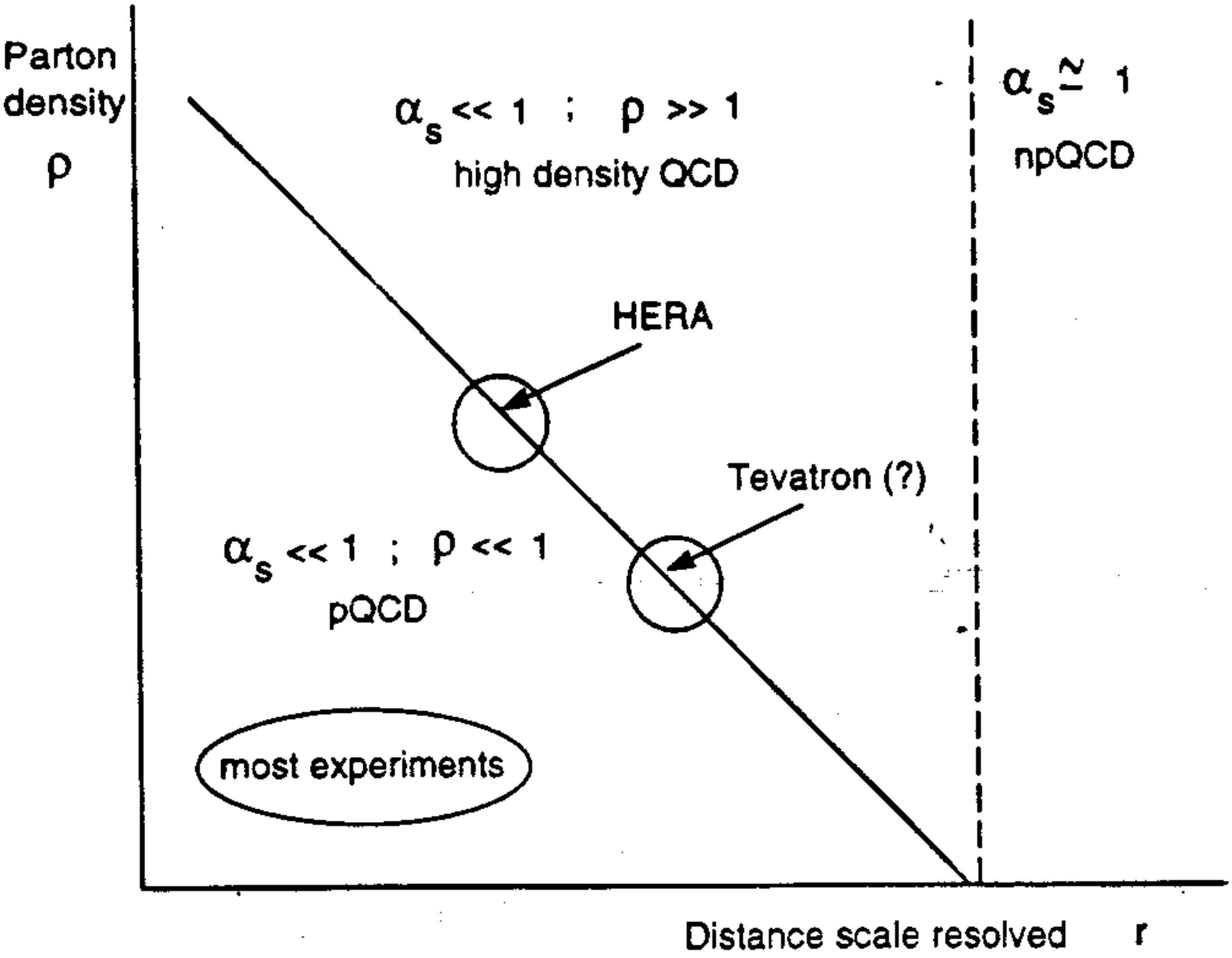

Fig. 1 The map of QCD. ρ is the density of partons (gluons) in transverse plane and r is the distances resolved in an experiment.

2. *The region of large distances (npQCD region).*

Here we have to deal with the confinement problems of QCD, since $\alpha_s(1/r^2) \gg 1$. In this kinematical region we need to use nonperturbative methods such as lattice calculation [6] or QCG Sum Rules [7]. The progress here is remarkable but all developed methods cannot yet be applied to scattering processes. **3** *The region of small distances but high parton density (hdQCD region).*

Here we have a unique situation in which the coupling constant α_s is still small but the density is so large that we cannot use the usual methods of perturbation theory.

In essence the theoretical problem here is also a nonperturbative one but the origin of the nonperturbative effects here is quite different from that in the previous region. Here we face the situation where we have to develop new methods that let deal with a dense relativistic system of gluons in a nonequilibrium state. Unfortunately we are only at the beginning of this road.

Fortunately, we can control theoretically this dense system of partons in some transition region on the border of the pQCD and hdQCD regions and here we can study this remarkable system of partons in great detail. Thus the right strategy is to approach this interesting kinematic region from the low density pQCD region.

In my opinion the interest to high energy scattring was revived only because moving from pQCD region to hdQCD region we can open new window to study a nonperturbative nature of QCD.

2.3 What fundamental problems could be solved?

This is why we want to list here the fundamental problems that we hope to solve penetrating high density QCD region. We hope:

1. to specify the kinematical region in which we can trust pQCD (GLAP evolution equation, gluon bremstrahlung, factorization theorem ...);

2. to find new collective phenomena for nonabelian theories such as QCD ;

3. to find the analytic solution of hd QCD which is nonperturbative but looks simpler than np QCD since $\alpha_s \ll 1$ here;

4. to develop methods with which build an effective theory for hd QCD.

We need the effective theory because we can use the Lagrangian of such a theory for exact calculation on lattice, for example, or even we can try to solve problem analytically.

3 How to penetrate the high density QCD region.

Access to this interesting kinematical region is actually easily achieved in our scattering processes. We know at least three ways to prepare a large density system of partons.

1. The first is given by nature, which supplies us with large and heavy nucleii. In ion-ion collisions we can already reach a very high density of partons at not so high energies, because the partons from different nucleons in a nucleus are freed.

2. The second relates to hard processes in hadron-hadron collisions or in deep inelastic scattering. These also give us access to a high density of partons because we expect a substantial increase in the parton density in the region of small Bjorken x. The experimental data from HERA show the significant increase of the deep inelastic structure function:

$$F_2(Q^2, x_B) \propto (\frac{1}{x_B})^{0.33} \; at \; Q^2 \sim 10\, GeV^2.$$

3. The third is to measure the event with sufficiently large multiplicity of produced particles, larger than the multiplicity in the typical inelastic (bias) event.

Thus we can formulate the ideal experiment for search of high density partonic system:

" The deeply inelastic scattering with nucleus at low x with special selection of events with large multiplicity of produced particles. "

4 Theory.

4.1 Method or two diabols that we are fighting with.

Accorgingly to our strategy we start with perturbative QCD in that region of Fig.1 where the density and coup[ling QCD constant (α_s) are small. Each observable (i.e gluon structure function) could be written in pQCD as following series:

$$xG(x, Q^2) \; = \; \Sigma_{n=0}\, C_n(\alpha_s)^n \cdot (L^n + a_{n-1} L^{n-1} ... a_0) \; , . \qquad (1)$$

We have two big problems with this perturbative series which are two our biggest enemies:

1. The natural small parameter α_s is compensated by large log (L). The value of L depends on the process and kinematic region. For example in deeply inelastic scattering (DIS):

$$L \; = \; \log Q^2 \quad at \quad Q^2 \gg Q_0^2 \; but \; x \sim 1$$

$$L = \log(1/x) \quad at \quad Q^2 \sim Q_0^2 \; and\, x \to 0$$
$$L = \log Q^2 \cdot \log(1/x) \; at \quad Q^2 \gg Q_0^2 \; and\, x \to 0$$
$$L = \log(1-x) \quad at \quad Q^2 \sim Q_0^2 \; and \; x \to 1$$

Of course it is not the full list of scales. The only that I would like to emphasize that L depends on the kinematic region. Thus to calculate $xG(x,Q^2)$ one cannot calculate only the Born Approximation but has to calculate the huge number of Feynman diagrams.

2. $C_N \to n! \;\; at \;\; n \gg 1$ It means that we are dealing with asymptotic series and we do not know the general rules what to do with such series. There is only one rule, namely to find the analytic function which has the same perturbative series. Sometimes but very rarely we can find such analytic function. In this case this is the exact solution of our problem. Mainly we develop some general approach based on Leading Log Approximation (LLA). The idea is simple. Let us find the analytic function that sums the series:

$$xG(x,Q^2)_{LLA} = \Sigma_{n=0} C_n \, (\alpha_s \cdot L)^n \; . \tag{2}$$

Ussualy we can write the equation for function $xG(x,Q^2)_{LLA}$. The most famous one, the GLAP evolution equation [3], sums eq.(2) if $L= \log Q^2$. The BFKL [8] equation gives the answer for eq.(2) in the case when L=log($1/x$). Using the solution of the LLA equation we built the ratio:

$$R(x,Q^2) \; \frac{xG(x,Q^2)}{xG(x,Q^2)_{LLA}} = \Sigma_{n=0} r^n = \Sigma_{n=0} c_n \cdot (L^{n-1} + a_{n-1} L^{n-2} + ... a_0) \; . \tag{3}$$

This ratio is also asymptotic serias but what we are doing we calculate this series term by term. Our hope is that the value of the next term will be smaller then the previous one ($\frac{r_n}{r_{n-1}} \ll 1$) for sufficiently large n. However we know that at some value of $n = N$ $\frac{r_N}{r_{N-1}} \sim 1$. The only that we can say about such situation in general that our calculation has intristic theoretical accuracy and the result of calculation should be presented in the form

$$R(x,Q^2) = \Sigma_{n=0}^{n=N-1} r_n \pm r_N \; . \tag{4}$$

How big is the value of N depends mostly on how well we chose the LLA and how well we established the value of scale L in the process of interest.

4.2 Present Theoretical Status = Regeneration of Reggeon Calculus.

What we are doing now approaching the high density QCD domain is really the regeneration of the old idea of Reggeon Calculus [9], namely we reduce the complicated problem of quark and gluon interaction in the dense parton system to the interaction of our "building bricks", so called Pomerons (see Fig. 2, where the structure of our approach is shown). It looks like in old, good time of Reggeon Dominancy but we have two new and very important ingradients:

1. The QCD Pomeron is not an invention but naturally appears in perturbative QCD in the leading $log(1/x)$ approximation (LL (x)A) to the scattering amplitude at high energy (the Balitski - Fadin -Kuraev - Lipatov (BFKL) equation [8]). It means that in a restricted kinematical region the BFKL Pomeron describes the high energy interaction within a certain guaranteed theoretical accuracy. This fact makes it unavoidable that one should build an effective theory starting with the BFKL Pomeron.

2. The vertices of the Pomeron interactions are not phenomenological parameters but can be calculated in perturbative QCD. It turns out that only two of them are essential in the vicinity of the border between pQCD and hdQCD regions in Fig.1, namely triple Pomeron vertex (γ) and rescattering of two Pomerons (λ).

4.3 Correct degrees of freedom at low x.

During last two years a significant advance has been made in understanding the structure of the BFKL Pomeron. A.Mueller (1993) and N.Nikolaev with collaborators (but six months later)[10] constructed the partonic infinite momentum wave function of a hadron at low x and opened a new way in understanding of physical meaning and formal derivation of the BFKL equation as well as its generalization .

The main progress was related to the fact that they discovered the correct degrees of freedom for the region of small x in QCD. They showed that if we discuss the deeply inelastic processes not in terms of quark and gluons as we did before but introducing new degrees of freedom, namely the color dipole of the definite size (d), we still can use the simple probabalistic interpretation in the region of low x. It means that the physical meaning of the deep inelastic structure function is not the number of quark or gluons but the number of the color dipoles with the sizes larger than $1/Q$ ($d > \frac{1}{Q}$). It is worthwhile mentioning that at $x \sim 1$ we still can use our old interpretation as probability to find parton but as well as a new one.

This idea shed a light on all technicalities of the BFKL equation which was the manifestation of the artistic skill in the calculation of Feynman diagrams and reduced the high art to normal physics understandable for a normal post-doc. I anticipate a big progress in the development of all kind problems related to high energy QCD based on Mueller approach.

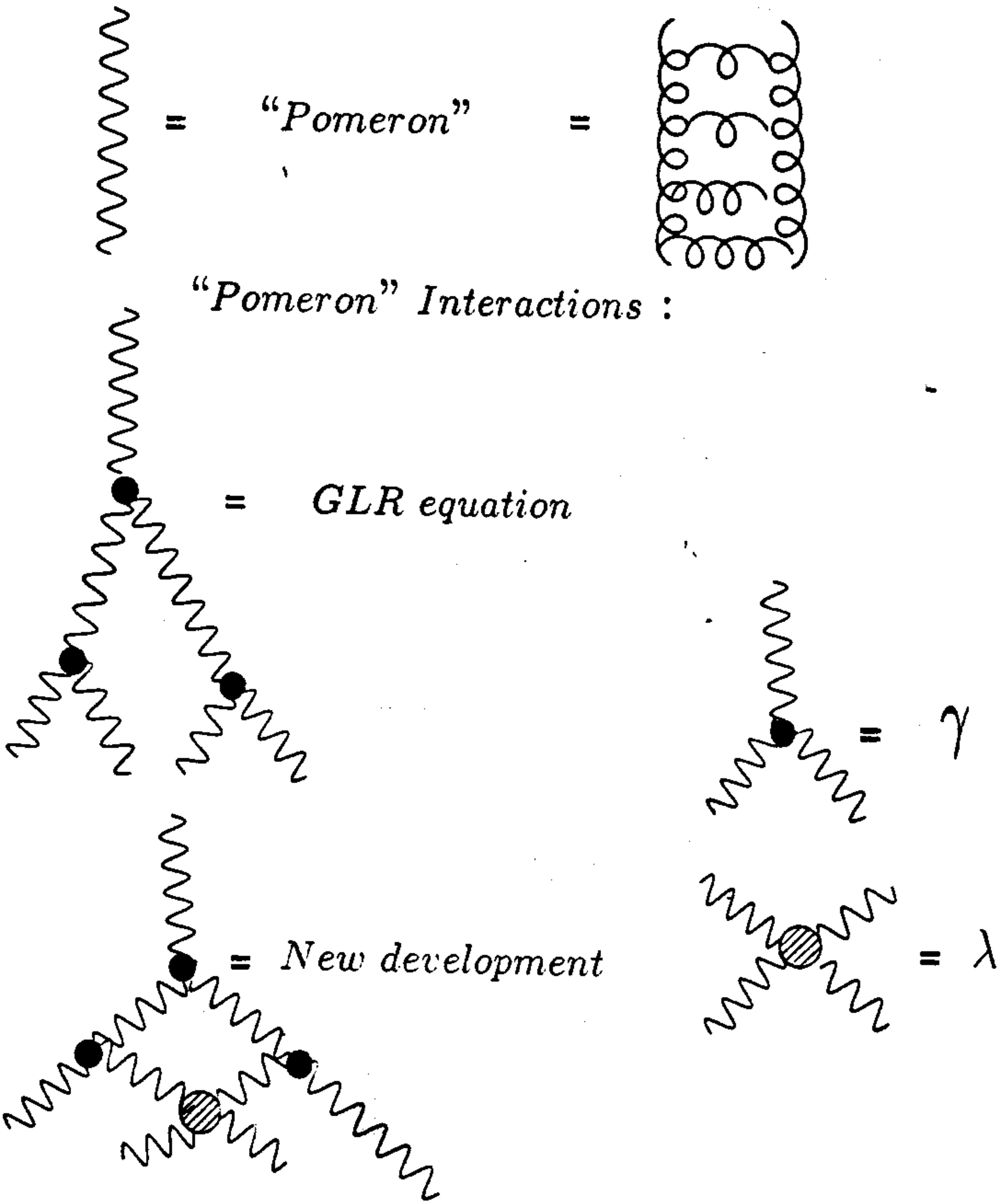

Fig. 2 " Pomeron" Calculus in QCD.

4.4 Shadowing Corrections.

However, we know that it is certainly not enough to understand better the BFKL equation or the Pomeron structure in general. Indeed, the BFKL equation leads to power-like increase of the total cross section for the deeply inelastic processes which violates the s-channal unitarity ($\sigma_{tot} < ln^2(1/x)$) even at small distances (large Q^2)[11]. It means that the problem of Pomeron - Pomeron interaction should be solved or in other words the diagrams of Figf.2 with Pomeron interactions have to be summed. The first attempt was made in GLR paper [11] where the evolution equation was written that sums "fan" diagrams (see Fig.2). Now we have reached better understanding of the main properties of the shadowing correction and their relation to high twist contributions to the deeply inelastic processes[13]. It is possible because we found the new small parameter that controls the accuracy of our calculations. It turns out that such new small parameter [11] is equal to

$$W = \frac{\alpha_s}{Q^2} \cdot \rho = \frac{\alpha_s}{Q^2} \cdot \frac{xG(x,Q^2)}{\pi R^2} \qquad (5)$$

where R is the radius of hadron.

The first factor in eq.(5) is the cross section for gluon absorption by a parton from the hadron. So it is clear that W has a very simple physical meaning, namely it is the probability of parton (gluon) recombination in the parton cascade. We can rewrite the unitarity constraint in the form $W \leq 1$. Thus W is the natural small parameter in our problem. It is worthwhile to note that W can be rewritten through the so called packing factor

$$PF = \langle r^2_{constituent} \rangle \cdot \rho \, . \qquad (6)$$

Indeed $W = \alpha_s \cdot PF$.

Using this small parameter we can resum the whole perturbative series (see above about phylosophy and strategy of resummation), rearrange it as the Reggeon - like diagrams for the Pomeron interaction and write down the equation [11]. The result of this resummation can be easily understood considering the structure of the QCD cascade in a fast hadron. Inside the cascade there are two processes that are responsible for the resulting number of partons:

$$Emission \ (1 \rightarrow 2); \ \ Probability \propto s\rho \ ; \qquad (7)$$

$$Annihilation \ (2 \rightarrow 1); \ \ Probability \propto 2_s r^2 \rho^2 \propto s^2 \frac{1}{Q^2} \rho^2 \, ,$$

where r^2 is the size of produced parton in the annihilation process. For deep inelastic scatterinq $r^2 \propto \frac{1}{Q^2}$.

It is obvious that at $x_B \sim 1$ only the production of new partons (emission) is essential since $\rho \ll 1$, but at $x_B \to 0$ the value of ρ becomes so large that the annihilation of partons that diminishes the total number of gluons enters into the game.

Finally this simple parton picture allows to write an equation for the density of partons that takes these processes properly into account. Indeed, the number of parton in a cell of the phase space ($\Delta y = \Delta \ln \frac{1}{x}, \Delta \ln Q^2$) increases due to emission and decreases as result of annihilation. As an outcome the particle balance for this cell looks as follows:

$$\frac{\partial^2 \rho}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} \rho - \frac{\alpha_s^2 \gamma}{Q^2} \rho^2 , \qquad (8)$$

or in terms of the gluon structure function $x_B G(x_B, Q^2)$

$$\frac{\partial^2 x_B G(x_B, Q^2)}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} x_B G(x_B, Q^2) - \frac{\alpha_s^2 \gamma}{Q^2 R^2} (x_B G(x_B, Q^2))^2 . \qquad (9)$$

Eq. (9) is the socalled GLR equation [11]. Unfortunately even now we need some complicated technique of summation of Feynman diagramms in W^n'th order of perturbation theory to calculate the value of γ [12] and to understand the kinematical region where we can trust the equation (9). The value of γ calculated in ref.[12] reads $\gamma = \frac{81}{16}$ *for* $N_c = 3$.

By now we have understood that the GLR equation has very restricted power and should be improved taking into account the BFKL Pomeron in the first term of the GLR equation (eq.(9 was written for the GLAP equation for the Pomeron) as well as rescattering of Pomerons (two Pomeron amplitude (λ)). This is the subject of current study.

5 Our expectations.

Now we can summarize what new phenomena we anticipated in the region of small x_B just before HERA started to operate.

1. Increase of the parton density.

Both evolution equations (GLAP and BFKL) predict an increase of the parton density:

$$\textbf{GLAP}: \quad x_B G(x_B, Q^2) \to e^{\sqrt{\frac{4\alpha_s}{\pi} \ln \frac{1}{x_B} \ln \frac{Q^2}{Q_0^2}}} ;$$

$$\mathbf{BFKL}: x_B G(x_B, Q^2) \rightarrow x^{-\omega_0} e^{-\frac{\ln^2 \frac{Q^2}{Q_0^2}}{\Delta \ln \frac{1}{x_B}}} \quad where \quad \omega_0 = \frac{4N_c}{\pi} \ln 2\alpha_s(Q_0^2) .$$

2. Growth of the typical transverse momentum of partons.

This property changes crucially the physics of deeply inelastic scattering at low x_B since the typical "hard" process occurs only at Q^2 larger than the mean value of the transverse momentum of partons. If Q^2 is smaller we face the very unusual situation where processes with the typical properties of "soft" interactions occur at small distances and can be treated by pQCD.

The growth of the typical parton transverse momentum is a common feature of the BFKL eqution as well as the GLR one. Indeed:

$$\mathbf{BFKL}: \ln \frac{\langle |p_t^2| \rangle}{Q_0^2} = \sqrt{\alpha_s(Q_0^2) 14\zeta(3) \ln \frac{1}{x_B}} ;$$

$$\mathbf{GLR}: \quad \ln \frac{\langle |p_t^2| \rangle}{Q_0^2} = \sqrt{a \ln \frac{1}{x_B}} .$$

The constant a has been calculated in ref. [11].

3. Saturation of the gluon density.

Directly from the GLR equation we can see that the parton (gluon) density reaches a limiting value at low x_B. Inspite of the fact that we cannot trust the GLR equation at very small values of x_B we believe that the saturation of the gluon density is a new phenomena which reflects the basic property of the parton cascade in hdQCD.

Fig.3 shows all our expectations in terms of the gluon structure function.

6 What density is large.

From GLR evolution equation which takes into account the screening (shadowing) correction one can estimate the maximum value of so called packing factor:

$$PF = \left\langle r^2_{constituent} \right\rangle \cdot \rho .$$

It turns out that for parton with $\langle r^2_{constituent} \rangle = \frac{1}{Q^2}$

$$(PF)_{max} = 0.21 \ for \ N_c = 3 .$$

However we need to know the value of radius R in the definition of the parton density through the deep inelastic structure function. At the moment we have two working hypothesis : i) $R = R_{hadron}$ and ii) $R \approx \frac{1}{3}R_{proton} < R_{proton}$. In the first case the parton density saturation starts from

$$x_B G(Q^2, x_B) > 150 \ at \ Q^2 = 10GeV^2$$

while in the second picture

$$x_B G(Q^2, x_B) > 15 \ at \ Q^2 = 10GeV^2 \ .$$

6.1 What x is small.

Using HERA data and the maximum value of the packing factor from the GLR equation we are able to estimate the value of x_B at which we expect the new physics related to the parton density saturation. For this purpose we use the simplest paramentrization for gluon deep inelastic structure function that describes the HERA data:

$$x_B G(Q^2, x_B) \ = \ 4(100 x_B)^{-0.33} \ at \ Q^2 == \ 10\, GeV^2.$$

From this oversimplify expression for $x_B G(Q^2, x_B)$ we get that the limiting packing factor our system can reach at $Q^2 = 10GeV^2$ at $x_B^{saturation} = 0.2\,10^{-7}$ or $x_B^{saturation} = 0.2\,10^{-4}$ for $R = R_{proton}$ and $R = \frac{1}{3}R_{proton}$ respectively.

For nucleus with the number of nucleons A the critical value of the packing factor decreases in $1 + A^{\frac{1}{3}} \cdot \frac{R^2}{R^2_{proton}}$ times. It results in an increase of the value of x_B at which the parton density reaches the saturation,namely

$$(x_B^{saturation})_A \ = \ (1 \ + \ A^{\frac{1}{3}} \cdot \frac{R^2}{R^2_{proton}})^3 \cdot (x_B^{saturation})_N \ .$$

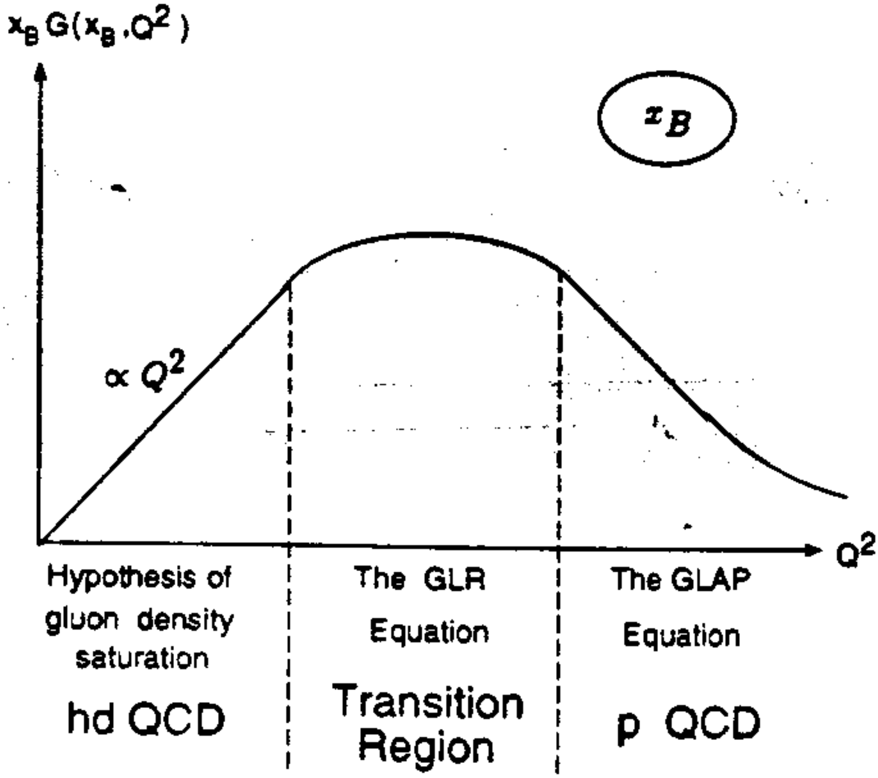

Fig. 3 The behaviour of $xG(x, Q^2)$ versus Q^2 at fixed x.

7 How to measure the high density event.

Let me list here the main ideas how to measure the new physics that we anticipate at high density system of partons:

1. The probability of double parton interaction should be large (of the order of the maximum value of the packing factor) about 20%.

The double parton interaction can be seen not only as cross section for production of two pair of hard jets with the same value of rapidity,but also as a cross section of the inclusive production of hadrons in the window of rapidity $y + \Delta y, y - \Delta y$ where y is the rapidity of a hard jet with transverse momentum p_t and $\Delta y = \ln \frac{p_t}{p_0}$, where p_0 is the transverse momentum of produced hadron. It could be also seen as a long range correlation in rapidity between produced hard jet and produced hadron which is not specially hard.

2. In the high density event we should see the Landau - Pomeranchuk suppression of the emission of gluons with transverse momentum smaller that the typical momentum $q_0(x_B)$ which can be found from the equation:

$$\frac{xG(q_0^2(x), x)}{q_0^2(x)\pi R^2} = (PF)_{max} . \qquad (10)$$

Such a suppression can be seen as the deviation from the factorization theorem for jets with $p_t \leq q_0(x)$.

3. . Decorrelation effect for jets with transverse momentum p_t of the order of q_0. The value of transverse momentum for such a jet is compensated not by one jet in the opposite direction but by a number of jets with average transverse momentum about q_0.

4. Polarization of produced hadrons allows us to measure the typical transverse momentum in the process since in the region of pQCD polarization should be equal to zero.

5. It is seen directly from eq.(9) that the saturation reaches in the system with small size at larger transverse momentum (smaller value of the gluon structure function). So this is why we have to create experimentally such compact system. We have three ideas how to confine the gluons in the disc of the small size:

A. to find a carrier of partons with small size. Even hadron could be such a carrier if the hypothesis of constituent quarks with small radius will be confirmed experimentally. However better to use the virtual photon or Pomeron. The last is not well theoretically defined object and what is Pomeron is one of the questions that the future experiment should answer. However even available experimentall information confirm the idea that

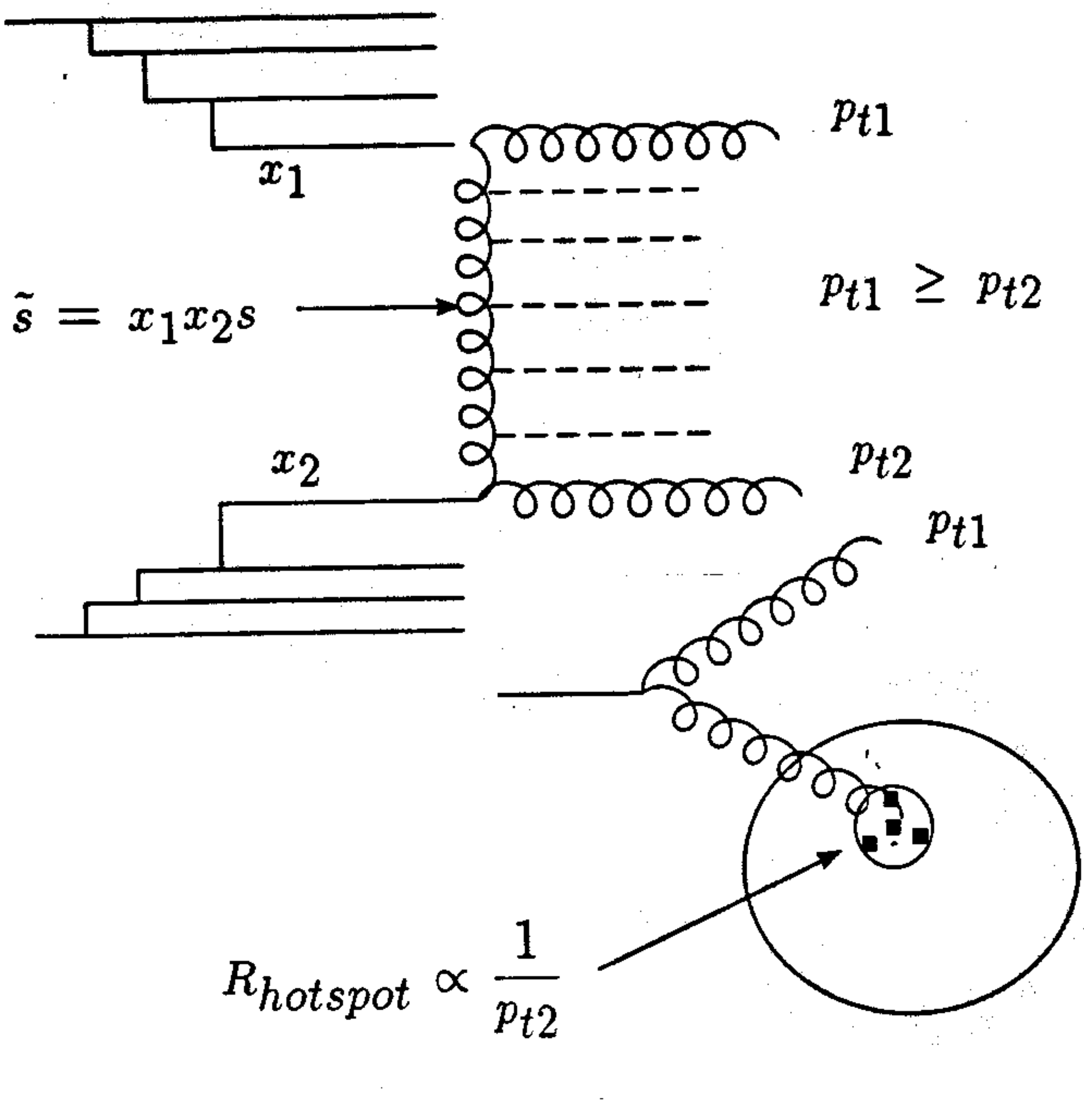

Fig.4. The Mueller - Navalet Process.

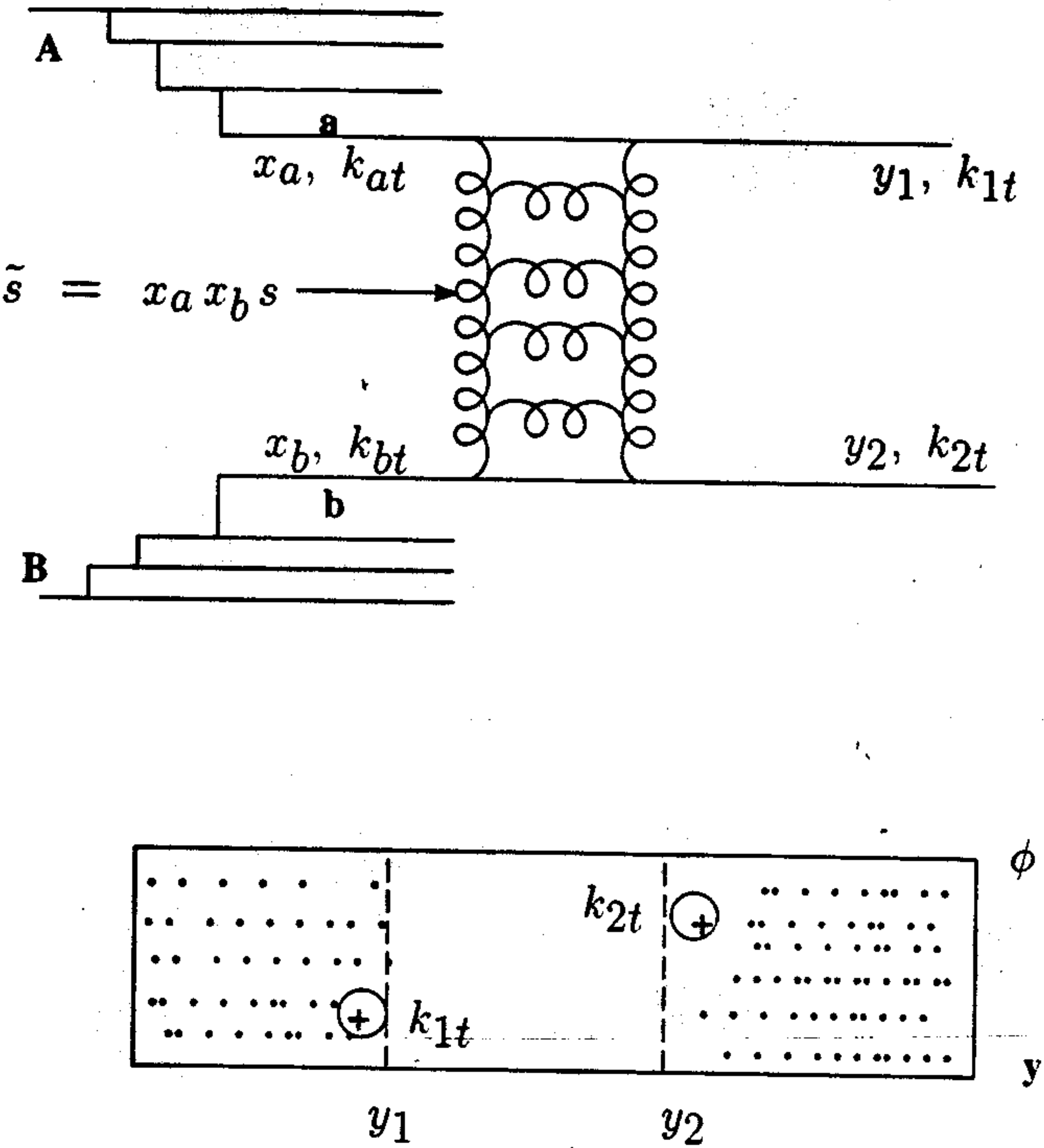

Fig. 5. Large Rapidity Gap Process.

Pomeron's size is much smaller that hadron one and of the order of $R_P \approx \sqrt{\alpha'_P} \sim 0.5GeV^{-1} \sim 0.1Fm$. Thus hard diffraction with Pomeron can give a good possibility to localize the parton system in small disc and to see high density phenomena in the most clear way.

B. to find the experiment (microscope) that can resolve the small part of the hadrom and investigate it in detail. This idea is realized in so called Mueller - Navalet process [14] or "hot spot" hunting [15] (see Fig.4).

This process allows us to measure the small $\propto \frac{1}{p_{t2}}$ part of the hadron and using the two jets with sufficiently large transverse momentum as a trigger e can study the system with large parton density in many details.

C. Bjorken [16] pointed out that the large rapidity gap (LRG) processes can give us new way to look inside the high density parton system. Indeed, due to intimate relation between inelastic processes and elastic one coming from new reggeon-like approach the process with the LRG such as two high transverse momentum jet production with rapidities y_1 and y_2 but without any hadron with rapidities $y_1 > y_h > y_2$ can be described as the exchange of "hard" Pomeron (see Fig.5). The properties of the " hard" Pomeron exchange is well known theoretically (see ref. [8]) and can be checked experimentally.

6. One of the way to measure the high parton density is to select the event with large multiplicity of produced hadrons. More discussion on this subject you can find in the report of Fermilab Working Group on QCD [17]

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