CBPF-NF-070/94

Non Linear Non Local Theory of Gravity

M. Novello and V.A. De Lorenci

Centro Brasileiro de Pesquisas Físicas - CBPF-CNPq-LAFEX
Rua Dr. Xavier Sigaud, 150
22290-180 - Rio de Janeiro-RJ, Brazil

ABSTRACT

Deser and Laurent (1968) explored an alternative linear theory for spin-two field using a nonlocal divergence-free projection operator on the matter energy-momentum tensor. Here, we extend this theory by including non-linearity of the gravitational field. We show that such extended model is indistinguishable from General Relativity, as far as the four standard tests of gravity are concerned.

Key-words: Non local, Non linear, Gravity

1 Introduction

Any theory that aims to be competitive as a possible description of gravitational interaction should provide an explanation at least as good as General Relativity (GR) does for the so-called four traditional tests.

Recently at the Angra Meeting[1] we have presented a non-local theory of gravity which was based on an ancient idea of Deser and Laurent (DL)[1].

The reason to come back to this theory is twofold:

- To analyse the non-local property of the energy-momentum distribution of the gravitational field;
- To improve Deser-Laurent model by introducing non-linearity on it.

The main appeal of DL model was related to the possibility of dealing with a consistent closed field theory without recurrence into the traditional geometrical scheme, which deals with the summation of an infinite series as it has been shown by many authors (see, for instance, Feynmann [2]). The problem, in a few words, can be summarized in the following way.

The source of spin-two field in Fierz-Pauli linear theory is the energy-momentum tensor $T_{\mu\nu}$ of matter. The coherence of this theory induces a conservation law:

$$T^{\mu\nu}{}_{,\nu} = 0.$$

Once gravity is not a ghost field, it possesses also an energy distribution. Then, by a mechanism of self-coherence, an infinite series appears, yielding naturally Einstein's geometrical formulation. DL found an alternative solution to this problem by considering a non-local scheme of compatibility.

Although DL model has been successful to describe some properties of the motion of a material particle in a gravitational field it contains a fatal drawback that appears when gravitational waves are present. The reason for this is precisely related to the linearity of DL model, that treats gravity as transparent to gravitons. So, in order to solve such difficulty and in the same vein to keep its good properties it seems that a very natural way should be the construction of a theory that incorporates both features:

- non locality;
- non linearity.

In this paper we exhibit a toy model that possesses such properties. We will limit our exam here only to observational consequences of the model leaving to a separate paper the presentation of its main theoretical framework. This is worth considering once the results that follows is that our model is compatible with observations, as far as the four standard tests of gravitational processes are concerned, as we will show in this paper.

[1]Invited talk at the XV Brazilian Conference on Particles and Field Theory, Angra dos Reis, August 1994.

2 The Model

Following the procedure outlined in DL paper we take as the true equations of motion for the gravitational field the non-local form[2]:

$$G_L^{\mu\nu} = -k(\hat{T}^{\mu\nu} + n\,\hat{t}^{\mu\nu}). \tag{1}$$

Deser-Laurent model corresponds to the case in which the adimensional constant n takes the value zero. Here we will set $n = 1$. $G_L^{\mu\nu}$ is the linear Fierz-Pauli expression for the spin-two field and $t^{\mu\nu}$ represents the corresponding Gupta [3] energy-momentum tensor, which is quadratic in the gravitational variable $\phi_{\mu\nu}$. A hat above a tensor, as it appears in the above equation, represents the result of the application of the operator $P_{\mu\nu}$ that projects any tensor on the divergenceless space. This means the following. Given any tensor V_μ we define its associated form $\hat{V}_\mu$ as

$$\hat{V}_\mu \equiv P_\mu{}^\alpha V_\alpha \tag{2}$$

such that the quantity $\hat{V}_\mu$ is divergence-free. The form of the non-local projector operator[3] is given by

$$P_\mu{}^\alpha \equiv \delta_\mu^\alpha - \Box^{-1}\partial^\alpha\partial_\mu$$

in which $\Box^{-1}$ is such that

$$\Box\,\Box^{-1} = 1.$$

The most general form of generating a divergenceless second order tensor is provided by the product

$$\hat{T}_{\mu\nu} \equiv (P_{\mu\alpha}\,P_{\nu\beta} + p\,P_{\mu\nu}\,\eta_{\alpha\beta} + q\,P_{\mu\nu}P_{\alpha\beta})T^{\alpha\beta}.$$

The constants p and q are for the time being free parameters. They will be fixed in order to fit the observations, as we shall see.

3 Material Particles

The simplest way to analyse the net consequences of the present model is to look into the gravitational effects on the motion of material particles that are obtained from the Lagrangian

$$\mathcal{L} = \frac{1}{2}\eta U_\mu U^\mu + \psi^{\mu\nu}\mathcal{T}_{\mu\nu} \tag{3}$$

in which U_μ is the four-velocity of the fluid particles, $\mathcal{T}_{\mu\nu}$ stands for the sum of matter and gravity (Gupta) energy-momentum tensors, η represents the Jacobian of the transformation from the fluid variables a^μ to the usual coordinates x^μ and $\psi^{\mu\nu}$ is nothing but the projected form $\hat{\phi}^{\mu\nu}$.

[2]We would like to call attention to the fact that we are using the background Minkowski metric $\eta_{\mu\nu}$ to lower and raiser indices. See latter on for the effective geometry $g_{\mu\nu}$ when gravity effects are taken into account.

[3]We follow Deser-Laurent and take here only the causal part of such operator. We postpone any further comment on this for a subsequent paper.

The effect of the interaction on the motion of the particles is given by the modification of the proper time through the expression:

$$(da^0)^2 = S(\phi)^{-1}\,(\eta_{\mu\nu} + 2\psi_{\mu\nu})\,dx^\mu\,dx^\nu \tag{4}$$

in which

$$S(\phi) \equiv 1 - \frac{2}{\eta}\psi^{\mu\nu}\,t_{\mu\nu}.$$

Let us follow DL and concentrate our exam in the static spherically symmetric configuration having a point-like source. In this case, the expression of $(da^0)^2$ reduces to[4]

$$\left(da^0\right)^2 = \left(1 - \frac{a}{r} - \frac{A}{r^2}\right)dt^2 - \left(1 + \frac{b}{r} + \frac{B}{r^2}\right)dr^2 - \left(1 + \frac{d}{r} + \frac{D}{r^2}\right)r^2 d\Omega, \tag{5}$$

in which

$$d\Omega \equiv d\theta^2 + sin^2\theta\,d\phi^2.$$

The coefficients that appear in this expression are combinations of the constants p and q given by:

$$a = -3(p+q)^2 - 2(p+q) + 1, \tag{6}$$

$$b = 3(p+q)^2 + 2(p+q) + 1, \tag{7}$$

$$d = \frac{3}{2}(p+q)^2 + \frac{5}{2}(p+q) + 1, \tag{8}$$

$$A = -\frac{15}{16}q^4 - \frac{3}{4}p^4 - \frac{57}{16}pq^3 - \frac{51}{16}p^3q - \frac{81}{16}p^2q^2 - \frac{11}{8}q^3 - \frac{21}{16}p^3 + -\frac{65}{16}pq^2 - 4p^2q - \frac{3}{16}(p+q)^2 + \frac{1}{4}(p+q), \tag{9}$$

$$B = \frac{15}{8}q^4 + \frac{3}{2}p^4 + \frac{57}{8}pq^3 + \frac{51}{8}p^3q + \frac{81}{8}p^2q^2 + \frac{29}{16}q^3 + \frac{15}{8}p^3 + +\frac{11}{2}pq^2 + \frac{89}{16}p^2q - \frac{9}{16}(p+q)^2 - \frac{1}{8}(p+q), \tag{10}$$

$$D = \frac{15}{16}q^3 + \frac{3}{4}p^3 + \frac{21}{8}pq^2 + \frac{39}{16}p^2q + \frac{17}{16}(p+q)^2 - \frac{1}{8}. \tag{11}$$

[4]It is understood that in order to arrive at this expression we used the solution of the field equations eq. (1). Note that eq. (5) reduces to DL's form in the case $A = B = D = 0$.

4 Observational Tests

By the same procedure as in the case of General Relativity, we obtain the following constants of motion E and L:

$$E \equiv \left(1 - \frac{a}{r} - \frac{A}{r^2}\right) \frac{dt}{da^0} \tag{12}$$

and

$$L \equiv \left(1 + \frac{d}{r} + \frac{D}{r^2}\right) r^2 \frac{d\varphi}{da^0}. \tag{13}$$

Defining the new variable $u \equiv \frac{1}{r}$, and using the above relations, the equation governing the shape of the orbit is provided by

$$\frac{d^2u}{d\varphi^2} + u = \frac{a}{2}\frac{E^2}{L^2} + \frac{1}{2}(2d - b)\frac{E^2 - 1}{L^2} + \left[\left(A + a^2 - ab + 2ad\right)\frac{E^2}{L^2} + (-B + 2D + {} +b^2 - 2bd + d^2)\frac{E^2 - 1}{L^2}\right] u + \frac{3}{2}(b - d)\, u^2. \tag{14}$$

Using eq. (14) we find the corresponding equation for a planetary orbit. Taking into account the approximation $E^2 \approx 1$, it follows

$$\frac{d^2u}{d\varphi^2} + u = \frac{a}{2}\frac{E^2}{L^2} + \left(A + a^2 - ab + 2ad\right)\frac{E^2}{L^2} u + \frac{3}{2}(b - d)\, u^2. \tag{15}$$

Using the approximation, valid for the solar system[5]

$$u \approx \frac{a}{2}\frac{E^2}{L^2}, \tag{16}$$

it follows[6]

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{L^2} a + 2\left(\frac{2A}{a} + 2a - \frac{b}{2} + \frac{5}{2}d\right) GMu^2. \tag{17}$$

The last term gives the perihelion precession of our present theory. This should be compared with the analogous result in General Relativity. Before this, however, it is more convenient to analyse the consequences of the remaining equations.

Limiting our analysis here to the path of light rays we have

$$\frac{d^2u}{d\varphi^2} + u = \frac{3}{2} GM\,(a + b)\, u^2. \tag{18}$$

[5]The reader may consult the quoted article of Deser et al for more details.

[6]From here on we will write explicitly the constants GM of our problem instead of working in a system of units in which they are made equal to 1. This is done here just for comparison with observations.

Using the values for a and b taken from eqs.(6) and (7) it follows identically that $a + b \equiv 2$. Substituting this value into the above eq. (18) we obtain the same value predicted by GR for the bending of light. Indeed,

$$\frac{d^2u}{d\varphi^2} + u = 3GMu^2.$$

The fourth test (radar delay time) is automatically satisfied. To proceed we need to specify the values of a and b. Let us examine a simple possible case by choosing

$$a = 1, \tag{19}$$

$$b = 1. \tag{20}$$

Coming back to the original equation for the perihelion precession results, in this case,

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{L^2} + 3GMu^2 + (4A + 5d)\, GMu^2.$$

By comparison with the analogous result in GR we are led to impose

$$4A + 5d = 0 \tag{21}$$

which provides the second[7] equation for the unknown parameters p and q. Solving the algebraic equation we obtain

$$p = -1, \tag{22}$$

$$q = \frac{1}{3}. \tag{23}$$

Coming back to the eq. (12) and using these values of p and q we obtain

$$E = \left(1 - \frac{2GM}{r}\right)\frac{dt}{da^0} \tag{24}$$

which is precisely the formula of GR for the red-shift.

From these results we conclude that the choice $p = -1$ and $q = \frac{1}{3}$ makes the present theory, as far as the four tests are concerned, indistinguishable from General Relativity.

References

[1] S. Deser and B. E. Laurent, *Gravitation Without Self-Interaction.* Ann. Phys., **50**, 76 (1968).

[2] R. P. Feynman, *Lectures on Gravitation.* California Inst. Techonology. (1962-3).

[3] S. N. Gupta, *Quantization of Einstein's Gravitational Field: Linear Approximation.* Proc. Phys. Soc., A **65**, 162 (1952).

[7] The other equation is provided by eq.(19) or eq.(20). Note that any one of these yields the same restriction on the parameters p and q.