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NON-EQUILIBRIUM FRIEDMANN COSMOLOGIES

by

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ABSTRACT

A uniform cosmological model filled with a fluid which possesses pressure and bulk viscosity is developed using extended thermodynamics. The Einstein and thermodynamic equations can be exactly integrated on Friedmann-like situation. One of the solutions is non singular: it stars from a steady state behavior and expands to a situation where viscosity dies out.

Key-words: Non-equilibrium; Thermodynamics; Cosmology; Relativistic.

I - INTRODUCTION

Since the pioneering work of Einstein, the material content of the universe has been represented by a relativistic perfect fluid. The hydrodynamic properties of this fluid are described and determined by Einstein's field equations. The best description of the properties of the galactic fluid is furnished by Friedmann's solution, where the thermodynamic properties of the fluid are described by a model in which the fluid is in equilibrium state. These two descriptions, the hydrodynamic and the thermodynamic, are in many aspects complementary and indpendent and allow us to equate the observational data.

In this work we treat the following two problems that derive from this standard model: (a) how the observed entropy per particle was produced and (b) how to avoid the singularity inherent in the standard model.

Solutions for these problems have been investigated in various papers.[1,2,3,4,5,6,7] In all these these works, the description of the thermodynamic properties of the fluid was modified by the introduction of heat production mechanisms (bulk and shear viscosities). A model for the thermodynamical properties of the fluid was set up using the principle of local equilibrium[8] and the standard equation of Gibbs-Duhem. The resulting theory which is not satisfactory for consistency reasons have been extensively examined[9,10]).

To avoid the inconsistencies of the thermodynamics of non-equilibrium processes based on the hypothesis of local equilibrium, new theories, known collectivelly as Causal Thermodynamics, were established[9,10,11].In these theories the principle of local equilibrium is abandoned and the dissipative variables are treated as independent dynamic variables, thereby avoiding the difficulties of the previous approaches.

In the present work we use a causal and covariant formulation of the dissipative processes[11]. This theory will be used to describe the representation the simple dissipative fluid which is the source of the curvature of the universe. We obtain and investigate exact solutions of the Einstein equations for this source which in turn is shown to be compatible with both the cosmological principle and observation.

II - FORMALISM

We start by contemplating the possibility that the source of our cosmological models is a dissipative fluid. Thus, as we will see, can be made compatible with the cosmological principle (of large-scale homogeneity) and with the observed high degree of isotropy. The absence of privileged directions in space reduces the stress-energy tensor of the fluid to be of the form

$$T_{\mu\nu} = \rho v_\mu v_\nu - (p+\pi) h_{\mu\nu} \qquad (1)$$

where π is the part of the isotropic pressure due to the bulk viscosity, ρ the energy density, and p the thermodynamic pressure.

Writing Einstein equations in terms of the kinematic parameters and of three-curvature of the hypersurface of simultaneity of the fluid, we obtain[16]

$$\dot{\theta} + \frac{\theta^2}{3} = -\frac{1}{2}\,[\rho + 3(p+\pi)] + \Lambda \tag{2}$$

$$\hat{R} = -\frac{2}{3}\,\theta^2 + 2\rho + 2\Lambda \tag{3}$$

$$\dot{\rho} = -\,(\rho+p+\pi)\,\theta \tag{4}$$

$$U_{\mu||\nu}U^{\nu} = a_{\mu} = 0 \tag{5}$$

where $\theta = V^{\alpha}{}_{||\alpha}$ is the expansion parameter, $\hat{R}$ is the three--curvature of the hypersurface of simultaneity associated with the fluid, a_{μ} the acceleration of the particles of the fluid, and Λ the cosmological constant.

The hydrodynamic description of the fluid must be completed by specifiyng its thermodynamic properties. Thus, we must complement the representation of the fluid with the entropy flux four vector s^{α} that describes the heat content of the fluid per unit volume and is determined by the generalized Gibbs-Duhem equation[11], the flux of particles per unit volume four vector N^{α} that determines the continuity equation, and the evolution equation for the bulk viscosity.For the models we are analyzing, which consider only first-order terms in the description of the viscosity, the equations that determine these objects reduce to the set

$$\tau_0\dot{\pi} + \pi = -\,\xi\theta \tag{6}$$

$$n\dot{s} = \frac{\pi^2}{\xi T} \tag{7}$$

$$\theta = -\,\frac{\dot{n}}{n} \tag{8}$$

where $n = U_{\alpha}W^{\alpha}$ is the number of particles per unit proper volume of the fluid, $s = U_{\alpha}S^{\alpha}$ is the specific entropy, ξ the bulk viscosity coefficient, and τ_0 the relaxation time associated with the viscous process.

To conveniently specify the source we must also define the equation of state for the viscous fluid, which we assume as

$$p = \lambda\rho \tag{9}$$

where λ is a parameter with values restricted by $0 \leq \lambda \leq 1$.

We now analyze two differente situations specified by different viscosity coefficients ξ : ξ = const. and ξ proportional to ρ . In both cases we take τ_0 = const. For simplicity, we restrict our analysis to the cases where $\dot{\hat{R}} = 0$. For the sake of clarity in the interpreation and to simplify the computations we define the Hubble constant as $H = \frac{1}{3}\theta$ and rewrite the dynamic equations fo the system using comoving and isotropic coordinates.

Case I ($\xi = \frac{2}{3}\alpha$)

Since in this case α is a constant, equations (2), (3), and (6) are written as

$$\rho = 3H^2 - \Lambda \tag{10}$$

$$p+\pi = \Lambda - 2\dot{H} - 3H^2 \tag{11}$$

$$\tau_0\ddot{H}+H[1+3\tau_0 H(\lambda+1)] + \frac{3}{2}(\lambda+1)H^2 - \alpha H - \frac{(\lambda+1)}{2}\Lambda = 0 \tag{12}$$

Equation (12), obtained from eq. (6) using the abovementioned specifications and from eqs. (9), (10), and (11), determines the scale function of the model R(t). Supposing that

$\dot{H} = \mathscr{H}(H)$ and changing the variables according to

$$y = \dot{H} \quad \text{and} \quad x = H \quad , \tag{13}$$

equation (12) can be written as

$$\tau_0 yy' + y[1+3\tau_0(\lambda+1)x] + \frac{3}{2}(\lambda+1)x^2 - \alpha x - \frac{(\lambda+1)}{2}\Lambda = 0 \tag{14}$$

where $y' = \frac{dy}{dx}$.

We use the power series method to integrate eq. (14); depending on the relations between the non null coefficients of the series expahsion we obtain two different solutions.

In the first case y(x) is given by

$$y(x) = \frac{1+4\alpha\tau_3}{12\tau_0(\lambda+1)} - \frac{x}{2\tau_0} \tag{15}$$

and the constants α, τ_0, λ and Λ are related by

$$\Lambda = \frac{1+4\alpha\tau_0}{12\tau_0{}^2(\lambda+1)^2} \tag{16}$$

In the second case, the cosntant α is necessarily zero and the model reduces to the ones already studied by Robertson, Walker, Friedmann, and DeSitter.

From (15) and (13) it follows that the scale parameter R(t) and Hubble's parameter for the new solution are given by

$$R(t) = R_0\, e^{2\tau_0(\lambda+1)\left[\Lambda t - \frac{C_1}{(\lambda+1)}\, e^{-t/2\tau_0}\right]} \tag{17}$$

$$H(t) = 2\tau_0\Lambda(\lambda+1) + C_1\, e^{-t/2\tau_0} \tag{18}$$

The production of entropy for this model is given by (7) and can be calculated once the temperature distribution is know. For the particular cases where $\lambda = \frac{1}{3}$ and $\lambda = 0$, the temperature is given respectively by[13]

$$T(t) = T_0 R^{-1} \tag{19}$$

and

$$T(t) = T_0 R^{-3} \tag{20}$$

In both cases $\dot{s} \to 0$ when $t \to -\infty$ and $\dot{s} \to \infty$ when $t \to \infty$. In these models the arrows of time determined by the expansion and by the production of entropy coincide; furthermore, the measured values for the matter density, Hubble constant and deceleration parameter can be used to determine the age of the universe, the integration constants C_1 and the cosmological constants Λ. The relaxation time τ_0 can then be determined by the measured value of the entropy per particle. This new result is a consequence of the use of the causal thermodynamics, since it introduces a new dynamic degree of freedom in the description of the dissipative fluid. The simplifications we have made, however, remove from this model the complexities of the real universe. The next model offers some improvements.

Case II ($\xi = \beta\rho$)

In a more realistic model, both the bulk viscosity coefficient ξ and the relaxation time τ_0 are functions of the internal energy and the specific volume of the fluid. In the models we study below keep on the relaxation time τ_0 constant and for the viscosity coefficient we assume the following expression:

$$\xi = \beta\rho \tag{20}$$

Here, following a procedure analogous to the one presented Case I, the equation that determines the dynamics of this new model is

$$\tau_0\ddot{H} + H[3\tau_0(\lambda+1)H+1] - \frac{1}{2} H^3\beta + \frac{3}{2} H'(\lambda+1)$$

$$+ \frac{3}{2} \beta H\Lambda - \frac{\Lambda}{2} (\lambda+1) = 0 \tag{21}$$

Maintaining as in Case I the same hypothesis $\dot{H} = \mathcal{H}(H)$ and using (1) again, we obtain

$$\tau_0 yy'+y[3\tau_0(\lambda+1)x+1] - \frac{9\beta}{2} x^3 + \frac{3}{2} (\lambda+1)x^2$$

$$+ \frac{3}{2} \beta\Lambda x - \frac{\Lambda}{2} (\lambda+1) = 0 \tag{22}$$

Using again the power series method to solve the differential equation, we find

$$y(x) = A_0 + A_1x + A_2x \tag{23}$$

where A_0, A_1 and A_2 are constant coefficient determined alternatively by the following two sets of algebraic relations:

$$\left.\begin{aligned} A_0 &= \frac{\Lambda}{4\eta} (\lambda+1)(3\eta+1) \\ A_1 &= -\frac{1}{\tau_0} \left(\frac{\eta+1}{3\eta+1}\right) \\ A_2 &= \frac{3}{4} (\lambda+1)(\eta-1) \end{aligned}\right\} \tag{24a}$$

$$\left.\begin{aligned} A_0 &= \frac{\Lambda}{4\eta}\,(\lambda+1)(3\eta-1) \\ A_1 &= -\,\frac{1}{\tau_0}\,\left(\frac{\eta-1}{3\eta-1}\right) \\ A_2 &= -\,\frac{3}{4}\,(\lambda+1)(\eta+1) \end{aligned}\right\} \qquad (24b)$$

where

$$\eta = \sqrt{1 + \frac{4\beta}{\tau_0^{\,2}(\lambda+1)^2}} \qquad (25)$$

These relations impose an interdependency between the constants β, λ, τ_0 and Λ for each separate set of relations. For the set (24a), it follows that Λ is determined by

$$\Lambda = \frac{16\eta^2}{3\tau_0^{\,2}(\lambda+1)^2(3\eta+1)^2(\eta+1)^2} \qquad (26)$$

and for (24b),

$$\Lambda = \frac{16\eta^2}{3\tau_0^{\,2}(3\eta-1)^2(\eta-1)^2(\lambda+1)^2} \qquad (27)$$

From (23) it follows directly that

$$t-t_0 = \frac{1}{A_2}\int \frac{dH}{(H + A_1/2A_2)^2 + \dfrac{4A_0A_2-A_1^{\,2}}{4A_1^{\,2}}} \qquad (28)$$

For the values of A_1, A_2 and A_3 determined by (24a), we have two different possibilities:

$$\frac{4A_0A_2 - A_1^{\,2}}{4A_2^{\,2}} = 0 \qquad (29a)$$

if $\eta = 2 + \sqrt{5}$, and

$$\frac{4A_0A_2 - A_1^2}{4A_2^2} < 0 \tag{29b}$$

for all $\eta > 1$ and $\eta \neq 2 + \sqrt{5}$.

For the values indicated by (29a), the functions H(t) and R(t) are determined by

$$H(t) = \frac{-1}{A_2 t} + \frac{|A_2|}{2A_2} \tag{30}$$

$$R(t) = \frac{R_0\, e^{\frac{|A_1|}{2A_2} t}}{(A_2 t)^{1/A_2}} \tag{31}$$

where R_0 is a constant of integration. The solution thus obtained has a singular point at $t = 0$.

The values indicated by (29b) determine however another solution with the functions H(t) and R(t) expressed by

$$H(t) = \frac{A_1}{\alpha A_2} - \sqrt{|B|}\ \text{tgh}\ (A_2\sqrt{|B|}\, t) \tag{32}$$

$$R(t) = R_0\, e^{\frac{|A_1|}{A_2} t}\ \text{sech}^{1/A_2}\ (\sqrt{|B|}\, A_2 t) \tag{33}$$

where R_0 is a constant of integration and B is given by

$$B = \frac{4A_0A_2 - A_1^2}{4A_2^2} \tag{34}$$

The function H satisfies the restriction

$$\left|\frac{H + A_1/2A_1}{\sqrt{|B|}}\right| < 1 \ .$$

This solution is non-singular.

The set of values determined by (24b) furnishes another solution. It is directly verified that in this case $G = 4A_2A_0 - A_1^2/4A_2^2$ is always negative and $\Lambda > 0$. In this case $H(t)$ and $R(t)$ are determined by

$$H(t) = \sqrt{|G|}\ \text{cotgh}\,(\sqrt{|G|}\ |A_2|t) - \frac{|A_1|}{2|A_2|} \tag{35}$$

$$R(t) = R_0\ e^{-\frac{A_1}{2A_2}t}\ \text{senh}^{1/|A_2|}\,(\sqrt{|G|}\ A_2 t) \tag{36}$$

where R_0 is a constant of integration and $H(t)$ is restricted by the inequality $\left|\frac{A + A_1/A_2}{\sqrt{|G|}}\right| > 1.$

We have obtained three different solutions, one non--singular and two singular, which we represent graphically below:

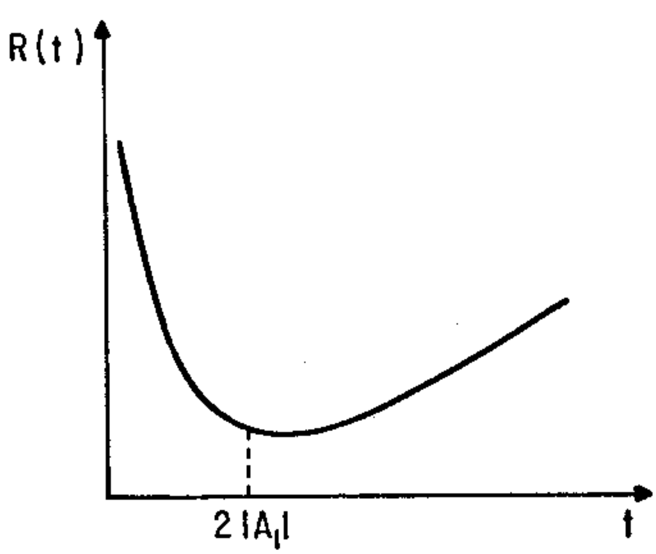

FIG.1

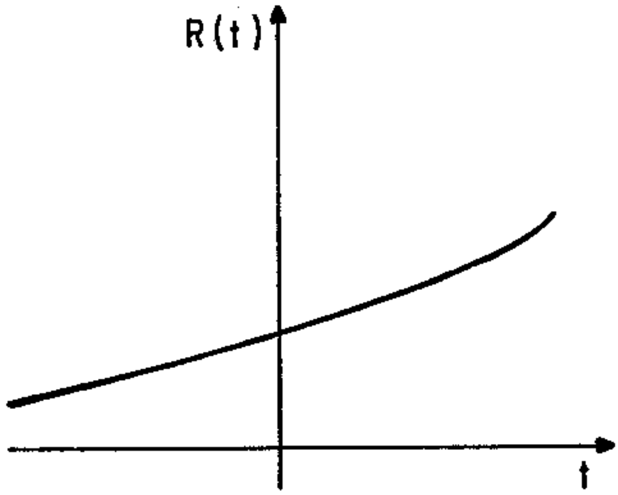

FIG.2

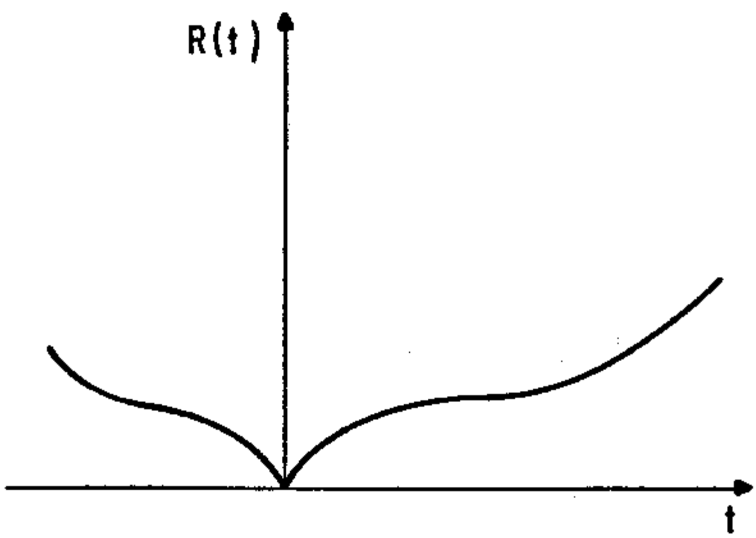

FIG.3

Following the same procedure as in Case I, we can compute the production of entropy for the solutions we obtained. As we shall see below, the first solution does not satisfy the weak energy condition, and therefore should not be considered a physical solution. The third solution has a singularity for $t = 0$ and violates the weak energy condition; therefore it should not be considered either. For the solution (33), supposing that the temperature is determined by (19) and (20), we can compute the production of entropy, and the result is such that $\dot{s} \to 0$ when $t \to -\infty$ and $\dot{s} \to \infty$ when $r \to \infty$. This result shows that, conveniently adjusting the free parameters of this solution, it is possible to make it reproduce the observational data for ρ, H, and the entropy per particle s.

III - ENERGY CONDITIONS

It has been sugested that every solution associated with a physical system must necessarily satisfy the strong and the weak energy conditions[14]. The weak energy condition is expressed by

$$T^{\mu\nu}V_{\mu}V_{\nu} \geq 0 \quad , \tag{37}$$

for an arbitrary time-like vector V^{μ}.

The models represented by solutions (17) and (33) satisfy this condition for all values of t. In solution (33), this condition is satisfied because η is restricted to the interval $1 < \eta \leq 4$. The models represented by the solutions

(31) and (36) do not satisfy this condition for certain time intervals.

The strong energy condition establishes that the energy flux is also timelike. For the source of the models we have studied, this condition is written as $\rho \geq |\rho+\pi|$. Using the field equations, it can be equivalently rewritten as
$-\rho \leq \dot{H}(t) \leq$.

The solutions expressed by (17) and (33) satisfy this relation only for $1 < \eta \leq 4$.

Below we give graphic representations of the matter-energy density $\rho(t)$ for the solutions that satisfy both energy conditions.

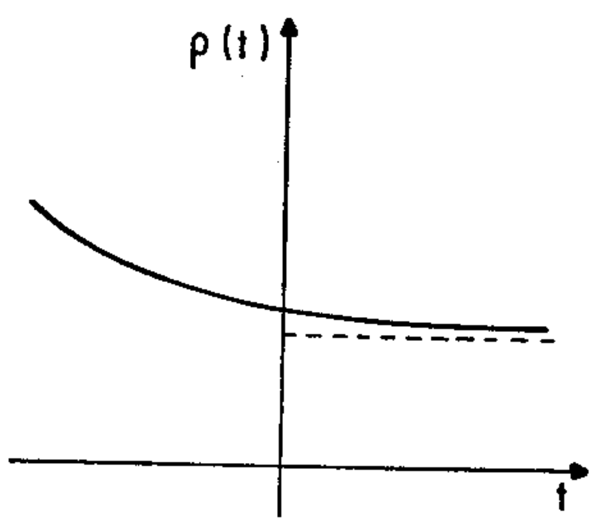

FIG. 4

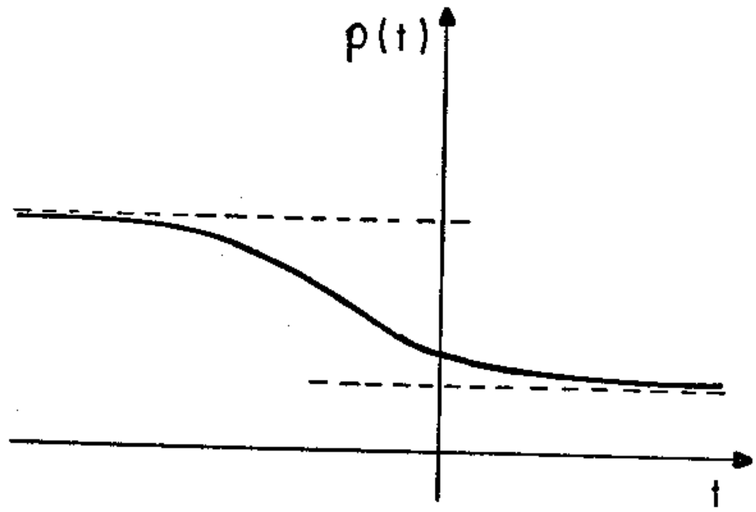

FIG.5

In Fig. 4 we observe that, although the expansion is not null, ρ is asymptotically constant. This can be explained if we interpret the bulk viscosity as the macroscopic phenomenological description of the microscopic phenomenon of particle production by the gravitational field[15]. In this case the singularity is removed to $t = -\infty$.

In Fig. 5 we verify that the density ρ is asymptotically constant in both directions of the t axis and the solution is non-singular.

IV - CONCLUSION

In this work, without deviating from a purely classical context, we have tried to find a way to solve some of the problems inherent in the standard model, especially the initial singularity and the high entropy per particle. We have closely followed other attempts, as in Murphy[7], Heller[5] and Novello[15]. The basic difference we have introduced in our model concerns the thermodynamic properties of the source. To describe the source, we have used causal thermodynamics, and the resulting model is different from its predecessors in that it has an additional dynamic variable. This additional dynamic degree of freedom leads to a differential equation of the third order for the scale factor R(t) that determines the geometry of the model, while in all the previous works the differential equations obtained are always of the second order. The conclusions of Pavon and Fustero[12] concerning the relaxation time and the production of entropy in a cosmological model do not hold, since we have used a different background. Two of the solutions we have obtained, both compatible with dust ($\lambda = 0$) and radiation ($\lambda = 1/3$), are non--singular and explain the origin of the observed high entropy per particle. With this extremely simplified and abstract model, we do not intend to describe the real process to which the material content of the universe is subject. We only intend to show that, from a purely classical point of view, taking into account only the bulk viscosity of the fluid, it is possible, without violating any physical law, to set up a model for the universe which is both compatible with the observation and non-singular.

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