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SUPERSYMMETRIC GAUGE INVARIANT INTERACTION REVISITED

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Abstract

A supersymmetric Lagrangian invariant under local U(1) gauge transformations is written in terms of a non-chiral superfield which substitute the usual vector supermultiplet together with chiral and anti-chiral superfields. The Euler equations allow us to obtain the off-shell version of the usual Lagrangian for supersymmetric quantum-electrodynamics (SQED).

Key-words: Supersymmetric QED.

1. Introduction

The superfields introduced by Salam and Strathdee [1] provide an elegant and compact description of supersymmetry representation. They are defined over the eight-dimensional space whose points z^M are represented by $(x^m, \theta^\mu, \bar{\theta}^{\dot{\mu}})$ where $x^m (m=0,1,2,3)$ denotes the usual space-time coordinates and the Weyl spinors θ^μ, $\bar{\theta}^{\dot{\mu}}$ are anticommuting Grassmann's variables with $\mu, \dot{\mu} = 1,2$. We are going to use the same notations and conventions of reference [2].

Superfields have a general power series expansion in θ and $\bar{\theta}$ given by

$$F(x,\theta,\bar{\theta}) = f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \tag{1.1}$$

$$+ \bar{\theta}\bar{\theta}n(x) + \theta\sigma^m\bar{\theta}v_m + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta}d(x)$$

and transforms as

$$\delta F = (\xi Q + \bar{\xi}\bar{Q})\ F \tag{1.2}$$

under a supersymmetry transformation with parameters ξ^α, $\bar{\xi}_{\dot{\alpha}}$, where Q_α, $\bar{Q}^{\dot{\alpha}}$ are the differential operators

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^m_{\alpha\dot{\beta}}\,\bar{\theta}^{\dot{\beta}}\,\partial/\partial x^m \tag{1.3}$$

$$\bar{Q}^{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - i\theta^\alpha\sigma^m_{\alpha\dot{\beta}}\,\varepsilon^{\dot{\beta}\dot{\alpha}}\partial/\partial x^m$$

Usually some constraints are introduced on superfields and the most common ones are

$$\bar{D}_{\dot{\alpha}}\phi = 0 \tag{1.4}$$

$$D_{\alpha}\phi^{+} = 0 \tag{1.5}$$

$$V^{+} = V \tag{1.6}$$

where D_α and $\bar{D}_{\dot{\alpha}}$ are the usual covariant derivatives:

$$D_{\alpha} = \frac{\partial}{\partial\theta^{\alpha}} + i\sigma^{m}_{\alpha\dot{\beta}}\,\bar{\theta}^{\dot{\beta}}\,\frac{\partial}{\partial x^{m}} \tag{1.7}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^{\beta}\sigma^{m}_{\beta\dot{\alpha}}\,\frac{\partial}{\partial x^{m}} \tag{1.8}$$

ϕ, ϕ^+ and V are called chiral, anti-chiral and vector superfields, respectively, and they have been used to construct supersymmetric gauge invariant Lagrangians [2,3].

Projection operators P_1, P_2 [4] can be introduced with the following properties

$$P_1\phi^{+} = \phi^{+} \tag{1.9}$$

$$P_2\phi = \phi \tag{1.10}$$

$$P_1\phi = P_2\phi^{+} = 0 \tag{1.11}$$

Explicitly they are given by

$$P_1 = \frac{D^2\bar{D}^2}{16\Box} \tag{1.12}$$

$$P_2 = \frac{\bar{D}^2 D^2}{16\,\Box} \tag{1.13}$$

But, as we can see, these operators do not sum to identity. There is another operator P_3 so that

$$P_1 + P_2 + P_3 = 1\,, \tag{1.14}$$

which is given by

$$P_3 = -\frac{D^\alpha \bar{D}^2 D_\alpha}{8\,\Box} \tag{1.15}$$

It is a matter of algebraic calculations to find a superfield ϕ_{NC} (called non-chiral) with the following constraints:

$$P_3 \phi_{NC} = \phi_{NC}\,, \tag{1.16}$$

$$P_1 \phi_{NC} = P_2 \phi_{NC} = 0 \tag{1.17}$$

This can be done by applying the operator P_3 on a general superfield like the one given by (1.1).

So ϕ, ϕ^+ and ϕ_{NC} form a complet set of superfields [5] whose power series expansion in θ, $\bar{\theta}$ we write as

$$\phi(x,\theta,\bar{\theta}) = A(x) + \sqrt{2}\,\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^m\bar{\theta}\partial_m A(x) +$$

$$+ \frac{i}{\sqrt{2}}\,\theta\theta\bar{\theta}\bar{\sigma}^m\partial_m\psi(x) + \frac{1}{4}\,\theta\theta\bar{\theta}\bar{\theta}\,\Box A(x) \tag{1.18}$$

$$\phi^+(x,\theta,\bar\theta) = A^*(x) + \sqrt{2}\,\bar\theta\bar\psi(x) + \bar\theta\bar\theta F^*(x) - i\theta\sigma^m\bar\theta\partial_m A^*(x) +$$

$$+ \frac{i}{\sqrt{2}}\,\bar\theta\bar\theta\theta\sigma^m\partial_m\bar\psi(x) + \frac{1}{4}\,\theta\theta\bar\theta\bar\theta\,\Box A^*(x) \qquad (1.19)$$

$$\phi_{NC}(x,\theta,\bar\theta) = C(x) + \theta\chi(x) + \bar\theta\bar\chi(x) + \theta\sigma^m\bar\theta v_m(x) -$$

$$-\frac{i}{2}\,\theta\theta\bar\theta\bar\sigma^m\partial_m\chi(x) - \frac{i}{2}\,\bar\theta\bar\theta\theta\sigma^m\partial_m\bar\chi(x) - \frac{1}{4}\,\theta\theta\bar\theta\bar\theta\,\Box C(x) \qquad (1.20)$$

As we observe the non-chiral superfield contains among its components the vector field v_m and satisfies the same constraint relation (1.6) of a vector superfield V. The superfield ϕ_{NC} is simpler than V[2] but here v_m satisfies the equation $\partial^m v_m = 0$. This will not be a problem since we will use a gauge transformation for this supermultiplet and then the new vector field v'_m will no longer satisfy this constraint.

The purpose of this work is to present a general supersymmetric Lagrangian invariant under an Abelian gauge transformation, using the ϕ_{NC} superfield, together with chiral and anti-chiral superfields. This is done in the next section. We will see that using the Wess-Zumino gauge and the Euler equations for the auxiliary fields of the chiral and anti-chiral superfields one can recover the off-shell version of the usual Lagrangian for supersymmetric quantum electrodynamics (SQED) [2].

2. Gauge invariant interaction

We can write the following supersymmetric interaction invariant under local U(1) gauge transformations,

$$S = \int d^4x\, d^2\theta\, d^2\bar\theta\, [\frac{1}{4}W^\alpha W_\alpha\,\delta(\bar\theta) + \frac{1}{4}\,\bar W_{\dot\alpha}\bar W^{\dot\alpha}\,\delta(\theta) + \phi_1^+ e^{g\phi_{NC}}\phi_1 + \phi_2^+ e^{-g\phi_{NC}}\phi_2 +$$

$$+ m(\phi_1\phi_2\,\delta(\bar\theta) + \phi_1^+\phi_2^+\,\delta(\theta)) + \xi\phi_{NC}] \qquad (2.1)$$

where

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha \phi_{NC} \tag{2.2}$$

$$\bar{W}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}\phi_{NC} \tag{2.3}$$

$$\bar{D}_{\dot{\alpha}}\phi_1 = 0 \tag{2.4}$$

$$\bar{D}_{\dot{\alpha}}\phi_2 = 0 \tag{2.5}$$

and

$$\delta(\theta) = \theta^\alpha\theta_\alpha$$

$$\delta(\bar{\theta}) = \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} \tag{2.6}$$

if one assigns the following transformation law for our superfields

$$\phi_1' = e^{-g\Lambda}\phi_1 \tag{2.7}$$

$$\phi_2' = e^{g\Lambda}\phi_2 \tag{2.8}$$

$$\phi_{NC}' = \phi_{NC} + (\Lambda+\Lambda^+) \tag{2.9}$$

where Λ and Λ^+ are chiral and anti-chiral superfields respectively, i.e.,

$$\bar{D}_{\dot{\alpha}}\Lambda^+ = 0 \tag{2.10}$$

$$D_\alpha \Lambda^+ = 0 \tag{2.11}$$

The term $\xi\phi_{NC}$ allow us to breack supersymmetry spontaneously through the Fayet-Iliopoulos mechanism [6].

One can also add the term $-\frac{1}{8\alpha}\int d^2\theta d^2\bar\theta(\bar D^2\phi_{NC})(D^2\phi_{NC})$ which yields a piece proportional to $(\partial_m v^{m'})^2$ that is the usual covariant gauge fixing term for the v'_m field.

The transformation (2.9) in terms of component fields reads

$$\begin{aligned}\phi'_{NC} = {} & (C+B+B^*) + \theta(\chi+\sqrt{2}\,\phi) + \bar\theta(\bar\chi+\sqrt{2}\,\bar\phi) + \\ & + \theta\theta M + \bar\theta\bar\theta M^* + \theta\sigma^m\bar\theta[v_m + i\partial_m(B-B^*)] + \\ & - \frac{i}{2}\theta\theta\bar\theta\bar\sigma^m\partial_m(\chi-\sqrt{2}\,\phi) - \frac{i}{2}\bar\theta\bar\theta\theta\sigma^m\partial_m(\bar\chi-\sqrt{2}\,\bar\phi) + \\ & - \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\Box\,(C-B-B^*)\end{aligned} \tag{2.12}$$

We can choose B(x) and $\phi(x)$ so that the first three terms of (2.12) are gauged away (Wess-Zumino gauge).

Redefining the component fields we have in this gauge

$$\begin{aligned}\phi'_{NC} = {} & \theta\sigma^m\bar\theta v'_m(x) + i\theta\theta\bar\theta\bar\lambda(x) - i\bar\theta\bar\theta\theta\lambda(x) + \\ & + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x) + \theta\theta M(x) + \bar\theta\bar\theta M^*(x)\end{aligned} \tag{2.13}$$

where $v'_m = v_m + i\,\partial_m(B-B^*)$

The difference between ϕ'_{NC} and the usual superfield V is the presence of the terms $\theta^2 M$ and $\bar\theta^2 M^*$.

The powers of ϕ_{NC} in this gauge are:

$$\phi'^{\,2}_{NC} = \theta\theta\bar{\theta}\bar{\theta}(MM^* - \frac{1}{2}\, v'_m v^{m'}) \qquad (2.14)$$

$$\phi'^{\,3}_{NC} = 0 \qquad (2.15)$$

It is easy to check that the terms $\theta^2 M$ and $\bar{\theta}^2 M$ do not contribute in the kinetic part of the Lagrangian.

In components, the action (2.1) becomes

$$\begin{aligned} S = \int d^4x\,[\, & \frac{D^2}{2} - \frac{1}{4}\, v^{mn'} v'_{mn} - i\lambda\sigma^m\partial_m\bar{\lambda} \; + \\ & + F_1F_1^* + F_2F_2^* + A_1^*\Box A_1 + A_2^*\Box A_2 \; + \\ & + i(\partial_n\bar{\psi}_1\bar{\sigma}^n\psi_1 + \partial_n\bar{\psi}_2\bar{\sigma}^n\psi_2) \; + \\ & + g v^{m'}(\frac{1}{2}\bar{\psi}_1\bar{\sigma}_m\psi_1 - \frac{1}{2}\bar{\psi}_2\bar{\sigma}_m\psi_2 + \frac{i}{2}A_1^*\partial_m A_1 \; + \\ & - \frac{i}{2}\partial_m A_1^* A_1 - \frac{i}{2}A_2^*\partial_m A_2 + \frac{i}{2}\partial_m A_2^* A_2) \; + \\ & - \frac{ig}{\sqrt{2}}(A_1\bar{\psi}_1\bar{\lambda} - A_1^*\psi_1\lambda - A_2\bar{\psi}_2\bar{\lambda} + A_2^*\psi_2\lambda) \; + \\ & \frac{g}{2} D(A_1^*A_1 - A_2^*A_2) \; + \\ & + g^2(\frac{1}{2}MM^* - \frac{1}{4}v'_m v^{m'})(A_1^*A_1 + A_2^*A_2) \; + \\ & + g(F_1^* M A_1 + A_1^* M^* F_1 - F_2^* M A_2 - A_2^* M^* F_2) \; + \\ & + m(A_1F_2 + A_2F_1 - \psi_1\psi_2 - \bar{\psi}_1\bar{\psi}_2 + A_1^*F_2^* + A_2^*F_1^*) + \frac{1}{2}\xi D] \end{aligned}$$

(2.16)

where $v'_{mn} = \partial_m v'_n - \partial_n v'_m$.

This expression is equivalent to that of reference [2] for supersymmetric quantum electrodynamics.

The Euler equations for the auxiliary fields F_1, F_2 and M are:

$$F_1 + g A_1 M + m A_2^* = 0 \tag{2.17}$$

$$F_2 - g A_2 M + m A_1^* = 0 \tag{2.18}$$

$$\frac{1}{2} g^2 M (A_1^* A_1 + A_2^* A_2) + g A_1^* F_1 - g A_2^* F_2 = 0 \tag{2.19}$$

The substitution of equations (2.17) and (2.18) into equation (2.19) yields that $M = 0$. So M can be eliminate using the equations of motion for F_1 and F_2 while this degree of freedom is used to eliminate the component field which is the coefficient of $\theta\theta$ in the supermultiplet V.

Then setting $M = 0$ we can write

$$\begin{aligned} S = \int d^4x [& \frac{D^2}{2} - \frac{1}{4} v^{mn'} v'_{mn} - i\lambda\sigma^m \partial_m \bar{\lambda} + \\ & + A_1^* \Box A_1 + A_2^* \Box A_2 + F_1 F_1^* + F_2 F_2^* + i [\partial_m \bar{\psi}_1 \bar{\sigma}^m \psi_1 + \\ & + \partial_m \bar{\psi}_2 \bar{\sigma}^m \psi_2] + g v^{m'} [\frac{1}{2} \bar{\psi}_1 \bar{\sigma}_m \psi_1 + \\ & - \frac{1}{2} \bar{\psi}_2 \bar{\sigma}_m \psi_2 + \frac{i}{2} A_1^* \partial_m A_1 - \frac{i}{2} \partial_m A_1^* A_1 + \\ & - \frac{i}{2} A_2^* \partial_m A_2 + \frac{i}{2} \partial_m A_2^* A_2] + \\ & - \frac{ig}{\sqrt{2}} (A_1 \bar{\psi}_1 \bar{\lambda} - A_1^* \psi_1 \lambda - A_2 \bar{\psi}_2 \bar{\lambda} + A_2^* \psi_2 \lambda) + \\ & + \frac{gD}{2} (A_1^* A_1 - A_2^* A_2) - \frac{1}{4} g^2 v'_m v^{m'} (A_1^* A_1^* + A_2^* A_2^*) + \\ & + m (A_1 F_2 + A_2 F_1 - \psi_1 \psi_2 - \bar{\psi}_1 \bar{\psi}_2 + A_1^* F_2^* + A_2^* F_1^*) + \frac{1}{2} \xi D] \end{aligned} \tag{2.20}$$

This expression is the same of that of reference [2].

Then we have obtained a formulation of SQED using chiral, anti-chiral and non-chiral superfields which belong to spaces [5] spanned by the superfields $F_1(x,\theta,\bar{\theta}) = P_1F(x,\theta,\bar{\theta})$, $F_2(x,\theta,\bar{\theta}) = P_2F(x,\theta,\bar{\theta})$ and $F_3(x,\theta,\bar{\theta}) = P_3F(x,\theta,\bar{\theta})$ respectively and where $F(x,\theta,\bar{\theta})$ is the most general expression for a superfield (eq.(1.1)).

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References

[1] A. Salam and J. Strathdee, Phys. Rev. D11 (1975) 1521;

[2] J. Wess, "Supersymmetry and Supergravity", published by the Princeton University Press;

[3] P. Fayet and S. Ferrara, Physics Report 32C, (1977) 249;

[4] M.T. Grisaru, M. Roček and W. Siegel, Nucl. Phys. B159 (1979) 429;

[5] Susumu Okubo, to appear in the Proceedings of "Meeting on High Energy Theory", May 9-10, 1983, at University of Toronto.

[6] P. Fayet and J. Iliopoulos, Phys. Lett. 51B (1974) 461.