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ASYMPTOTIC BEHAVIOUR OF PHYSICAL AMPLITUDES IN A FINITE FIELD THEORY

by

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ABSTRACT

Using the N=4 super-Yang-Mills theory softly broken by supersymmetric N=1 mass terms for the matter superfields, we compute the one-loop chiral + chiral → antichiral + antichiral scattering amplitude directly in superspace. By suitable choices of the mass parameters, one can endow the model with a hierarchy of light and heavy particles, and the decoupling of the heavy sector from the light-light physical amplitude is studied. We also analyze the high-energy limit of the cross-section for a two physical scalar scattering and find a (logs) behaviour, which then respects the Froissart bound.

Key-words: Finiteness; High-energy behaviour; Supersymmetric gauge theories.

1. INTRODUCTION

Renormalization of wave functions, masses and coupling constants by the ultraviolet infinities encountered in perturbative calculations in quantum field theory has always been an enigma. With the advent of supersymmetry this situation has changed for the better. A large number of supersymmetric quantum field theories that are free from ultraviolet divergences have been discovered. The first to be proved ultraviolet finite to all orders in perturbation theory was the N=4 super-Yang-Mills theory [1]. The gauge and chiral matter superfields are in the adjoint representation of the gauge group G. Another class of finite field theories are theories with N=2 supersymmetry [2]. The matter representations (R) are chosen to satisfy the group theoretic relation

$$C_2(G) - \sum_R C_2(R) = 0$$

where G is the gauge group and C_2 is the second Dynkin index. Finally the N=3 super-Yang-Mills theory [3] was also shown to be finite with the heop of harmonic superspace techniques [4]. It has been proved further that the N=4 and N=2 finite theories remain ultraviolet finite in the presence of explicit mass terms for the fermions and scalars of the theory. These mass terms are required to satisfy the mass relation

$$\sum_{s=0,\frac{1}{2}} (-1)^{2s+1} \ (2s+1) \ m_s^2 = 0$$

Depending on whether the gauginos have mass terms or not, the N=4 and N=2 finite theories have either no supersymmetry or N=1 supersymmetry.

None of the aforementioned finite theories contain gravity. More recently it has been demonstrated that ten--dimensional local field theories for extended one-dimensional objects are very likely to be another example of ultraviolet finite field theories with the important feature of containing the gravitational interaction. These theories are the type I (N=1) superstrings [5] and the heterotic strings [6]. One important aspect of this development is that consistency of the theory demands the gauge group to be either SO(32) or $E_8 \times E_8$. It is interesting to note that the N=4 super-Yang-Mills theory emerges from the open type I superstring as an effective theory in the limit of zero slope and zero radius of compactification of the extra six dimensions. In this limit all the massive models of the string acquire infinite masses.

Just like in ordinary quantum field theory, it is of interest to calculate physical quantities in these ultraviolet finite field theories. In conventional field theories, bare masses and couplings are not physically measurable quantities. In ultraviolet finite theories, bare masses and bare couplings are in principle measurable. Quantum corrections give finite renormalizations of the bare quantities. Inherent in these corrections are energy scale at which new physics occurs. The energy scale isolate the theory into an "effective" theory decoupled from new physics at low energies. This and related issues are discussed in what follows. As a specific example we take the N=4 super-Yang-Mills theory with soft breaking N=1 supersymmetric mass terms for the matter multiplets. We calculate one loop corrected two particle scattering amplitudes in the

N=4 theory. This is done through the use of superfields from which all two particle scattering amplitudes are extracted. In the past, superfields have only been employed to discuss the formal aspects of field theory e.g. renormalization. Considering the number of diagrams to be evaluated in component form for two particle scattering processes, the merits of using superfields for calculating physical processes becomes only too transparent. We start with the superfield scattering $\phi\phi \to \bar{\phi}\bar{\phi}$ in the preceeding section.

The outline of our work is as follows. In Section 2, we organize and compute all contributions to the $\phi\phi \to \bar{\phi}\bar{\phi}$ scattering while working in superspace. Our results and their consequences for the decoupling of the heavy sector and the asymptotic hehaviour of cross-sections are the subject of Section 3.

2. SUPERSPACE CALCULATION OF $\phi\phi \to \bar{\phi}\bar{\phi}$ SCATTERING

The action of the N=4 super-Yang-Mills theory in terms of N=1 superfields ϕ_i and V in the adjoint representation of the gauge group G is:

$$S = \mathrm{tr}\Big[\int d^4x\, d^4\theta\, e^{-gV}\, \bar{\phi}_i\, e^{gV}\, \phi_i \; +$$

$$+ \frac{1}{64g^2}\int d^4x\, d^4\theta\, W^{\alpha}W_{\alpha} \; +$$

$$+ \frac{ig}{3!}\int d^4x\, \varepsilon_{ijk}(d^2\theta\phi_i[\phi_j,\phi_k] \; +$$

$$+ d^2\bar{\theta}\ \bar{\phi}_i[\bar{\phi}_j,\bar{\phi}_k]))] \quad , \tag{2.1}$$

where

$$W_\alpha \equiv D^{-2}(e^{-gV} D_\alpha e^{gV}).$$

To the action (2.1), we add the following gauge invariant N=1 mass terms for the chiral superfields ϕ_i:

$$S_m = -\frac{1}{2}\mathrm{tr}\,[\int d^4x\ d^2\theta\ \phi_i\ m_{ij}\ \phi_j + h.c.] \tag{2.2}$$

In the presence of these mass terms the N=4 theory possesses a residual N=1 supersymmetry and remains finite. This method ensures that the finiteness condition relating masses is satisfied, i.e.,

$$\sum_{\substack{\text{all}\\ \text{scalars}}} m^2_{\text{scalars}} = 2 \sum_{\substack{\text{all}\\ \text{fermions}}} m^2_{\text{fermions}} \tag{2.3}$$

Without loss of generality the mass matrix m_{ij} in eq. (2.2) is taken to be diagonal,

$$m_{ij} = m_i\delta_{ij} \tag{2.4}$$

Our rotation and conventions are the same as in ref. [7]. The Feyman rules we use are given in ref. [8].

The tree-level diagrams describing the superfield scattering are shown in figure 1.

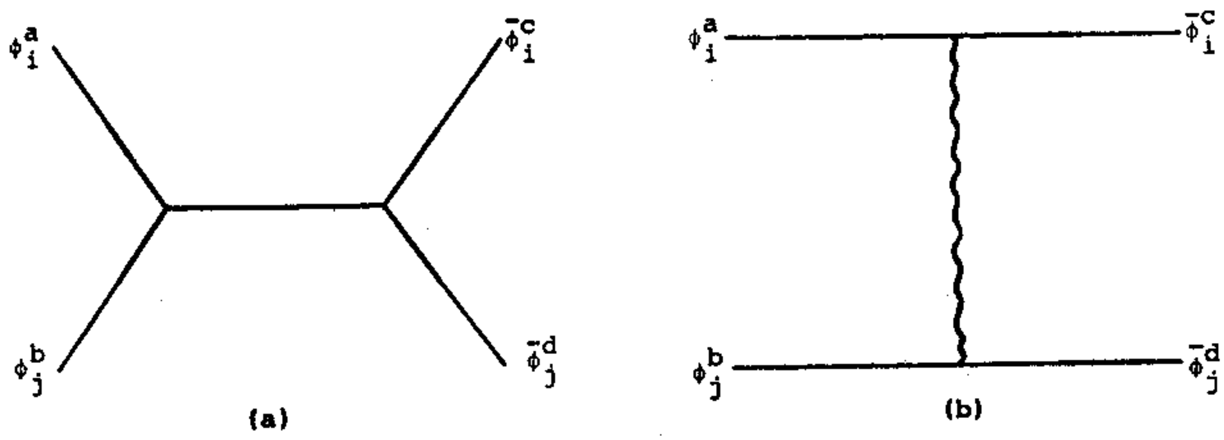


Fig. 1

Tree-level graphs describing the $\phi\phi \rightarrow \bar{\phi}\bar{\phi}$ scattering.

As in ref. [9], we also consider the case in which all particles are distinct, simply to avoid introducing additional crossed diagrams in the calculation; considering them would not change our final conclusions. Next, we draw all the Feynman graphs that contribute to the scattering we are going to study. They include one-particle irreducible diagrams and also graphs which contain one loop corrections to propagators and vertices. The introduction of masses brings new graphs contributing to a particular Green function, as new propagators, namely $\langle T\phi\phi\rangle$ and $\langle T\bar{\phi}\bar{\phi}\rangle$, show up with respect to the massless case. Though some of the new graphs may individually diverge, the overall contribution to the Green's function under consideration is finite. These new graphs can be easily individuated from our answers, as they carry multiplicative mass factors in the amplitudes. The latter have been considered in ref. [10].

The one-loop corrections to the diagrams in fig. 1 can be divided into the following parts:

(i) one-loop self-energy insertions for each chiral and anti-chiral superfield in the diagrams of fig. 1. The diagrams contributing to the one-loop self-energy of ϕ are shown in figs. 2 and 3.

Fig. 2
One-loop corrections to the $\Phi\Phi$ - propagators.

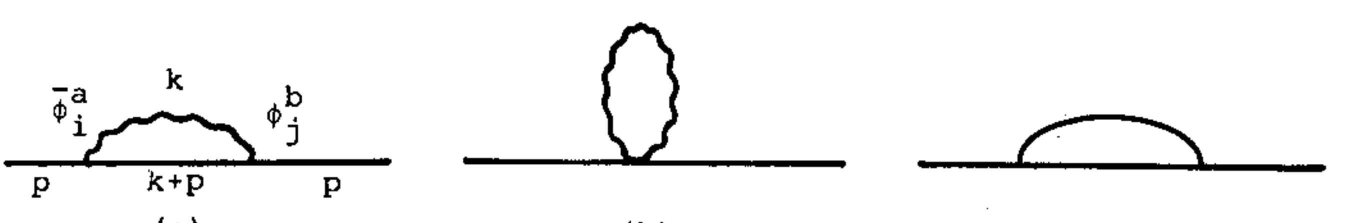


Fig. 3
One-loop corrections to the $\bar{\Phi}\Phi$ - propagators.

The contributions of fig. 2 vanish due to the algebra of the covariant derivatives. The contributions of fig. 3 are:

$$- \mu^{\varepsilon} g^2 \; C_2(G)\delta_{ab}\delta_{ij}\int \frac{d^4p}{(2\pi)^4}\int \frac{d^Dk}{(2\pi)^D}\,\frac{1}{k^2}\,\frac{1}{(k+p)^2+m_i^2} \cdot \tag{2.5}$$

$$\cdot \int d^4\theta \; \overline{\Phi_i^a}(p;\theta,\bar\theta)\;\Phi_j^b(p;\theta,\bar\theta), \quad \text{for fig. (3a)},$$

zero (due to the dimensional regularization scheme) for fig. 3b) and

$$\frac{1}{2}\,\mu^{\varepsilon} g^2 \; C_2(G)\delta_{ab}\delta_{ij}\int \frac{d^4p}{(2\pi)^4}\int \frac{d^Dk}{(2\pi)^D}\Big[\frac{1}{k^2+m_k^2}\,\frac{1}{(k+p)^2+m_\ell^2} + \tag{2.6}$$

$$+ \frac{1}{k^2+m_\ell^2}\cdot\frac{1}{(k+p)^2+m_k^2}\Big]\int d^4\theta\,\overline{\Phi_i^a}(p;\theta,\bar\theta)\;\Phi_j^b(p;\theta,\bar\theta),$$

with $k \neq \ell \neq i$, for fig. (3c).

In expressions (2.5) and (2.6), $C_2(G)$ is a Casimir coefficient defined in terms of the group structure constants so that

$$f_{amn} f_{bmn} = C_2(G)\,\delta_{ab} \tag{2.7}$$

and μ is an arbitrary parameter with the dimensions of mass which has been introduced to keep the coupling constant, G, dimensionless.

(ii) one-loop self-energy insertions for the vector superfield V in fig. 1. The diagrams contributing to the one-loop self-energy of V are shown in fig. 4.

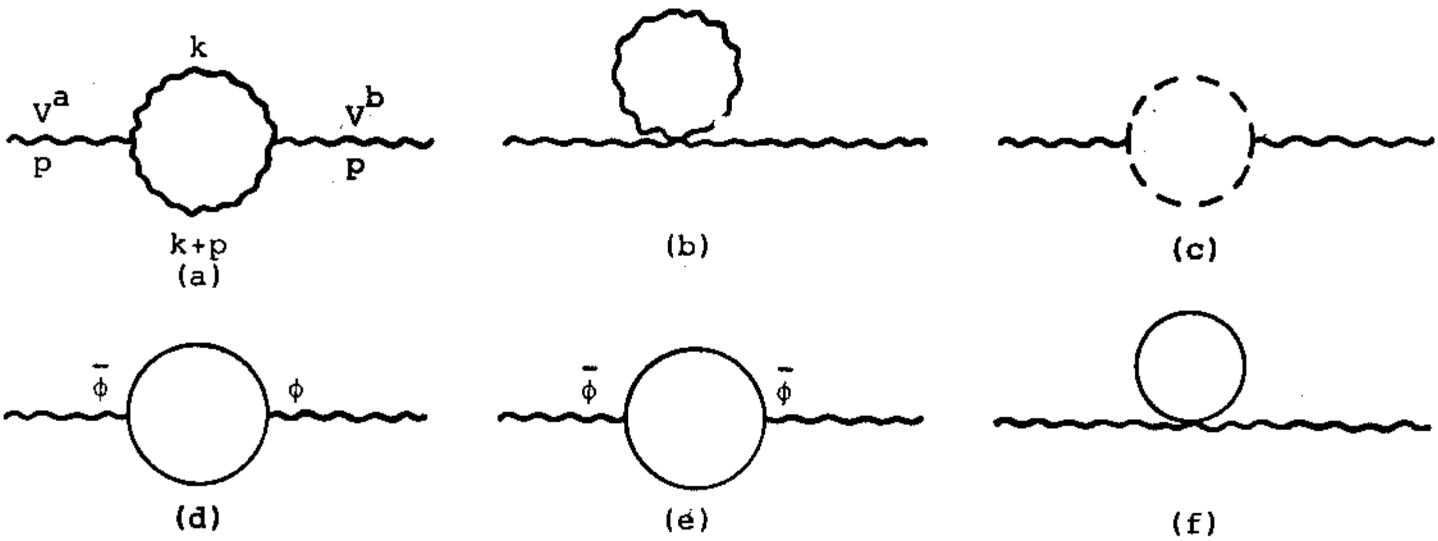


Fig. 4
One-loop vector superfield self-energy diagrams.

Their contributions are:

$$\frac{1}{2}\,\mu^{\varepsilon} g^2 C_2(G)\,\delta_{ab}\int\frac{d^4p}{(2\pi)^4}\int\frac{d^Dk}{(2\pi)^D}\,\frac{1}{k^2}\,\frac{1}{(k+p)^2}\int d^2\theta\; V^a(-p,\theta,\bar{\theta})\;\cdot$$

$$\cdot\;\left(-\frac{5}{2}\,\pi_{1/2}+\frac{1}{2}\,\pi_0\right)V^b(p,\theta,\bar{\theta}) \qquad (2.8)$$

for fig. (4a), zero for fig. (4b),

$$\frac{1}{2}\,\mu^{\varepsilon} g^2 C_2(G)\,\delta_{ab}\int\frac{d^4p}{(2\pi)^4}\int\frac{d^Dk}{(2\pi)^D}\,\frac{1}{k^2}\,\frac{1}{(k+p)^2}\int d^2\theta V^a(-p,\theta,\bar{\theta})\;\cdot$$

$$\cdot\;\left(-\frac{1}{2}\,\pi_{1/2}-\frac{1}{2}\,\pi_0\right)V^b(p,\theta,\bar{\theta}) \qquad (2.9)$$

for fig. (4c), where the projection operators π_0 and $\pi_{1/2}$ are

$$\pi_0 = \frac{1}{16\Box}(D^2\bar{D}^2 + \bar{D}^2D^2) \tag{2.10}$$

and

$$\pi_{1/2} = -\frac{1}{8\Box} D^{\alpha}\bar{D}^2 D_{\alpha} \quad , \tag{2.11}$$

$$\mu^{\varepsilon}g^2C_2(G)\delta_{ab}\int\frac{d^4p}{(2\pi)^4}\frac{d^Dk}{(2\pi)^D}\sum_i\frac{1}{k^2+m_i^2}\frac{1}{(k+p)^2+m_i^2}\int d^4\theta \quad .$$

$$V^a(-p;\theta,\bar{\theta})[-k^2+\frac{1}{2}k^{\alpha\dot{\beta}}D_{\alpha}(p)\bar{D}_{\dot{\beta}}(p)+\frac{1}{16}D^2(p)\bar{D}^2(p)] \quad . \tag{2.12}$$

$V^b(p;\theta,\bar{\theta})$, where $k^{\alpha\dot{\beta}} \equiv k.\sigma^{\alpha\dot{\beta}}$.

for fig. (4d). This contribution also survives in the case of massless N = 4 super-Yang-Mills theory.

$$-\mu^{\varepsilon}g^2C_2(G)\delta_{ab}\int\frac{d^4p}{(2\pi)^4}\frac{d^Dk}{(2\pi)^D}\sum_i m_i^2\frac{1}{k^2+m_i^2}\frac{1}{(k+p)^2+m_i^2}\int d^4\theta \quad .$$

$$V^a(-p;\theta,\bar{\theta})V^b(p;\theta,\bar{\theta}) \tag{2.13}$$

for fig. (4e) and

$$\mu^{\varepsilon} g^2 C_2(G)\,\delta_{ab} \int \frac{d^4p}{(2\pi)^4} \int \frac{d^Dk}{(2\pi)^D} \sum_i \frac{1}{k^2+m_i^2} \int d^4\theta V^a(-p;\theta,\bar{\theta}).V^b(p;\theta,\bar{\theta}). \tag{2.14}$$

for fig. (4f).

By then adding up the contributions of all diagrams depicted in fig. 4 we obtain

$$\frac{1}{2}\mu^{\varepsilon} g^2 C_2(G)\,\delta_{ab} \int \frac{d^4p}{(2\pi)^4} \int \frac{d^Dk}{(2\pi)^D} [-3\frac{1}{k^2(k+p)^2} + \sum_i \frac{1}{k^2+m_i^2} \cdot \frac{1}{(k+p)^2+m_i^2}] \int d^4\theta V^a(-p;\theta,\bar{\theta}) p^2 \pi_{1/2}(p) V^b(p;\theta,\bar{\theta}) , \tag{2.15}$$

To get expression (3.15), we have used that

$$\int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2+m_i^2} \frac{1}{(k+p)^2+m_i^2} k_a = -\frac{1}{2} p_a \int \frac{d^Dk}{(2\pi)^D} \cdot \frac{1}{k^2+m_i^2} \frac{1}{(k+p)^2+m_i^2} , \tag{2.16}$$

which can readily be obtained by shifting variables twice and by performing a symmetric integration.

All diagrams representing the self-energy insertions to $\phi\phi \to \bar{\phi}\bar{\phi}$ scattering are shown in fig. 5.

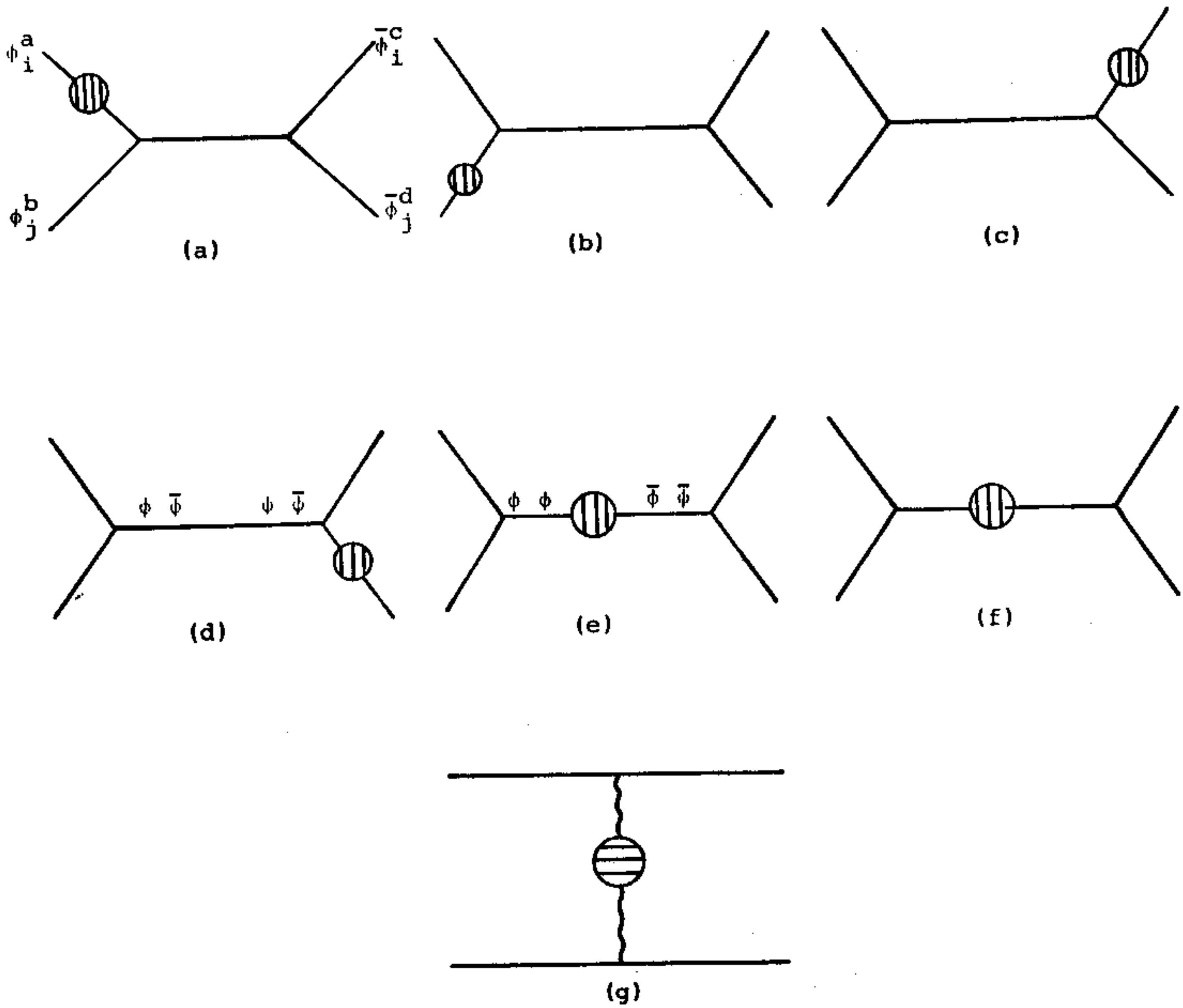


Fig. 5
Tree-level graphs corrected by one-loop self-energy insertions.

Their evaluation gives the following contributions:

$$g^4\varepsilon_{ijj'}\,\varepsilon_{ijj'}\,f_{abe}\,f_{cde}\,C_2(G)\,\frac{p_i^2}{p_i^2+m_i^2}\,\frac{1}{(p_1+p_2)^2+m_i^2}\,\cdot$$

$$\cdot\,[\frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2+m_k^2}\,\frac{1}{(k+p_1)^2+m_\ell^2}+(m_k\leftrightarrow m_\ell)\,+$$

$$-\int d^4k\,\frac{1}{k^2}\cdot\frac{1}{(k+p_1)^2+m_i^2}\,]\int d^4\theta\,\phi_i^a\,\phi_j^b\,\bar\phi_i^c\,\bar\phi_j^d \qquad ,\ (2.17)$$

$k\neq\ell\neq i$, for fig. (5a). The answer for the supergraphs of figs. (5b), (5c) and (5d) can be directly read off from the answer of fig. (5a) by suitably exchanging the external momentum and the internal indices. For figs. (5e), (5f) and (5g) we obtain respectively

$$(-g\varepsilon_{ii'j}f_{aa'b})(-g^2C_2(G))(-g\varepsilon_{ii'j}f_{ca'd})\,.$$

$$\cdot\{\frac{1}{(p_1+p_2)^2+m_{i'}^2}\cdot\frac{1}{(p_1+p_2)^2+m_{i'}^2}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{1}{(k+p_1+p_2)^2+m_{i'}^2}\,\cdot$$

$$\cdot\,(p_1+p_2)^2\int d\theta\,\phi_i^a\phi_j^b\bar\phi_i^c\bar\phi_j^d+(-g\varepsilon_{ii'j}f_{aa'b})\,(\tfrac{1}{2}g^2k_1)\,\cdot$$

$$\cdot\,(-g\varepsilon_{ii'j}f_{aa'd})\,\frac{1}{(p_1+p_2)^2+m_{i'}^2}\cdot\frac{1}{(p_1+p_2)^2+m_{i'}^2}\cdot\int\frac{d^4k}{(2\pi)4}\,\cdot$$

$$\cdot\,[\frac{1}{k^2+m_k{}^2}\cdot\frac{1}{(k+p_1+p_2)^2+m_\ell^2}+\frac{1}{k^2+m_\ell{}^2}\cdot\frac{1}{(k+p_1+p_2)^2+m_k{}^2}]\,.$$

$$\cdot (p_1+p_2)^2 \int d^4\theta \; \phi_i^a \phi_j^b \bar\phi_i^c \bar\phi_j^d \} \text{ for } k \neq \ell \neq i, \qquad (2.18)$$

$$(-g\varepsilon_{ii'j}\, f_{aa'b})(-g\varepsilon_{ii'j}\, f_{ca'd})(-g^2 C_2(G)) .$$

$$\cdot \{ \frac{m_{i'}}{(p_1+p_2)^2+m_{i'}^2} \cdot \frac{m_{i'}}{(p_1+p_2)^2+m_{i'}^2} \cdot \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \cdot$$

$$\cdot \frac{1}{(k+p_1+p_2)^2 + m_{i'}^2} \cdot \int d^4\theta \phi_i^a \phi_j^b \bar\phi_i^c \bar\phi_j^d +$$

$$+(-g\varepsilon_{ii'j}\, f_{aa'b})\; (-g\varepsilon_{ii'j} f_{ca'd})(\tfrac{1}{2}g^2 C_2(G)) . \;.$$

$$\cdot \frac{m_{i'}}{(p_1+p_2)^2+m_{i'}^2} \cdot \frac{m_{i'}}{(p_1+p_2)^2+m_{i'}^2} \int \frac{d^4k}{(2\pi)^4} [\frac{1}{k^2+m_k^2} \cdot$$

$$\cdot \frac{1}{(k+p_1+p_2)^2+m_\ell^2} + (m_k \leftrightarrow m_\ell)] \int d^4\theta \phi_i^a \phi_j^b \bar\phi_i^c \bar\phi_j^d \} \qquad (2.19)$$

for $k \neq \ell \neq i'$

and

$$(igf_{ca'a})(igf_{da'b}) \frac{(-1)}{(p_1-p_3)^2} \cdot \frac{(-1)}{(p_1-p_3)^2} \cdot$$

$$\cdot \int \frac{d^4k}{(2\pi)^4} [\frac{-3}{k^2 (k+p_1-p_3)^2} + \sum_\ell \frac{1}{k^2+m_\ell^2} \cdot \frac{1}{(k+p_1-p_3)^2+m_\ell^2}] \cdot$$

$$\cdot \; (p_1-p_3)^2 \cdot (\tfrac{1}{2}g^2 k_1) \int d^4\theta \phi_i^a \phi_j^b \bar\phi_i^c \bar\phi_j^d \; . \qquad (2.20)$$

(iii) one-loop vertex corrections to the diagrams in fig. 1. There are two types of vertices; the $V\phi\bar{\phi}$ vertex and the $\phi\phi\phi$ vertex. The one loop corrections are shown in fig. 6 and fig. 7.

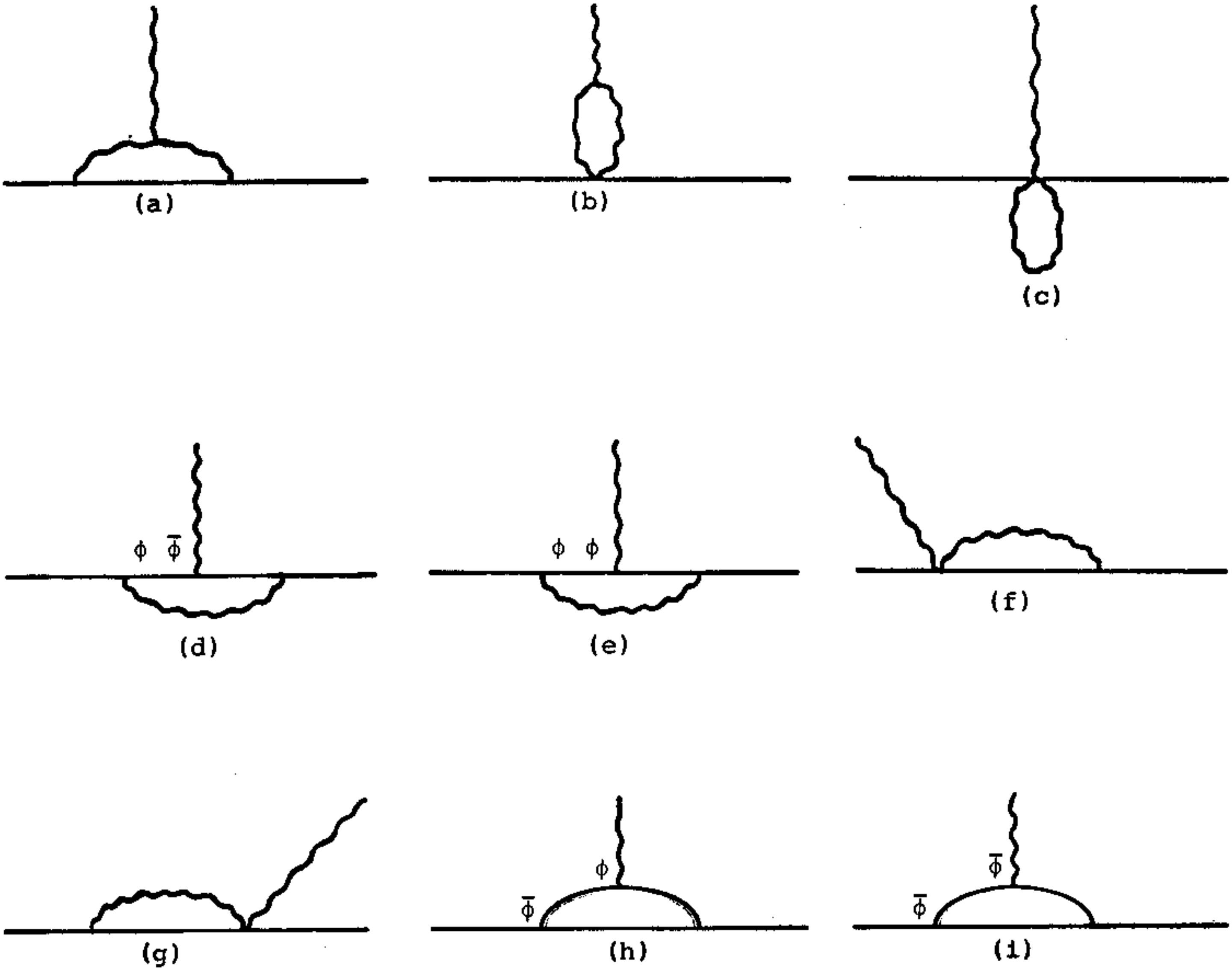


Fig. 6

One-loop corrections to the $\bar{\Phi}\Phi V$ - proper vertex.

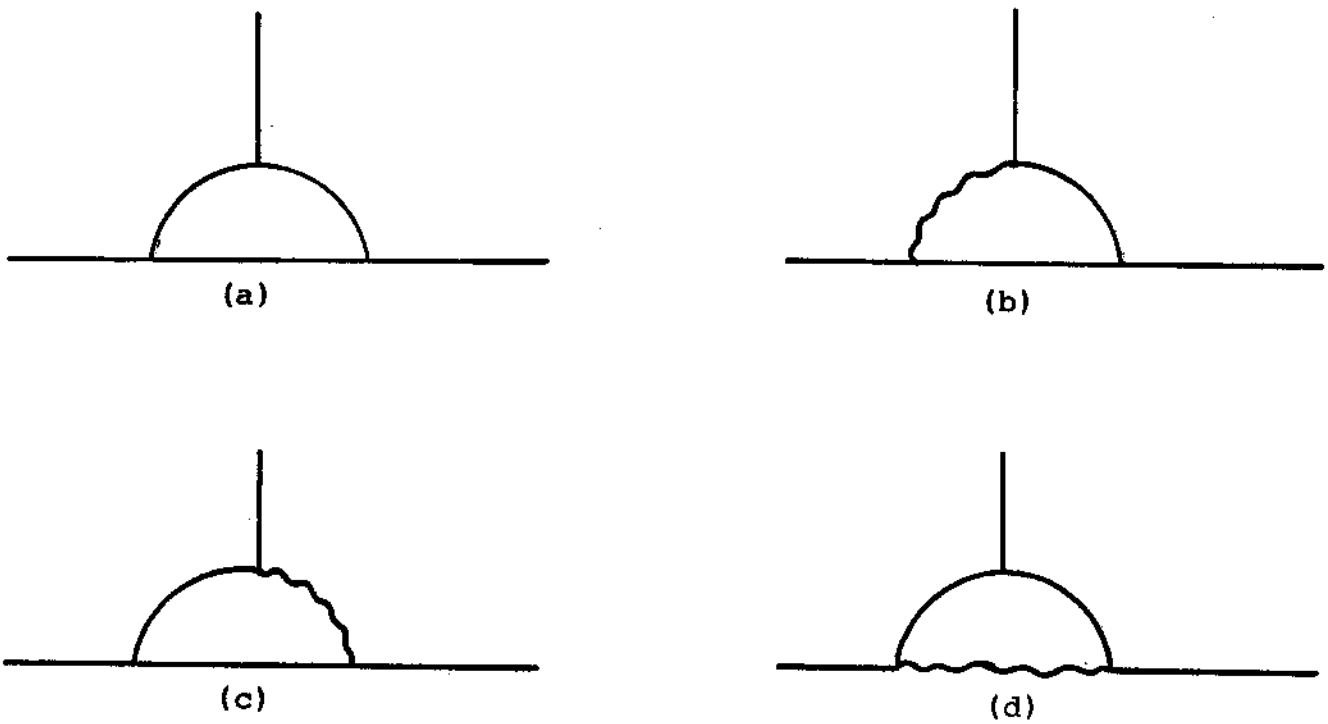


Fig. 7
One-loop contributions to the Φ^3 - proper vertex.

The contributions of the individual diagrams are:

$$\int\frac{d^Dk}{(2\pi)^D}\,\frac{1}{k^2+m_i^2}\,\frac{1}{(k+p)^2}\,\frac{1}{(k+q)^2}\int d^4\theta\,\{\frac{1}{32}\,[D^2(-p)\bar{\Phi}_i^a]\,.$$

$$.\,[D^2(q)\Phi_j^c] + \frac{1}{8}(p+q-2k)^{\alpha\dot{\beta}}[D_{\dot{\beta}}(-p)\bar{\Phi}_i^a][D(q)\Phi_j^c] +$$

$$+\frac{1}{2}\,[k(p+q)-p.q]\,\bar{\Phi}_i^a\;\Phi_j^c\,\}\,V^b \qquad (2.21)$$

for fig. (6a), (6b) and (6c) both possess a vanishing group-

-theoretical factor,

$$\int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2} \frac{1}{(k+p)^2+m_i^2} \frac{1}{(k+q)^2+m_i^2} \int d^4\theta \; \{- \frac{1}{32} \, .$$

$$\cdot \, [\bar{D}^2(-p)\bar{\Phi}_i^a][D^2(q)\Phi_i^c] - \frac{1}{4} \, (p+q+k)^{\alpha\dot{\beta}} [\bar{D}_{\dot{\beta}}(-p)\bar{\Phi}_i^a] \cdot [D_\alpha(q)\Phi_j^c] \; +$$

$$+ \frac{1}{2} \; [(p+q+k)^2+m_i^2]\bar{\Phi}_i^a \; \Phi_j^c \} \, V^b \qquad (2.22)$$

for figs. (6d) and (6e),

$$- \frac{3}{4} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2+m_i^2} \frac{1}{(k+p)^2} \frac{1}{(k+q)^2} \cdot [(k+p)^2+(k+q)^2] \int d^4\theta \bar{\Phi}_i^a V^b \Phi_j^c \qquad (2.23)$$

for fig. (6f) and (6g),

$$\int \frac{d^Dk}{(2\pi)^D} [\frac{1}{k^2+m_k^2} \frac{1}{(k+p)^2+m_\ell^2} \frac{1}{(k+q)^2+m_\ell^2} + (m_k \leftrightarrow m_\ell)] \quad .$$

$$\cdot \int d^4\theta \{- \frac{1}{32} [\bar{D}^2(-p_i^a)\bar{\Phi}_j^a][D^2(q)\Phi_j^c] + \frac{1}{4} k^{\alpha\dot{\beta}} [\bar{D}_{\dot{\beta}}(-p)\bar{\Phi}_i^a] . [D_\alpha(q)\Phi_j^c] \; +$$

$$+ \frac{1}{2} k^2 \bar{\Phi}_i^a \Phi_i^c \} V^b \qquad (2.24)$$

for fig. (6h) and

$$\frac{1}{2} \int \frac{d^Dk}{(2\pi)^D} [\, m_\ell^2 \, \frac{1}{k^2+m_k^2} \; \frac{1}{(k+p)^2+m_\ell^2} \; \frac{1}{(k+q)^2+m_\ell^2} \; +$$

$$+ \; (m_k \leftrightarrow m_\ell)] \int d\theta \bar{\Phi}_i^a V^b \Phi_j^c \, , \qquad (2.25)$$

where $k \neq \ell \neq i$,

for fig. (6i).

The contribution of fig. (7a) vanishes due to the algebra of the covariant derivatives.

$$(p-q)^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(k+p)^2 + m_1^2} \frac{1}{(k+q)^2 + m_2^2} \int d^2\theta \Phi_1^a(-p;\theta,\bar{\theta}) \cdot$$

$$\cdot \Phi_2^b(q;\theta,\bar{\theta}) \Phi_3^c(p-q;\theta,\bar{\theta}) \qquad (2.26)$$

for fig. (7b),

$$p^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m_2^2} \frac{1}{(k+p)^2 + m_3^2} \frac{1}{(k+q)^2} \int d^2\theta \Phi_1^a \Phi_2^b \Phi_3^c \qquad (2.27)$$

for fig. (7c) and

$$q^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m_1^2} \frac{1}{(k+p)^2} \frac{1}{(k+q)^2 + m_3^2} \int d^2\theta \Phi_1^a \Phi_2^b \Phi_3^c \qquad (2.28)$$

for fig. (7d)

with the common factor $\frac{1}{2} g^3 C_2(G) \varepsilon_{ijk}$ and integrals over p and q.

All vertex correction insertions to $\Phi\Phi \rightarrow \bar{\Phi}\bar{\Phi}$ are shown in fig. 8.

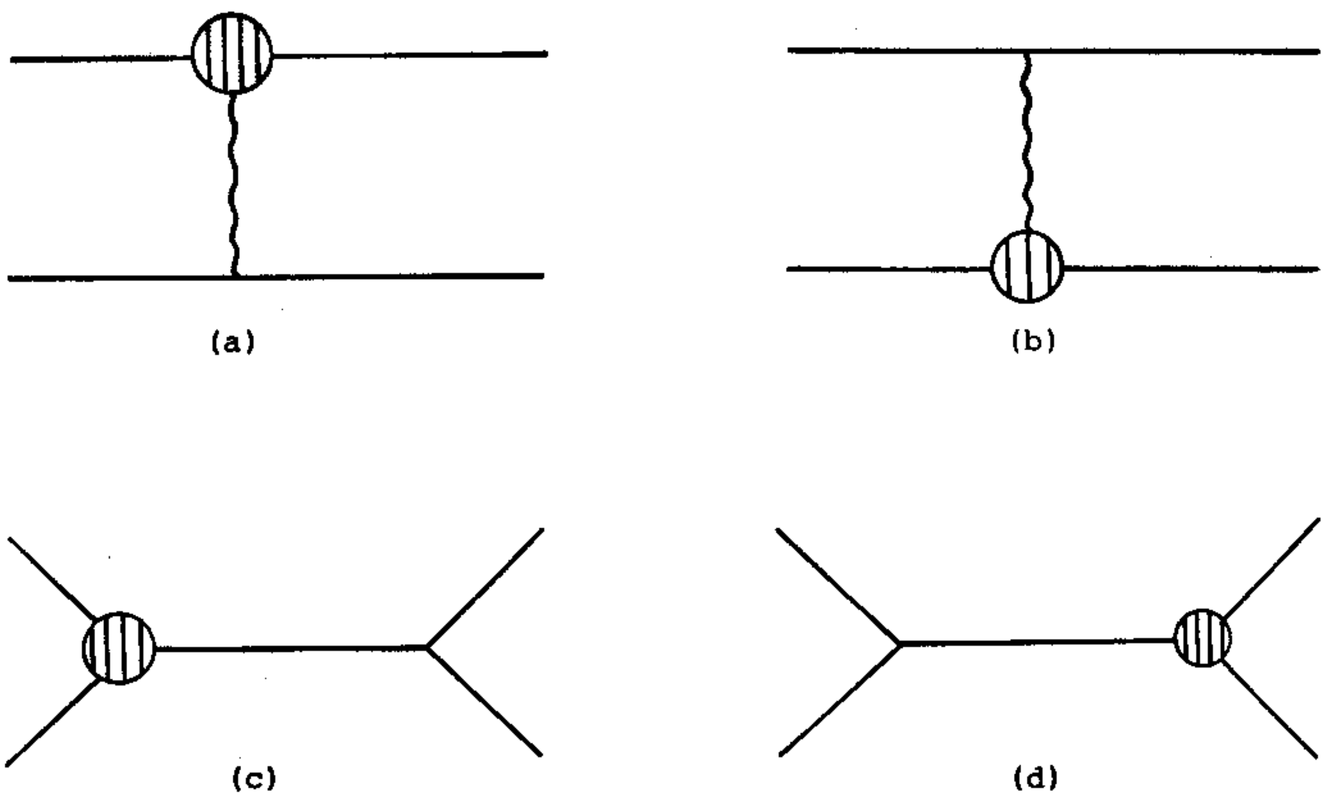


Fig. 8
Supergraphs with one-loop corrected propagators and vertices.

Their evaluation gives the following contributions:

Fig. (8a):

$$-g^4 C_2(G) f_{da'b} f_{ca'a} \{ \frac{1}{(p_1-p_3)^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+m_i^2} \cdot \frac{1}{(k-p_1)^2}$$

$$\cdot \frac{1}{(k-p_3)^2} \cdot [\frac{1}{32} \int d^4\theta (\bar{D}^2 \phi_i^c)(D^2 \phi_i^a) \phi_j^b \bar{\phi}_j^d - \frac{1}{8}(p_1 + p_3 + 2k)^{\alpha\dot{\beta}} \cdot$$

$$\cdot \int d^4\theta (\bar{D}_{\dot{\beta}} \bar{\phi}_i^c)(D_\alpha \phi_i^a) \phi_j^b \bar{\phi}_j^d - \frac{1}{2} k \cdot (p_1 + p_3) + p_1 \cdot p_3 \int d^4\theta \phi_i^a \phi_j^b \bar{\phi}_i^c \bar{\phi}_j^d] +$$

$$+ \frac{1}{(p_1 - p_3)^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(k-p_1)^2 + m_i^2} \cdot \frac{1}{(k-p_3)^2 + m_i^2} \cdot$$

$$\cdot\left[-\frac{1}{32}\int d^4\theta(\bar{D}^2\bar{\Phi}_i^c)(D^2\phi_i^a)\phi_j^b\bar{\phi}_j^d+\frac{1}{4}(p_1+p_3-k)^{\alpha\dot{\beta}}\int d^4\theta(\bar{D}_{\dot{\beta}}\bar{\Phi}_i^c)(\bar{D}_\alpha\phi_i^a)\phi_j^b\bar{\phi}_j^d+\right.$$

$$+\frac{1}{2}\left((p_1+p_3-K)^2+m_i^2\right)\int d^4\theta\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d \quad +$$

$$+\frac{1}{(p_1-p_3)^2}\left(-\frac{3}{4}\right)\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k-p_1)^2}\cdot\frac{1}{(k-p_3)^2}\cdot\frac{1}{k^2+m_i^2}\cdot$$

$$\cdot\quad[(k-p_1)^2+(k-p_3)^2]\int d^4\theta\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\Phi}_j^d \quad +$$

$$+\frac{1}{(p_1-p_3)^2}\int\frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2+m_k^2}\cdot\frac{1}{(k-p_1)^2+m_\ell^2}\cdot\frac{1}{(k-p_3)^2+m_\ell^2}\right. \quad +$$

$$+\left.(m_k\leftrightarrow m_\ell)\right]\cdot\left[-\frac{1}{32}\int d^4\theta(\bar{D}^2\bar{\Phi}_i^c)(D^2\phi_i^a)\phi_j^b\bar{\phi}_j^d\right. +$$

$$+\frac{1}{4}K\alpha\dot{\beta}\int d^4\theta(\bar{D}_{\dot{\beta}}\bar{\Phi}_i^c)(D_\alpha\bar{\phi}_i^a)\phi_j^b\bar{\phi}_j^d+\frac{1}{2}k^2\int d^4\theta\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d \quad +$$

$$+\frac{1}{(p_i-p_3)^2}\int\frac{d^4k}{(2\pi)^4}\left[m_\ell^2\frac{1}{k^2+m_k^2}\cdot\frac{1}{(k-p_1)^2+m_\ell^2}\cdot\frac{1}{(k-p_3)^2+m_\ell^2}\right. \quad +$$

$$\left.+(m_k\leftrightarrow m_\ell)\right]\int d^4\theta\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d\ \} \tag{2.19}$$

for $k \neq \ell \neq i$ (but k or ℓ may be equal to j);

Fig. (8b):

the same answer as above, by suitably exchanging momenta and internal indices;

Fig. (8c):

$$\left(\frac{1}{2} g^3 C_2(G) \varepsilon_{ii'j} f_{aa'b}\right)\left(-g \varepsilon_{ii'j} f_{ca'd}\right) \cdot \frac{1}{(p_1+p_2)^2+m_{i'}^2} \cdot$$

$$\cdot \int \frac{d^4k}{(2\pi)^4} \left[(p_1+p_2)^2 \frac{1}{k^2} \frac{1}{(k+p_1)^2+m_i^2} \cdot \frac{1}{(k-p_2)^2+m_j^2} + \right.$$

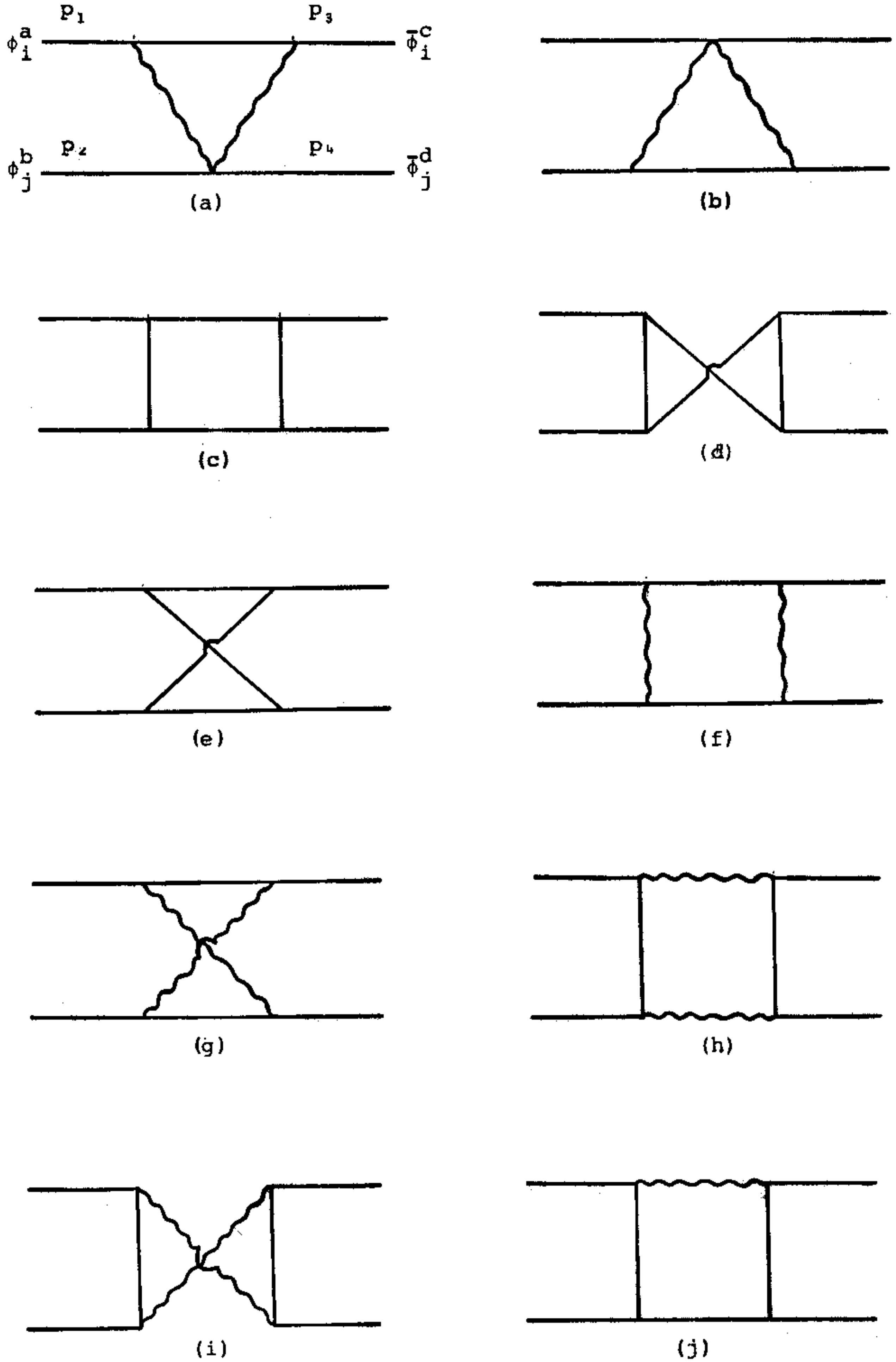
$$+ p_1^2 \cdot \frac{1}{k^2+m_j^2} \cdot \frac{1}{(k+p_1)^2+m_{i'}^2} \cdot \frac{1}{(k-p_2)^2} + $$

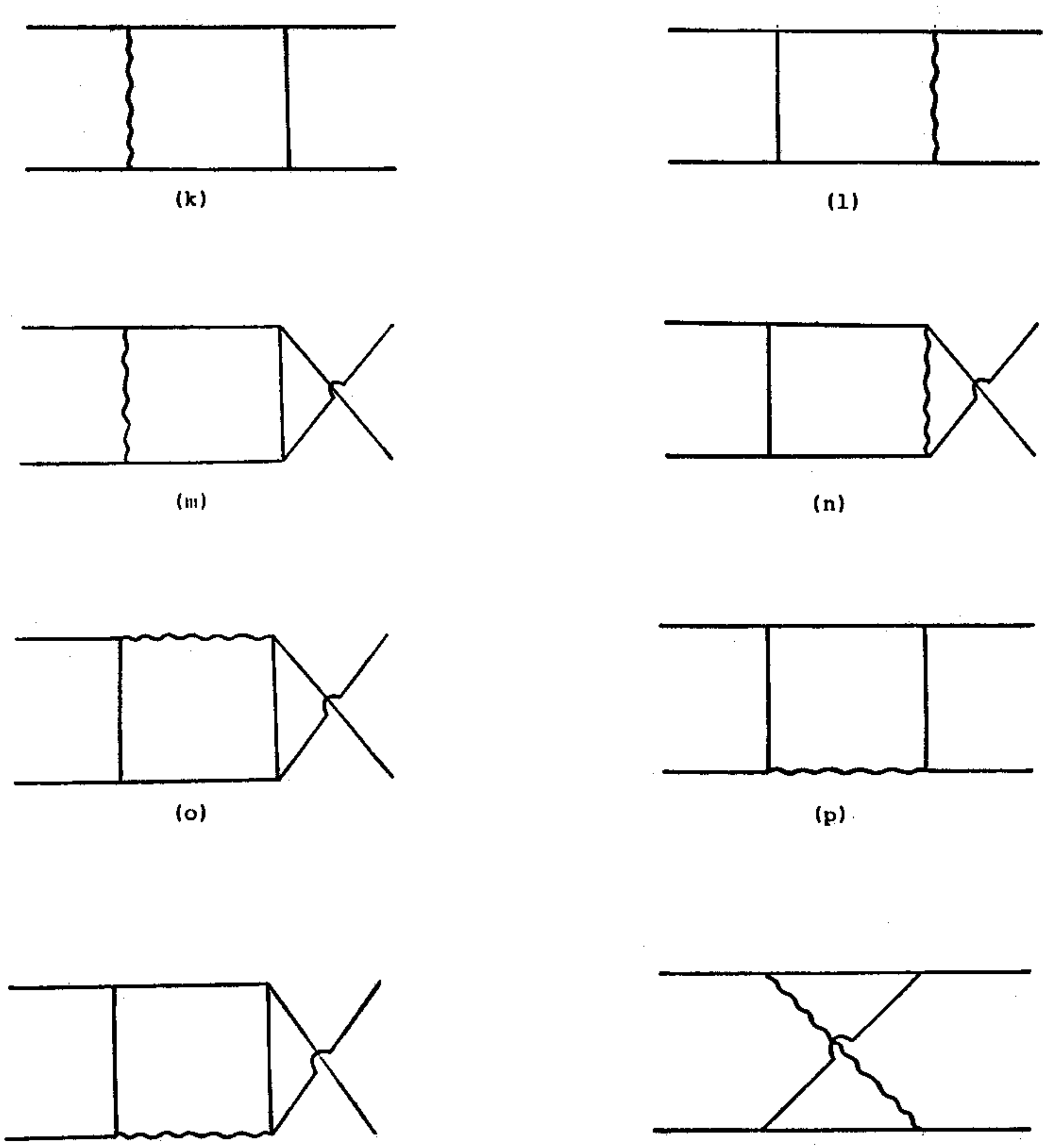
$$\left. + p_3^2 \; \frac{1}{k^2+m_i^2} \cdot \frac{1}{(k+p_1)^2} \cdot \frac{1}{(k-p_2)^2+m_{i'}^2} \right] \int d^4\theta \, \phi_i^a \phi_j^b \bar{\phi}_i^c \bar{\phi}_j^d \quad ; \qquad (2.30)$$

Fig. (8d):

it is just the complex conjugate of fig. (8c).

(iv) the rest of the one loop corrections to $\phi\phi \to \bar{\phi}\bar{\phi}$ are shown in fig. 9.


p_1
p_3
ϕ_i^a
$\bar{\phi}_i^c$
p_2
p_4
ϕ_j^b
$\bar{\phi}_j^d$
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)



(k) (l)

(m) (n)

(o) (p)

(q) (r)

Fig. 9

One particle irreducible graphs contributing finite corrections to the $\phi\phi\to\bar{\phi}\bar{\phi}$ scattering.

Their evaluation gives the following contributions:

Fig. (9a):

$$\frac{g^4}{2}\left(\Lambda^{abdc}+\Lambda^{adbc}\right)\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{1}{(k+p)^2+m_i^2}\,\cdot$$

$$\cdot\,\frac{1}{(k+p_1-p_3)^2}\int d^4\theta\;\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d\;; \qquad (2.31)$$

Fig. (9b):

$$\frac{g^4}{2}\left(\Lambda^{acdb}+\Lambda^{acbd}\right)\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{1}{(k+p)^2+m_j^2}\,\cdot$$

$$\frac{1}{(k+p_2-p_4)^2}\int d^4\theta\;\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d \qquad (2.32)$$

Fig. (9c):

$$g^4\,\Lambda^{abdc}\,\varepsilon_{ii'i''}\,\varepsilon_{ii'i'''}\,\varepsilon_{jj'i''}\,\varepsilon_{jj'i'''}\int\frac{d^4k}{(2\pi)^4}\,\frac{m_{i''}}{k^2+m_{i''}^2}\,\cdot$$

$$\frac{1}{(k+p_1)^2+m_{i'}^2}\cdot\frac{m_{i'''}}{(k+p_1-p_3)^2+m_{i'''}^2}\cdot\frac{1}{(k-p_2)^2+m_{j'}^2}\int d^4\theta\,\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d$$

$$(2.33)$$

Fig. (9d):

$$g^4 \ \Lambda^{abcd} \ \varepsilon_{ii'i''} \ \varepsilon_{ij'j''} \ \varepsilon_{ji'j''} \ \varepsilon_{jj'i''} \int \frac{d^4k}{(2\pi)^4} \cdot \frac{m_{i''}}{k^2+m_{i''}^2} \cdot$$

$$\frac{1}{(k+p_1)^2+m_{i'}^2} \cdot \frac{m_{j''}}{(k+p_1-p_4)^2+m_{j''}^2} \cdot \frac{1}{(k-p_2)^2+m_{j'}^2} \int d^4\theta \phi_i^a \phi_j^b \bar{\phi}_i^c \bar{\phi}_j^d \ ; \qquad (2.34)$$

Fig. (9e):

$$g^4 \ \Lambda^{acbd} \ \varepsilon_{ii'i''} \ \varepsilon_{ii'j''} \ \varepsilon_{jj'j''} \ \varepsilon_{jj'j''} \int \frac{d^4k}{(2\pi)^4} \cdot \frac{1}{k^2+m_{i''}^2} \cdot$$

$$\frac{1}{(k+p_1)^2+m_{i'}^2} \cdot \frac{1}{(k+p_4-p_2)^2+m_{j''}^2} \cdot \frac{1}{(k+p_4)^2+m_{j'}^2} \cdot$$

$$\cdot \{-k^2 \int d^4\theta \phi_i^a \phi_j^b \bar{\phi}_i^c \bar{\phi}_j^d + \int d^4\theta \bar{D}^2 [(D^2\phi_i^a)\bar{\phi}_i^c] \ \phi_j^b \bar{\phi}_j^d \ +$$

$$-4k^{\alpha\dot{\beta}} \int d^4\theta \bar{D}_{\dot{\beta}} [(D_\alpha \phi_i^a)\bar{\phi}_i^c] \ \phi_j^b \bar{\phi}_j^d \ \} \ ; \qquad (2.35)$$

Fig. (9f):

$$g^4 \ \Lambda^{abdc} \int \frac{d^4k}{(2\pi)^4} \cdot \frac{1}{k^2} \cdot \frac{1}{(k+p_1)^2+m_i^2} \cdot \frac{1}{(k+p_1-p_3)^2} \cdot$$

$$\cdot \frac{1}{(k+p_1-p_3-p_4)^2+m_j^2}[-(p_1+p_2)^2]\int d^4\theta\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d \quad ; \tag{2.36}$$

Fig. (9g):

$$g^4\Lambda^{adbc}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\frac{1}{(k+p_1)^2+m_i^2}\cdot\frac{1}{(k+p_1-p_3)^2}\cdot\frac{1}{(k+p_1+p_2-p_3)^2+m_j^2}\cdot$$

$$\{-(k+p_1+p_2-p_3)^2\int d^4\theta\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d+\int d^4\theta\bar{D}^2[(D^2\phi_i^a)\bar{\phi}_i^c]\phi_j^b\bar{\phi}_j^d +$$

$$-4(k+p_1+p_2-p_3)^{\alpha\dot{\beta}}\int d^4\theta\bar{D}_{\dot{\beta}}[(D_\alpha\phi_i^a)\bar{\phi}_i^c]\phi_j^b\bar{\phi}_j^d\} \quad ; \tag{2.37}$$

Fig. (9h):

$$g^4\Lambda^{acdb}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\cdot\frac{m_i}{(k+p_1)^2+m_i^2}\cdot\frac{1}{(k+p_1+p_2)^2}\cdot$$

$$\cdot\frac{m_i}{(k+p_3)^2+m_i^2}\int d^4\theta\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d \quad ; \tag{2.38}$$

Fig. (9i):

$$g^4\Lambda^{adcb}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\cdot\frac{m_i}{(k+p_1)^2+m_i^2}\cdot\frac{1}{(k+p_1+p_2)^2}\cdot$$

$$\cdot\frac{m_i}{(k+p_4)^2+m_i^2}\int d^4\theta\phi_i^a\phi_j^b\bar{\phi}_i^c\bar{\phi}_j^d \quad ; \tag{2.39}$$

Fig. (9j):

$$g^4\Lambda^{acdb}\varepsilon_{jj'i'}\varepsilon_{jj'j''}\int\frac{d^4k}{(2\pi)^4}\left(-\frac{1}{k^2}\right)\frac{1}{(k+p_1)^2+m_{i'}^2}\cdot$$

$$\cdot \frac{1}{(k+p_1+p_2)^2+m_{j'}^2} \cdot \frac{1}{(k+p_1+p_2-p_4)^2+m_{j''}^2} \{ -(k+p_1+p_2-p_4)^2 \int d^4\theta \phi_i^a \phi_j^b \bar{\phi}_i^c \bar{\phi}_j^d +$$

$$+ \int d^4\theta (D^2\phi_i^a) \phi_j^b \bar{\phi}_i^c \bar{\phi}_j^d - 4(k+p_1+p_2-p_4)^{\alpha\dot{\beta}} \int d^4\theta D_{\dot{\beta}} [(D_\alpha \phi_i^a) \bar{\phi}_i^c] \phi_j^b \bar{\phi}_j^d \}. \tag{2.40}$$

Fig. (9k):

it vanishes due to the algebra of covariant derivatives;

Fig. (9l);

it vanishes due to the algebra of covariant derivatives;

Fig. (9m):

$$-g^4 \Lambda^{adcb} \varepsilon_{ji'j''} \varepsilon_{ij'j''} \int \frac{d^4k}{(2\pi)^4} \left(-\frac{1}{k^2}\right) \frac{m_{i'}}{(k+p_1)^2 + m_{i'}^2} \cdot$$

$$\cdot \frac{m_{j''}}{(k+p_1-p_4)^2[(k+p_1-p_4)^2+m_{j''}^2]} \cdot \frac{m_{j'}}{(k-p_2)^2+m_{j'}^2} \int d^4\theta (D^2\phi_i^a) \phi_j^b \bar{\phi}_i^c \bar{\phi}_j^d \ ; \tag{2.41}$$

Fig. (9n):

it vanishes due to the algebra of covariant derivatives;

Fig. (9o):

$$g^4 \varepsilon_{jj'i'} \varepsilon_{ij'i''} \Lambda^{abcd} \int \frac{d^4k}{(2\pi)^4} \left(-\frac{1}{k^2}\right) \frac{1}{(k+p_1)^2 + m_{i'}^2} \cdot$$

$$\cdot \frac{1}{(k+p_1+p_2)^2+m_{j'}^2} \cdot \frac{1}{(k+p_1+p_2-p_3)^2+m_{i''}^2} \cdot \{-(k+p_1+p_2-p_3)^2 \cdot$$

$$\cdot \int d^4\theta \phi_i^a \phi_j^b \bar{\phi}_i^c \bar{\phi}_j^d + \int d^4\theta \bar{D}^2 [(D^2\phi_i^a) \bar{\phi}_i^c] \phi_j^b \bar{\phi}_j^d +$$

$$-4(k+p_1+p_2-p_3)^{\alpha\dot\beta}\int d^4\theta\ \bar D_{\dot\beta}[(D_\alpha\phi_i^a)\bar\phi_i^c]\ \phi_j^b\bar\phi_j^d\} \qquad (2.42)$$

Fig. (9p):

$$g^4\ \varepsilon_{ii'i''}\ \varepsilon_{ii'j'}\ \Lambda^{acdb}\int\frac{d^4k}{(2\pi)^4}\cdot\frac{1}{k^2+m_{i''}^2}\cdot\frac{1}{(k+p_1)^2+m_{i'}^2}\cdot\frac{(-1)}{(k-p_2)^2}$$

$$\frac{1}{(k+p_1-p_3)^2+m_{j'}^2}\ \{(-k^2)^2\int d^4\theta\phi_i^a\phi_j^b\bar\phi_i^c\bar\phi_j^d +$$

$$+\int d^4\theta\bar D^2[(D^2\phi_i^a)\bar\phi_i^c]\ \phi_j^b\bar\phi_j^d - 4k^{\alpha\dot\beta}\int d^4\theta\bar D_{\dot\beta}[(D_\alpha\phi_i^a)\bar\phi_i^c]\phi_j^b\bar\phi_j^d\}; \qquad (2.43)$$

Fig. (9q):

$$g^4\ \varepsilon_{ji'j''}\ \ \varepsilon_{ii'j'}\ \Lambda^{badc}\int\frac{d^4k}{(2\pi)^4}\ \left(\frac{-1}{k^2}\right)\ \frac{1}{(p_2+k)^2+m_{j'}^2}\cdot$$

$$\frac{1}{(p_1+p_2+k)^2+m_{i'}^2}\cdot\frac{1}{(p_1+p_2+k-p_4)^2+m_{j''}^2}\ \{-(p_2+k)^2\int d^4\theta\phi_i^a\phi_j^b\bar\phi_i^c\bar\phi_j^d +$$

$$+\int d^4\theta\bar D^2[(D^2\phi_i^a)\bar\phi_i^c\ ]\phi_j^b\bar\phi_j^d - 4(p_2+k)^{\alpha\dot\beta}\int d^4\theta\bar D_{\dot\beta}[(D_\alpha\phi_i^a)\bar\phi_i^c\ ]\phi_j^b\bar\phi_j^d\}\quad ; \qquad (2.44)$$

Fig. (9r):

$$g^4 \varepsilon_{ii'j''} \varepsilon_{jj'j''} \Lambda^{acbd} \int \frac{d^4k}{(2\pi)^4} \left(-\frac{1}{k^2}\right) \frac{m_{i'}}{(p_i+k)^2 + m_{i'}^2} \cdot$$

$$\cdot \frac{m_{j'}}{(p_4+k)^2 + m_{j'}^2} \int d^4\theta \phi_i^a \phi_j^b \bar{\phi}_i^c \bar{\phi}_j^d \quad . \tag{2.45}$$

k_1 and Λ_{abcd} are group-theoretical quantities so defined that

$$\frac{1}{2} k_1 f_{abc} = f_{a'ab'} f_{b'bc'} f_{c'ca'} \tag{2.46a}$$

$$\Lambda_{abcd} \equiv f_{a'ab'} f_{b'bc'} f_{c'cd'} f_{d'da'} \tag{2.46b}$$

with

$$\Lambda_{abcd} = \Lambda_{badc} = \Lambda_{cdab} = \Lambda_{dcba} \tag{2.46c}$$

In evaluating the diagrams contributing to $\phi\phi \to \bar{\phi}\bar{\phi}$ no use was made of the equations of motion for the superfields ϕ and $\bar{\phi}$. The sum of all the diagrams gives the off-shell four-point function $\phi\phi\bar{\phi}\bar{\phi}$. It is to be noted that the sum of the individual contributions in figs. 5 and 8 is ultraviolet finit. However all contributions in fig. 9 are individually ultraviolet convergent.

3. PHYSICAL AMPLITUDES, DECOUPLING AND HIGH-ENERGY BEHAVIOUR

To read off the physical processes described by the whole sit of graphs calculated in the previous section, the first step is to express our results in the component form. By parametrizing a chiral superfield by the θ-expansion

$$\phi(x,\theta,\bar{\theta}) = \exp i(\theta\sigma\bar{\theta}.\partial)(z + \theta\psi + \theta^2 h), \qquad (3.1)$$

where z is a physical complex scalar, ψ is a physical fermion and h is the complex auxiliar component of ϕ, the above superspace integrals can be worked out in their component version.

Once this is done the elimination of the auxiliary fields through then tree-level equations of motion is the next step towards the obtention of the physical processes accomodated by the scattering $\phi\phi \to \bar{\phi}\bar{\phi}$. The h_i^a's are to be eliminated through

$$h_{ia} = m_i z_{ia} + k_1 \epsilon_{ijk} f_{abc} z_{jb} z_{kc} \qquad (3.2)$$

One can verify that the two-particle scatterings with the supergraphs of the previous section taken into account are the following:

scalar + scalar → scalar + scalar

scalar + scalar → fermion + fermion

scalar + fermion → scalar + fermion

fermion + fermion → fermion + fermion

and

fermion + fermion → scalar + scalar

As our working example, we shall focus our attention on the purely scalar scattering just to avoid polarization factors which do not alter the essence of the conclusions we shall draw. By projecting into components and making use of the equations of motion (3.2), we can individuate all contributions to this process. If then we use the on-shell condition for these scalars, we finally get the physical amplitude for the purely scalar scattering under consideration.

In possess of the results obtained in the previous section, the first question we would like to discuss is that of decoupling. As already mentioned, the mass parameters of (2.2) can be so chosen that a hierarchy of light and heavy sectors is introduced into the model. Let us take for instance, $m_1 = m_2 = 0$ and m_3 very heavy, and let us suppose that the supergraphs of figures 8 and 9 carry the zero mass superfields in their external legs. So the heavy sector contributes to this process only by circulating inside loops. By then taking the limit $m_3 \to \infty$ and keeping only those contributions that diverge with m_3. We find that the graphs of fig. 8 exhibit a $\log \dfrac{m_3^2}{(p_1+p_2)^2}$ behaviour, in agreement with expectations based on power-counting together with Lorentz and gauge invariance. As for the graphs of fig. 9 we find that they are all suppressed by powers of the heavy mass. The net conclusion is that the heavy mass effects signal a violation of decoupling as physical amplitudes for the light-particle scattering diverge in the limit the heavy mass goes to infinity. This therefore confirms the results of reference [11]. To give a meaning to the effective light theory one has to absorb the heavy mass effects into a redefinition of the parameters of the Lagrangian through a finite renormalization, as already discussed in reference [11].

Coming now back to the physical scalar scattering, we would like to discuss the question of the high-energy limit of the cross-section for such a process in the framework of a finite theory in order to compare with the usual $\log^2$behaviour of ordinary renormalizable theories [12]. The first step we followed was the derivation of the large $s(\equiv(p_1+p_2)^2)$ behaviour of the physical scattering amplitudes. For this, we have used the kind of technique described in reference [13]. We found that their overall asymptotic limit grows as an (s log s) for $s \to \infty$. To get this behaviour we had not only derived the high s-limit of the loop integrals appearing in the supergraph answers but also took into account eventual powers of s arising when projecting the scalar-scalar scattering in components. From this, and by using the optical theorem, which relates the forward scattering amplitude to the total cross section for the process [14], we conclude that the physical scalar scattering is described by a cross-section which behaves as a (log s). This behaviour does respect the well-known Froissart bound for the cross-section which based on unitarity and in a completely model-independent way states that the cross section should be bound by a $(\log s)^2$. This raises again the discussion on the possibility of defining an effective coupling parameter which accounts for the effects of some physical process. Though in the case of a finite theory no regularization is needed and no subtraction point has to be introduced, the definition of an effective coupling constant out of the bare one should be done at a certain energy scale, E. In the usual case, where the Froissart bound governs the high s-behaviour of the cross-section, one would find a $(\log E)^{1/2}$. In our case, we get a different behaviour, as conjectured by Namazil, Salam and Stralhdee in reference [15]. Our analysis reveals a factor $(\log E)^{1/4}$ in the definition of the effective coupling parameter defined in such a way to take into account the scalar-scalar scattering of the N=4 model.

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