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RANGE-ENERGY FOR HEAVY IONS IN CR-39

by

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Abstract

Range-Energy relations in CR-39, for ions from He to Ar, are obtained after their effective nuclear charge. Comparison with earlier calculations and numerical results in the energy range 0.1 to 200 MeV/Nucleon are also given.

Key-words: CR-39; Range-energy; Heavy-ions.

1 INTRODUCTION

Range-Energy relations have played an unreplacable role in the identification of ionizing radiation as well as in the design of experimental shields and protections against their penetration. Demands in aerospace projects, in cosmic ray and general heavy ion research fostered many revisions and new tables on the subject ¹⁻⁸.

New detectors were also developed, particularly those for ionographic registration known as solid state nuclear track detectors (SSTD). They are particularly suited for low intensity heavy ion work owing to their low sensitivity to background of less ionizing radiation. Among those detectors, CR-39, a polymer of dimethylglycol, has progressively been raised to an outstanding position owing to its transparency to visible light and small value of critical losses.

This paper is an attempt to settle range-energy relations for CR-39, aiming at its uses as a spectrometer for heavy ions; calculations can be extended to other media of interest in a straightforward way. Comparison with earlier

relations¹¹ as well as a short discussion on restricted energy losses are also given.

2 ENERGY LOSSES AND RANGES.

Among the available schemes to deal with heavy ions⁶⁻¹¹, that of "range extension"^{10,11} met with wider acceptance in ionography. It is an adaptation of a procedure formerly suggested by Barkas¹², to be used in connection with nuclear emulsions, improved later on by Heckman et al.¹³

Under that scheme the ranges for any ion are obtained by adding a "range extension" to the calculated track length of an ideal proton (one whose charge is the same at all velocities), to account for the increase in pathlength owing to progressive charge neutralization, and next scaling to any particular ion by multiplying with the factor $(M/Z)^2$, M and Z being nucleon and charge numbers of the ion.

That procedure is affected by the need for an ideal proton range which has to be obtained, after all, from extrapolations of observed data at velocities where charge

neutralization is presumably negligible. Precision of calculations is also difficult to assess as Heckman's data for emulsions are scaled to other media by making use again of the ideal proton's curve. Those questions encouraged us to revise the whole procedure through a different optics, by abandoning the "range extension" methodology in favour of an alternate one based upon the effective nuclear charge.

We begin by defining the effective charge for an ion of charge number Z through:

$$\left(\frac{dE}{dx}\right)_{\text{Ion}}^* = Z^2 \left(\frac{dE}{dx}\right)_{\text{p}}^* \quad (1)$$

where Z^* is the effective nuclear charge, $\left(\frac{dE}{dx}\right)_{\text{p}}^*$ is the average rate of energy loss for an ideal proton and $\left(\frac{dE}{dx}\right)_{\text{Ion}}^*$ is the average rate of energy loss for ions of effective charge Z^* at the same velocity as the reference protons.

One has, as well:

$$\left(\frac{dE}{dx}\right)_{\text{p}}^* = z \left(\frac{dE}{dx}\right)_{\text{p}}^* \quad (2)$$

where z is the effective charge for protons and $\left(\frac{dE}{dx}\right)_{\text{p}}^*$ is their average rate of energy loss. From (1) and (2) one obtains:

$$\left(\frac{dE}{dx}\right)_{\text{Ion}}^* = \left(\frac{Z}{z}\right)^2 \left(\frac{dE}{dx}\right)_{\text{p}}^* \quad (3)$$

After determining the effective nuclear charges of the ion and the proton, the average rate of energy loss for any ion will be given by (3); ideal proton contributions are thus eliminated.

As to tables of average rates of energy loss for protons, one of the required items in (3), we found Janni's tabulations⁵ best suited to our purposes. Except for data in the energy range 20 KeV to 1 MeV, taken from the available experimental data base, all other are calculated from first principles, using well accepted formulations, with parameters fixed in a clean and transparent way. To be able to use equ. (3) one needs values of $(dE/dx)_P$ in the medium of interest. For CR-39 (C¹² -H¹⁸ -O⁷) we used additivity law to combine Janni's data for carbon hydrogen and oxygen within the centesimal composition above. Although this procedure to obtain average rates of energy loss in compound media is not in general recommended, owing to failure of additivity law, inaccuracies are tolerable at moderate to high velocities⁵ and, at low velocities, our factor involving the ratios of the effective charges becomes dominant.

We are then left with the problem of determining effective charges as a function of velocity for ions and protons. Many empirical solutions to that problem have been suggested and used with a fair success^{4,6,12,13}; however we preferred to follow our own way.

In choosing those functions we kept closely with the following directives: 1) both Z^* and z^* are similar functions of the reduced velocity of the ion, $137\beta z^{-2/3}$ as shown by Heckman et al¹³ for heavy ions; 2) those functions have the same general form as used successfully by other authors⁴ to fit empirical data; 3) when (Z/z) is carried into (3) with $Z=1$, the equation is satisfied identically; 4) in the case of heavy ions Z^* is extended beyond $E/M=0.5$ (for protons $z^*=1$ for $E>0.5$ MeV) by means of a function preserving all requirements above and mathematical continuity.

We begin with the effective charge for protons as a function of reduced velocity, by fitting data given by Janni⁵ for H, C and O with:

$$z^* = 1 - \exp\left(-\sum_i A_i x^i\right), \quad 0.001 < E(\text{MeV}) < 0.5 \quad (4)$$

$$z^* = 1, \quad 0.5 < E(\text{MeV})$$

where $x = 137\beta$. The resulting set of coefficients A_i is shown in Table I.

For ions we take:

$$Z^* = Z \left(1 - \exp\left(-\sum_i a_i x^i\right)\right), \quad 0.001 < E(\text{MeV/Nucleon}) < 0.5 \quad (5)$$

$$Z^* = Z(1 - \exp(b + cx)) \quad 0.5 < E(\text{MeV})$$

where $x = 137\beta Z^{-2/3}$. Coefficients a_i in (5) are so chosen

that they relate to the A_i in Table I through:

$$a_i = A_i Z^{2(i-1)/3}, \quad i=0 \dots 4 \quad (6)$$

while b and c are determined by requiring continuity of Z^* and its derivative at $E=0.5$ MeV/Nucleon. It results that $b = .7335Z^{-2/3}$

and $c = -1.7069$. The functions (Z^*/Z^2) , for the ions dealt with here are plotted in Figs. 1a and 1b.

With the effective charges as given by (4) and (5) directives above are approximately satisfied. However it

has been shown by Heckman et al¹³ that the exponent of Z appearing in those equations, which in turn comes from the definition of reduced velocity, is velocity dependent, tending to 1 as $\beta \rightarrow 1$. Therefore a residue of charge dependence not accounted for by (4) and (5) is expected to be present owing, at least, to that circumstance.

TABLE I

COEFFICIENTS OF THE POLINOMIAL EXPANSION IN Z^*

i	A_i
0	0.01096
1	-0.52953
2	0.19618
3	-0.23213
4	0.03074

The determination of the charge dependent residue goes through range calculations. Ranges are obtained by numerical integration of the inverse dE/dx function; we introduce a factor θ_Z to account for that residue:

$$R(E)_0 = \theta_Z \int_E^0 \frac{dE}{(dE/dx)_{\text{Ion}}} \quad (7)$$

The residue θ_Z is found by firstly scaling

the ranges of Carbon-, Nitrogen- Neon- and Argon-ions, calculated to CR-39, to standard nuclear emulsions by means of the proton curve for CR-39 (calculated) and for standard emulsion (from Janni's tables); secondly ion ranges thus obtained are fitted to those measured by Heckman et al¹³ in a MMQ calculation where θ is determined so as to minimize deviations. The values of θ obtained for each of the ions are then found as function of Z through a second MMQ fitting, with the result:

$$\theta = 0.92964 + 0.02989 Z \quad (8)$$

Figs. 2 and 3 show curves for ranges and for the function θ ; calculated ranges in figure 2 are within 10% or better from measured ones in all cases. Table II shows numerical values of ranges in CR-39 for selected ions from Helium to Iron and for E/M ranging from 0.1 to 200 MeV/Nucleon.

It has been suggested to workers in inorganic SSTD that the mechanism of energy deposition relevant to etchable damage formation depends upon the restricted energy loss (REL) rather than on the total average loss, dE/dx ¹⁴. We therefore found it worth while showing results on range

calculations to show also REL curves and to compare them with corresponding results obtained with the 'range extension' method.

Fig.4 shows the ranges in CR-39 obtained by us and those by Almasi and Somogyi¹¹ following range extension prescriptions; Fig. 5 shows REL($w = 1$ KeV) under both formulations. One can see that ranges are in fair agreement, exception of Iron ions, while REL values are slightly divergent also in the case of He. In the next section we will comment on this and other topics.

TABLE II

ION RANGES(μ) FOR ⁴He, ¹²C, ¹⁶O, ²⁰Ne, ²⁸Si, ⁴⁰Ar, ⁵⁶Fe

E/M	He	C	O	Ne	Si	Ar	Fe*
0.1	2.1	2.7	3.1	3.4	4.1	5.3	6.6
0.2	3.4	3.9	4.3	4.7	5.5	7.0	8.7
0.3	4.8	4.8	5.2	5.6	6.4	8.1	9.9
0.4	6.4	5.6	6.0	6.3	7.2	9.1	11.0
0.5	8.2	6.5	6.8	7.1	8.0	9.9	11.9
0.6	10.3	7.5	7.6	7.9	8.7	10.8	12.8
0.7	12.6	8.5	8.5	8.8	9.5	11.7	13.7

0.8	15.1	9.6	9.5	9.6	10.4	12.6	14.6
0.9	17.9	10.8	10.5	10.6	11.2	13.5	15.6
1.0	20.9	12.0	11.6	11.5	12.1	14.5	16.5
2.0	58.2	26.5	23.6	22.1	21.3	24.1	25.6
4.0	161	65.1	54.7	48.7	43.2	45.9	44.9
6.0	360	139	114	98.8	83.4	84.9	77.8
8.0	619	236	191	164	135	134	118
10	935	354	285	242	197	193	166
20	3152	1180	941	793	629	601	489
40	9559	3569	2840	2386	1877	1775	1408
60	22182	8274	6579	5523	4335	4087	3214
80	38632	14406	11453	9611	7538	7099	5566
100	58571	21839	17360	14567	11420	10751	8417
150	111569	41595	33061	27739	21740	20456	15995
200	191677	71456	56794	46648	37338	35126	27449

* extrapolated, unconfirmed data.

3 COMMENTS AND DISCUSSION

A word is to be said, firstly, about the limitations of this calculation. The reference experimental data

to which our results are shown to conform, consist in the measurement of ion ranges in nuclear emulsions of ref. 13. That set of measurements was chosen both because range measurements in nuclear emulsions are intrinsically better than in any SSTD and because it is the same set used to adapt the 'range extension' method to those detectors. Unfortunately that set does not include ions lighter than Carbon or heavier than Argon; therefore our numbers for Helium and Iron are tentative extrapolations, the case being the same for the calculations of Almasi and Somogyi¹¹.

As to the lighter particles group we have tried a comparison with data for the stopping cross section of alfa particles¹⁵; experimental points are converted into dE/dx for CR-39, by means of additivity rule, and plotted with our calculated curve in Fig.6. We find the agreement is fair to good, moreover if account is taken of difficulties associated with additivity already mentioned. Unfortunately, at the opposite extreme we do not have as good data to perform a reliable comparison so that results for Iron ions are but unconfirmed extrapolations.

Iron ions apart, the data for ranges in Fig.4 show fairly good agreement between both methods, range extension and effective charge calculations. We expect it to be so both because range extensions and effective charges are uniquely related^{12,13} and fiducial points are the same, taken from ref. 13; in a sense that agreement legitimates the uses of range extensions or nuclear effective charges as valid alternatives.

The same could be said about REL curves shown in Fig.5, if not for Iron ions, that diverge moderately and, perhaps, Helium ions. In the case of our calculation it is well possible that the charge residue as here determined is insufficient to account for all inaccuracies in effective charges. However we also notice the presence of source of errors in the treatment of ref. 11 that could be responsible for the discrepancy; if not for other equally important reasons, a hypothesis on effective charges has also to be done. Agreement between both sorts of REL calculations is however much better than expected on the basis of the different treatments adopted. We believe that by shifting the central problem from range extension measurements to effective charge calculations we open way to

further improvement, both because range extension measurements are difficult in SSTD and because one can profit from new experimental data on quantities related to effective charges that might help in its accurate determination.

CAPTIONS FOR FIGURES

Fig. 1a : Effective Charges for $\begin{matrix} 1 & 4 & 12 & 20 & 40 & 56 \\ \text{H, He, C, Ne, Ar, Fe} \end{matrix}$ in CR39, for $0 < E/M(\text{MeV/Nucleon}) < 1.0$

Fig. 2a : Effective Charges for $\begin{matrix} 1 & 4 & 12 & 20 & 40 & 56 \\ \text{H, He, C, Ne, Ar, Fe} \end{matrix}$ in CR39, for $1.0 < E/M(\text{MeV/Nucleon}) < 10$

Fig. 3 : Charge dependent residue (see text) as a function of Z.

Fig. 4 : Ranges in CR39 (solid lines); dotted lines represent ranges from ref. [11] .

Fig. 5 : $\text{REL}(w = 1 \text{ KeV})$ for $\begin{matrix} 4 & 12 & 20 & 56 \\ \text{He, C, Ne, Fe} \end{matrix}$ in CR39 (solid lines); dotted lines are corresponding curves from ref. [11].

Fig. 6 : Total rate of energy loss for alpha particles in CR39 (solid line); points are from ref. [15].

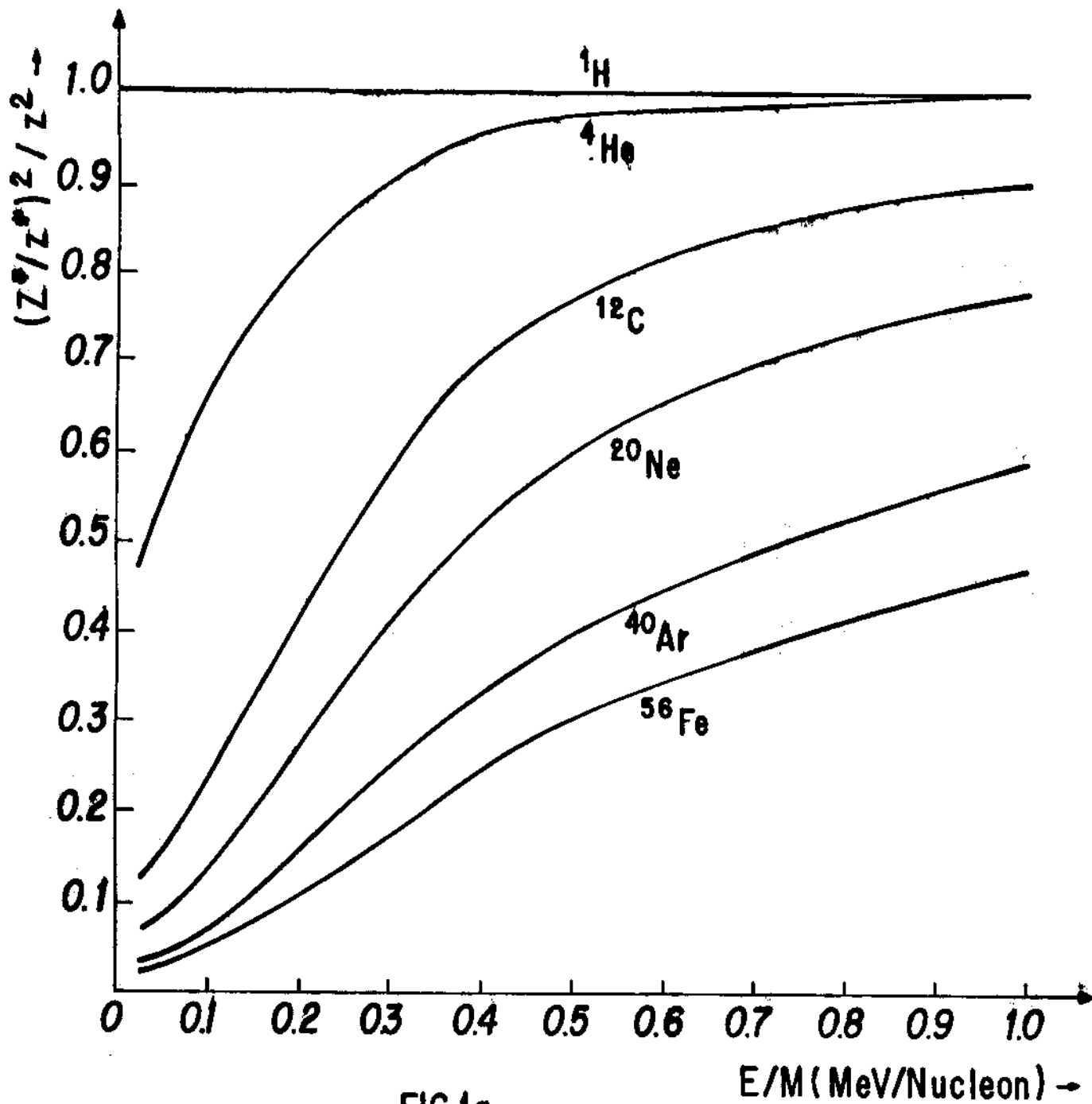


FIG.1a

 E/M (MeV/Nucleon) \rightarrow

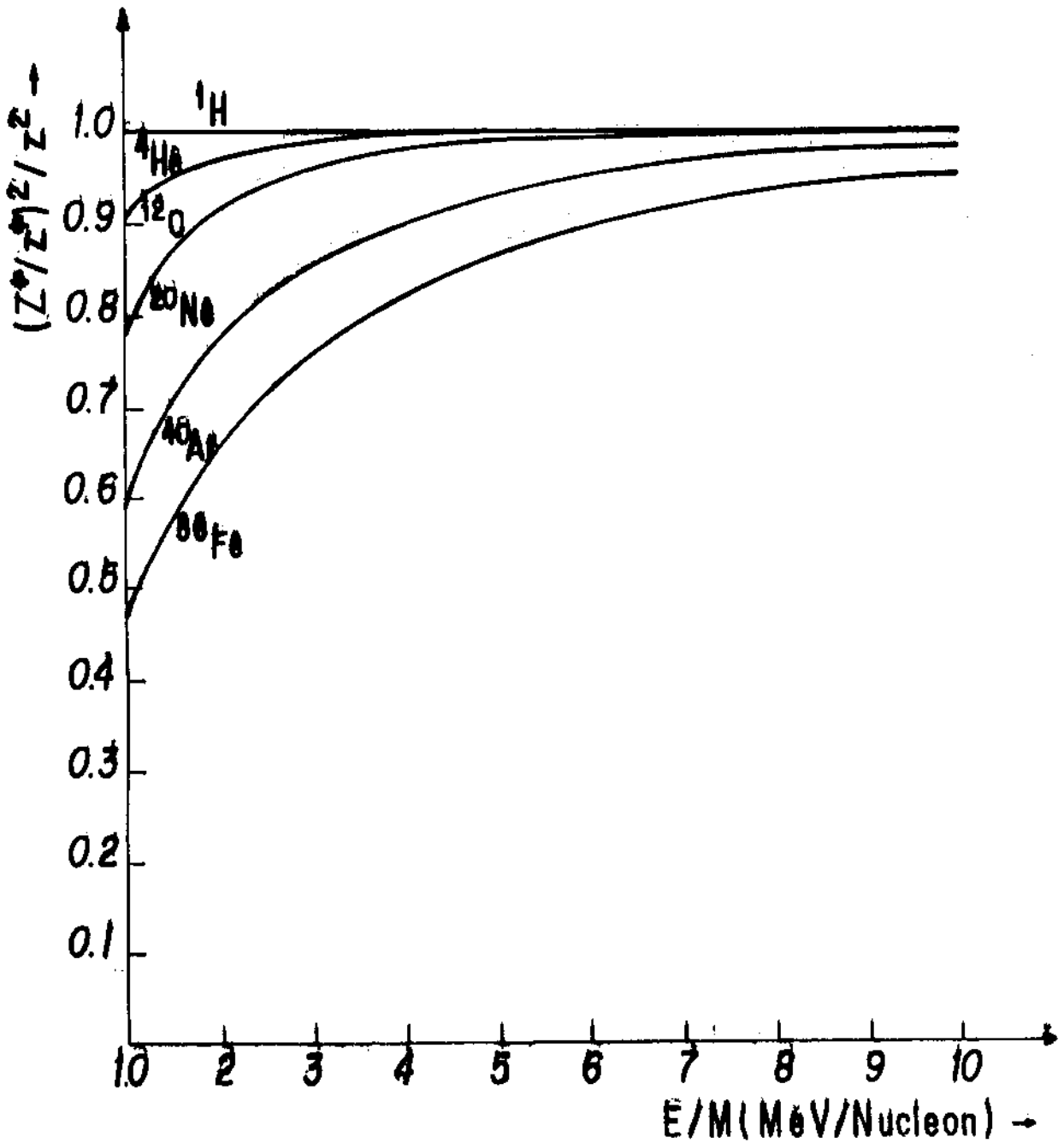


FIG. 1b

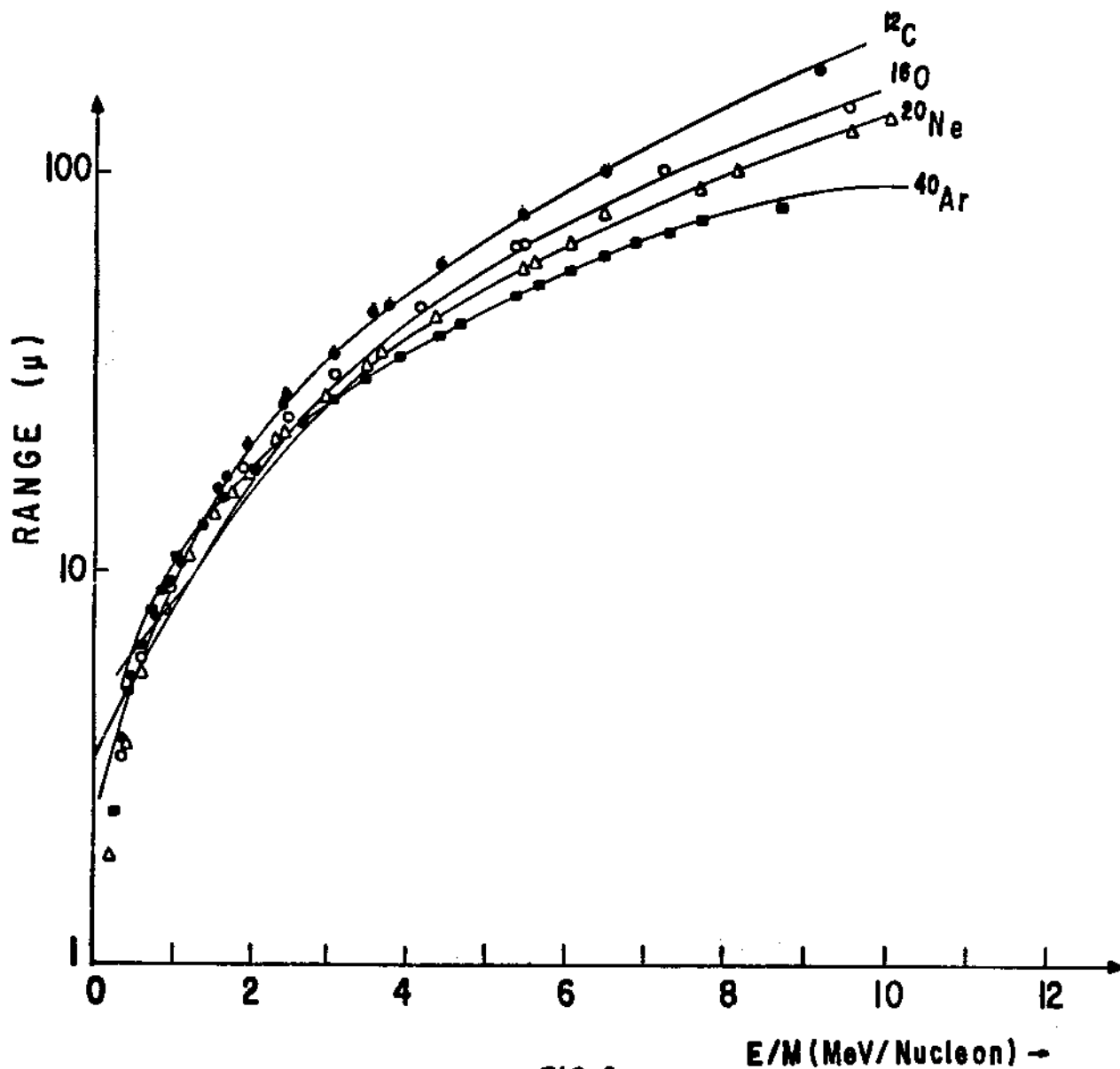


FIG. 2

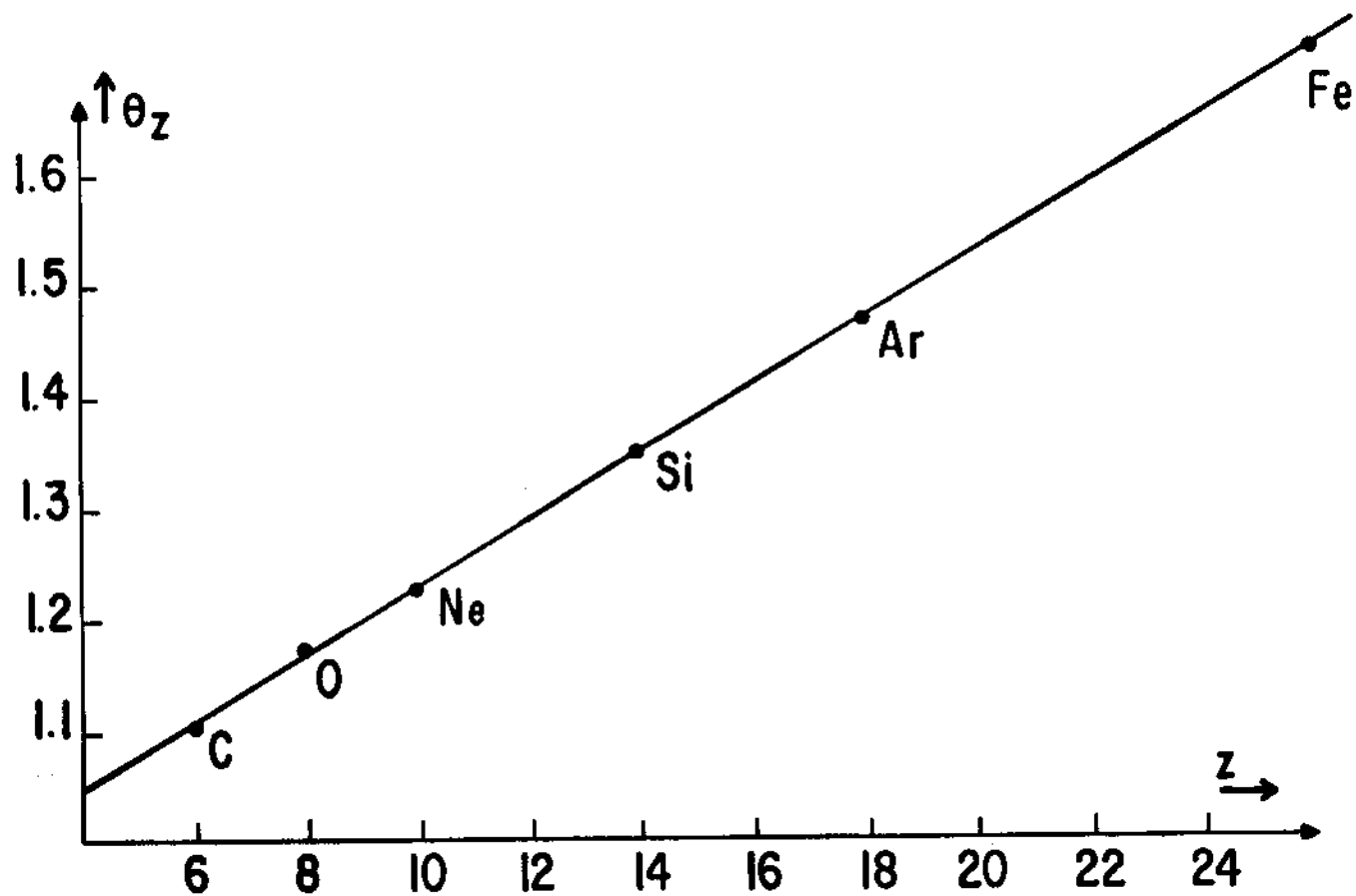


FIG. 3

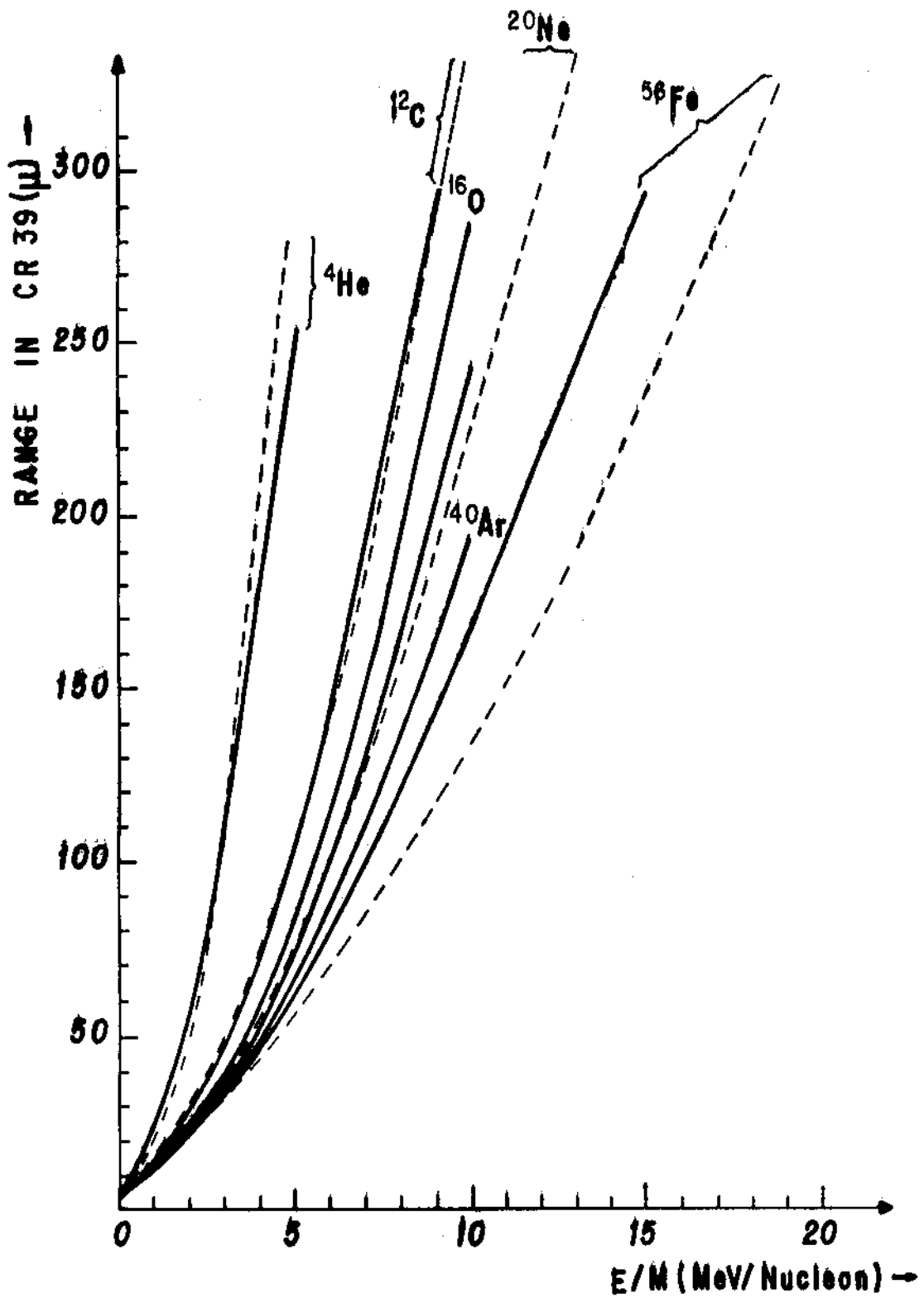


FIG. 4

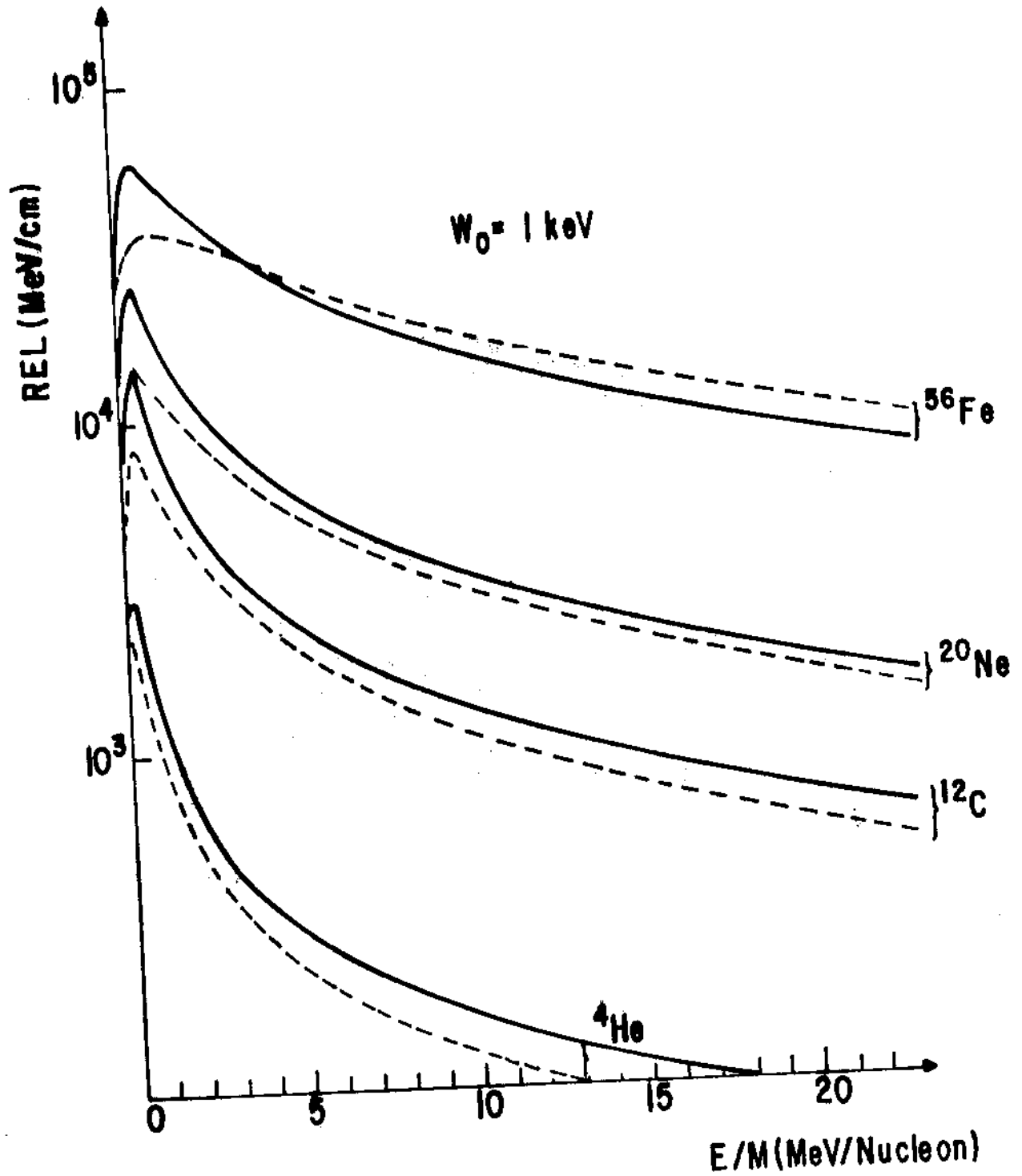


FIG. 5

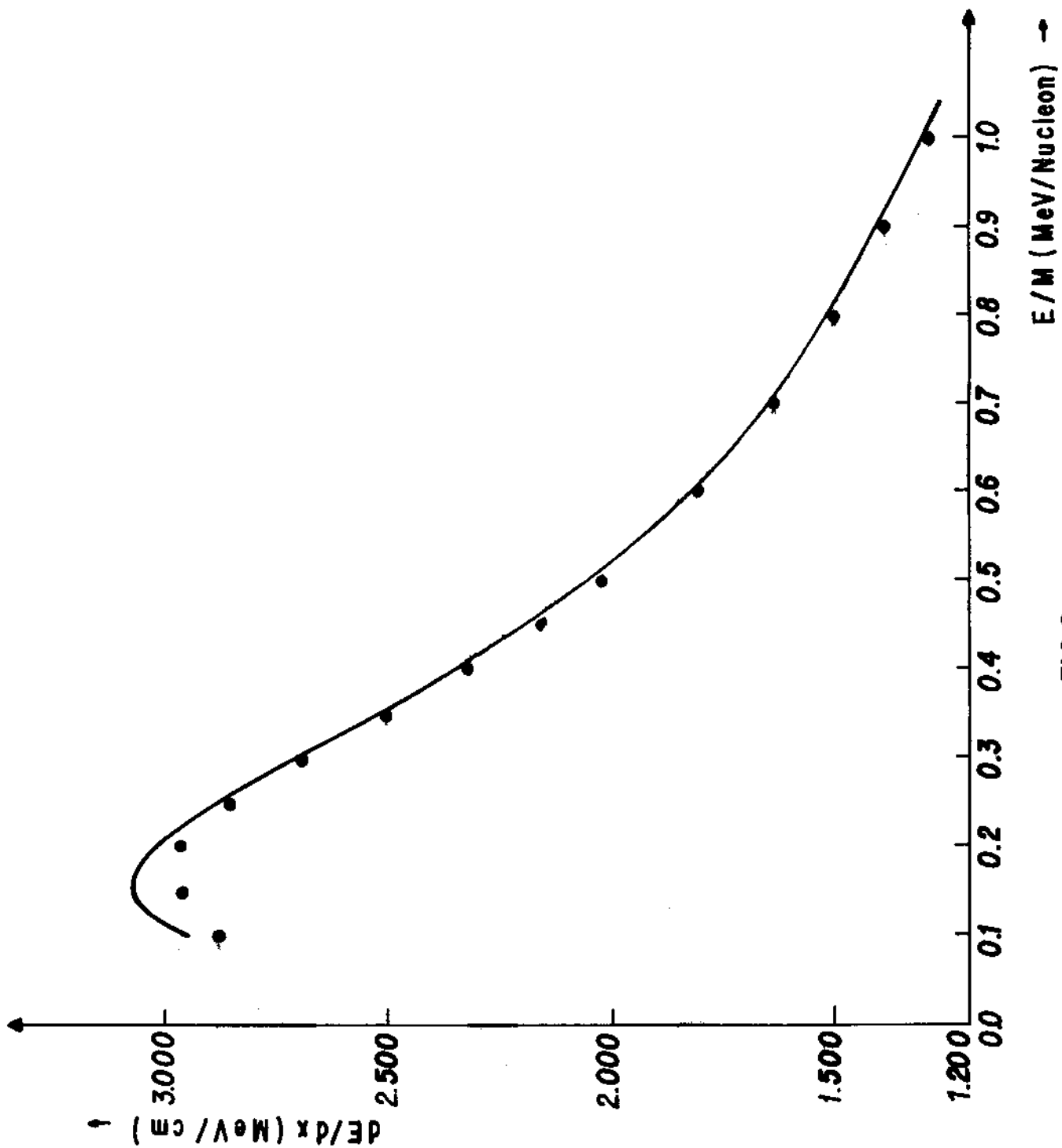


FIG.6

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