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MOST PROBABLE MASSES OF ISOTOPIC DISTRIBUTIONS AND SYSTEMATICS OF PHOTOSPALLATION YIELDS

By

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ABSTRACT

A Z-dependence of the most probable mass number A_{mp} for isotopic distributions of spallation residuals is proposed. The modified five-parameter semiempirical formula thus deduced seems to reproduce fairly well most of the experimental data of photo-spallation yields.

Key-words: Photospallation; Spallation; Isotopic distributions; Photo nuclear reactions.

Charge-Distribution-Mass-Distributions (isotopic distributions) of spallation residuals are described well enough by multiparametre semiempirical formulae($^{1-18}$) that, without any need of expensive and time consuming computer calculations (^{10,20}), have been proved to be suitable in reproducing the cross sections (or yields) of spallation residuals as a function of the mass- and atomic number (A_t,Z_t) of the target element and those of the product (A_i,Z_i), and of the nominal nucleon- and proton loss (ΔA_i,ΔZ_i).

Almost all these formulae contain, as a <u>free parametre</u>, the most probable (<u>mp</u>) mass, A_{mp}, of a given isotopic distribution on the mass-yield plane.

Due to the fact that $A_{mp}(Z)$ appears in the argument of a gaussian (or almost-gaussian) function, A_{mp} becomes an extremely critical parametre.

The aim of the present paper was to search for an analytical form of $A_{mp}(Z)$ that could give better results and cover much wider range of atomic numbers than do other formulae, especially in the case of photon-initiated spallation at intermediate energies (up to a few GeV).

Regardless of the experimental irradiation conditions, A_{mp} has always been shown to assume values which differ very little from the <u>A</u> values of the most abundant naturally occurring isotopes (with only a very few exceptions, within 0.5 to 1 A units) for any peculiar distribution (at fixed Z).

More precisely, A_{mp} seems to be very close to the average A value (A_{sw}) of the <u>n</u> existing stable masses for a given Z, <u>weighted</u> over the percent natural isotopic abundance w

$(\sum_{j=1}^{n} w_j = 1)$.

For each Z_i, $A_{mp}(Z_i)$ is consequently written as

$$(1) \qquad A_{mp}(Z_i) \cong A_{sw}(Z_i) = \sum_{j=1}^{n} [w_j A_{sj}(Z_i)] \quad ,$$

s standing for "stable", even if one or more A_{sj} are radioactive in nature.

Following the hypothesis analitically expressed by eq. (1), A_{sw} should represente centre of gravity of the n A_{sj} masses, and $A_{mp}(Z_i)$ the top of the distribution of isotopes having atomic number Z_i.

A careful regression analysis, with some data rejection, based on eq. (1) gave, from $Z = 6$ up to $Z = 83$, the following Z-dependence

$$(2) \qquad A_{sw(fit)} = \alpha Z^{\beta} \quad ,$$

with

$$(3) \qquad \begin{cases} \alpha = 1.590 \\ \beta = 1.103 \end{cases}$$

and a coefficient of correlation: $r^2 = 0.985$.

To verify the goodness of the fitting procedure and the homogeneity of the two-sample data $(A_{sw}, A_{sw(fit)})$ both parametric statistics were used.

Statistical Goodness of Fit (GOF) tests showed strong evidence as for homogeneity and correlation of the two samples were concerned (e.g., $R_{xy} \cong 1$, Spearman's $\rho = 0.999$, and reduced $\chi^2_{\nu=75} = 1.1$ as deduced from the Bartlett's criterion with a confidence interval of 90%).

A Student t-test (with $\nu = 74$ degrees of freedom and a probability of 0.975) was employed to obtain the errors effecting α and β

(4) $\alpha = 1.590 \pm 0.010$,

and

(5) $\beta = 1.103 \pm 0.009$.

From eqs. (1-5) we can write

(6) $A_{mp} \cong A_{fit} = (1.590 \pm 0.010) Z^{(1.103 \pm 0.009)}$,

(to avoid excess of symbols A_{fit} takes the place of $A_{sw,(fit)}$).

Table I shows a comparison between A_{sw} and A_{fit} ($\simeq A_{mp}$) for 13 values of Z (plus other two values out of systematics).

Plotted in Fig. 1 are the values of $A_{fit}(Z)$ (filled squares, from eq. (2)) and those of all naturally occurring isotopes A_s (open squares)*.

Following the hypotheses which eqs. (1) and (2) are based on, the filled squares of Fig. 1 should represent the bottom of the stability valley.

It is now very interesting to consider some of the A_{mp} dependences on Z proposed in (1-18,21) and in the present paper (eq. 2) in order to see how well they match the most abundant $A_s(Z_i)$.

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*There are, from ${}^{12}_{6}C$ to ${}^{209}_{33}Bi$, 274 natural elements for 76 Z values. About 15% of such element are radioactive, nevertheless they were included in the course of the analysis and in Fig.1, for they contribute to some extent to A_{sw}.

In doing this, we chose the R test, that is

$$R = \exp(\varepsilon) \quad , \tag{7}$$

with

$$\varepsilon = \left\{\frac{1}{m+n} \sum_{k=1}^{m} \sum_{i=i}^{n} [\ln(A_{s,ik}/A_{mp,ik})]^2\right\}^{1/2} = \tag{8}$$

$$= \left\{\frac{1}{274} \sum_{k=1}^{m} \sum_{j=i}^{n} [\ln(A_{s,jk}/A_{fit,jk})]^2\right\}^{1/2}$$

for grouped sets of m Z values (see later in Table II).

The quantity R takes, of course, values greater than unity. The greater the R value is, the larger is the difference $\ln A_{s,ik} - \ln A_{mp,ik}$ (or $A_{s,ik} - A_{mp,ik}$) and R allows a suitable test to be made in order to compare different A_{mp} dependences with each other.

Table II lists the results of such a comparison. Moreover, Fig. 2 shows the histogram of frequency distribution of $A_{mp}-A_s$ for the whole set of 274 A_s values ($6 \leq Z \leq 83$). A reduced $\chi^2 = 1.2$ was calculated.

At this point we are likely to analyse some expected changes of the values of parametres in our previous cross section formulae ([4,5,11]), due to the introduction of the present analytical form of A_{mp}.

As reference cross section formula for isotopic distribution we chose that reported as eq. (15) in Ref. ([11]), which is written as

$$\bar{\sigma}_i = a\bar{\sigma}_{\mathcal{N}} K_A \exp\,[-b(A-A_{mp}(Z_i))^2] \tag{9}$$

with

$$A_{mp} = cZ_i - d$$

and

$$(10) \qquad \begin{cases} a = 3.30\pm0.08 \\ \overline{\sigma}_N = 260\ \mu b \\ K_A = (1.67\pm0.03)A_t^{-(0.0580\pm0.0010)} \\ b = 0.243\pm0.005 \\ c = 2.28\pm0.07 \\ d = 2.18\pm0.09 \end{cases}$$

Cross section formula (9) gives the mean cross section <u>per</u> photon (energy range from about 0.2 GeV to about 1 GeV; $\overline{\sigma}_N$ = 260 μb for 0.3 GeV - 1. GeV range) of a spallation residual (A_i,Z_i) from a (A_t,Z_t) target. It contains the five parameters: <u>a</u> (a normalisation factor), K_A (slope of the yield-surface ridge (1,16-18) for isotopic distributions), b (which is related to the full-width-at-half-maximum $\Gamma = 2(\ln 2/6)^{1/2}$), and c and d (wich define A_{mp}). The $\overline{\sigma}_N$ is the mean cross section <u>per</u> photon of the elementary gamma-nucleon interaction(4,5).

If one substitutes A_{mp} of eq. (2) in eq. (9), the following equation is obtained for the natural log of the <u>normalised</u> cross-section (4,5) $\overline{\sigma}_i^{\,*}$

$$(11) \qquad \ln \overline{\sigma}_i^{\,*} = \ln[\overline{\sigma}_i/(K_A^{-\Delta Z_i} \times \overline{\sigma}_N)] = \ln(\zeta a) - \eta b(A_i - 1.590Z_i^{1.103})^2,$$

with ζ and η factors accounting for the changes in the parametres <u>a</u> and <u>b</u> (no change in K_A was espected, for K_A is much less sensitive to changes in A_{mp}).

A new regression analysis was thus carried out with a set of 112 measured mean cross section *per* photon for spallation residuals from V, Mn, Fe, and Co targets ([22,23]), and for ^{23}Na and ^{24}Na photoproduction from various targets ([4,5]).

The regression line gave

(12) $$\zeta a = 3.89 \pm 0.11$$

and

(13) $$\eta b = 0.247 \pm 0.007 \quad ,$$

(coefficient of determination $r^2 = 0.92$; Spearman's $\rho = -0.925$; reduced $\chi^2 = 1.71$).

The calculated trend of $\overline{\sigma}^*$ from eqs. (11), (12), and (13) is plotted in Fig. 3 (full line marked with *a*). Fig. 3 also reports $\overline{\sigma}^*$ from ([9,10]) (full line *b*) and the experimentally determined $\overline{\sigma}^*$ values.

Reduced χ^2 of 1.5 and 2.7 were obtained from statistical treatments of experimental data calculated ones from eq. (11) and eq. (9), respectively.

From eq. (11), eq. (12) and eq. (13), $\overline{\sigma}_i$ is written as

(14) $$\overline{\sigma}_i = 3.89\ \overline{\sigma}_N\ K_A^{-\Delta Z_i} \exp\left[-\ 0.247\ (A_i - 1.590\ Z_i^{1.103})^2\right] \ .$$

We tested eq. (14) further on by a set of cross sections *per* equivalent quantum (bremsstrahlung end-point energy E_0=1GeV) of gold isotopes from a ^{209}Bi target ([24,25]). Shown in Fig. 4 is the trend of eq. (14), modified for bremsstrahlung irradiations, and the experimental yields (note that the best-fit curve reaches its maximum a $A = A_s = 197$).

In concluding this note, we wish to put some stress on what follows:

i) eqs. (2) and (6) reproduce fairly well the A_{sw} and A_{mp} values (see Table I and Figs. 1 and 2);

ii) both Table II and Fig. 2 confirm eq. (6) as the most appropriate one to give a Z-dependence of A_{mp} (from Table II it is readily seen that eq. (2) gives the best result, also in consideration of the fact that it covers the whole range of Z; although very good results are also encountered for (12), especially for the ICSD-not-gaussian Rudstam's formula (12), the Z range is smaller than that covered by eq. (2));

iii) eq. (14) seems to reproduce experimental spallation yields with a fair approximation (reduced $\chi^2 = 1.5$; see also Figs. 3 and 4);

iv) all the gof tests furnish clear evidence for paired samples homogeneity and correlation.

The work is presently being carried on for isobaric distributions and charge-dispersion curves of photospallation residuals, also at photon energies above 1 GeV.

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FIGURE CAPTIONS

Fig. 1 - Trend of N (number of neutrons = A-Z) vs Z.

Open squares: $N_s(Z)$ for naturally occurring isotopes (mass A_s).

Filled squares: $N_{mp}(Z)$ (i.e. $A_{mp}-Z$) vs Z from eq. (2). For Z=43 (T_c) and Z=61 (p_m), and for $83 < Z < 90$, calculated values have only been plotted (the same for $Z > 92$). The dashed lines at Z=83 (Bi) indicate the upper limit of validity of eq. (2).

Fig. 2 - $A_{mp}-A_s$ frequency distribution (histogram) and best-fit gaussian frequency distribution (dashed curve). Histogram data: mode = 0.00 (dashed straight line), median = 0.42, mean = 0.56, $\Gamma = 6$, and area = 274 (a total of 274 elements having been considered). For the gaussian--shaped curve: $\mu = 0.533$ and $\Gamma = 5.83$.

Fig. 3 - Normalised cross section $\bar{\sigma}^*$ vs A_s-A_{mp}. Curve a represents $\bar{\sigma}_i^* = (\bar{\sigma}_i/K_A^{-\Delta Z_i}\,\bar{\sigma}_N a) = \zeta\exp[-0.247(A_{si}-A_{mp,i})^2]$, with $A_{mp,i} = 1.590\ Z_i^{1.103}$ and $\zeta = 1.18$ (see text). Curve b represents $\bar{\sigma}_i^* = (\bar{\sigma}_i/K_A^{-\Delta Z_i}\,\bar{\sigma}_w\ a) = \exp[-0.243(A_{si}-A_{mp,i})^2]$, with $A_{mp,i}(Z_i) = 2.28\ Z_i - 2.18$. Experimental points from Ref. (22) (open circles) and Ref. (23,4,5) (filled circles). As for the FWHM, $\Gamma_a = 3.35$ and $\Gamma_b = 3.38$ were obtained.

Fig. 4 - Yield of gold isotopes from ^{209}Bi at $E_0 = 1$ GeV.

Open circles: data of Ref. (24). Filled circles: data of Ref. (25). For ^{195}Au (filled circle in parentheses) some uncertainties in the decay scheme and difficulties in radioactivity measurments were met. The curve represent Eq. (14).

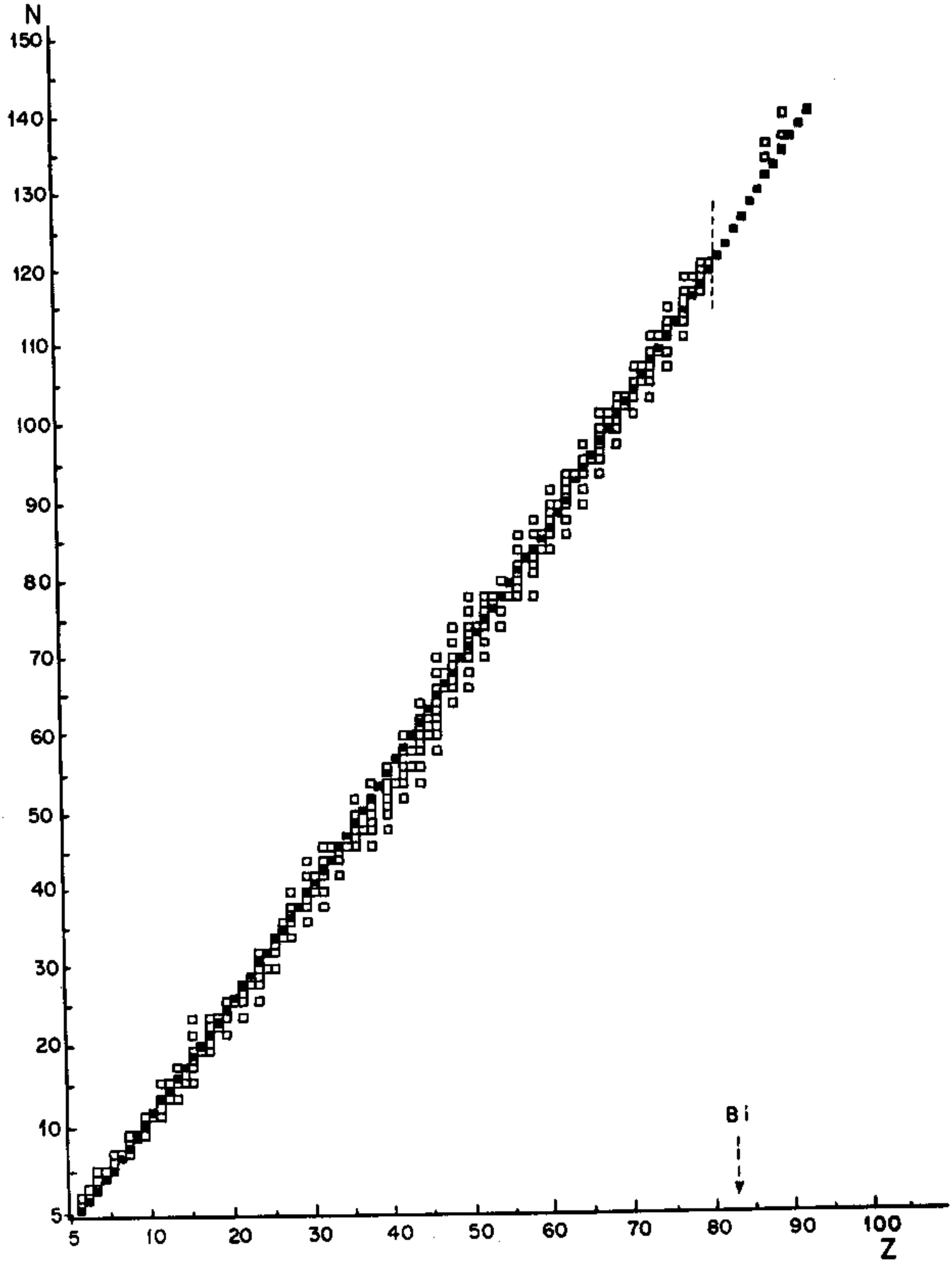
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Fig. 1

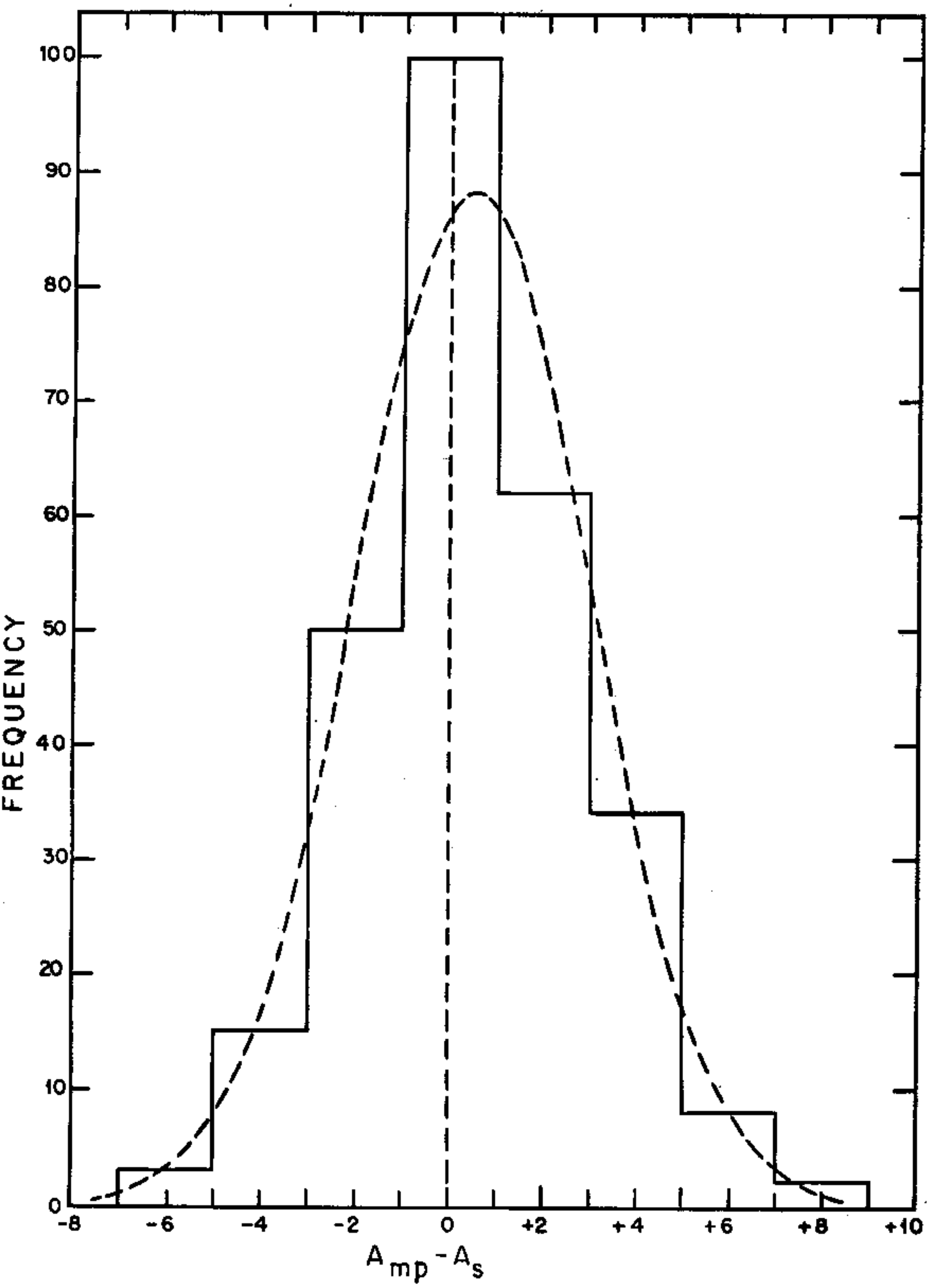
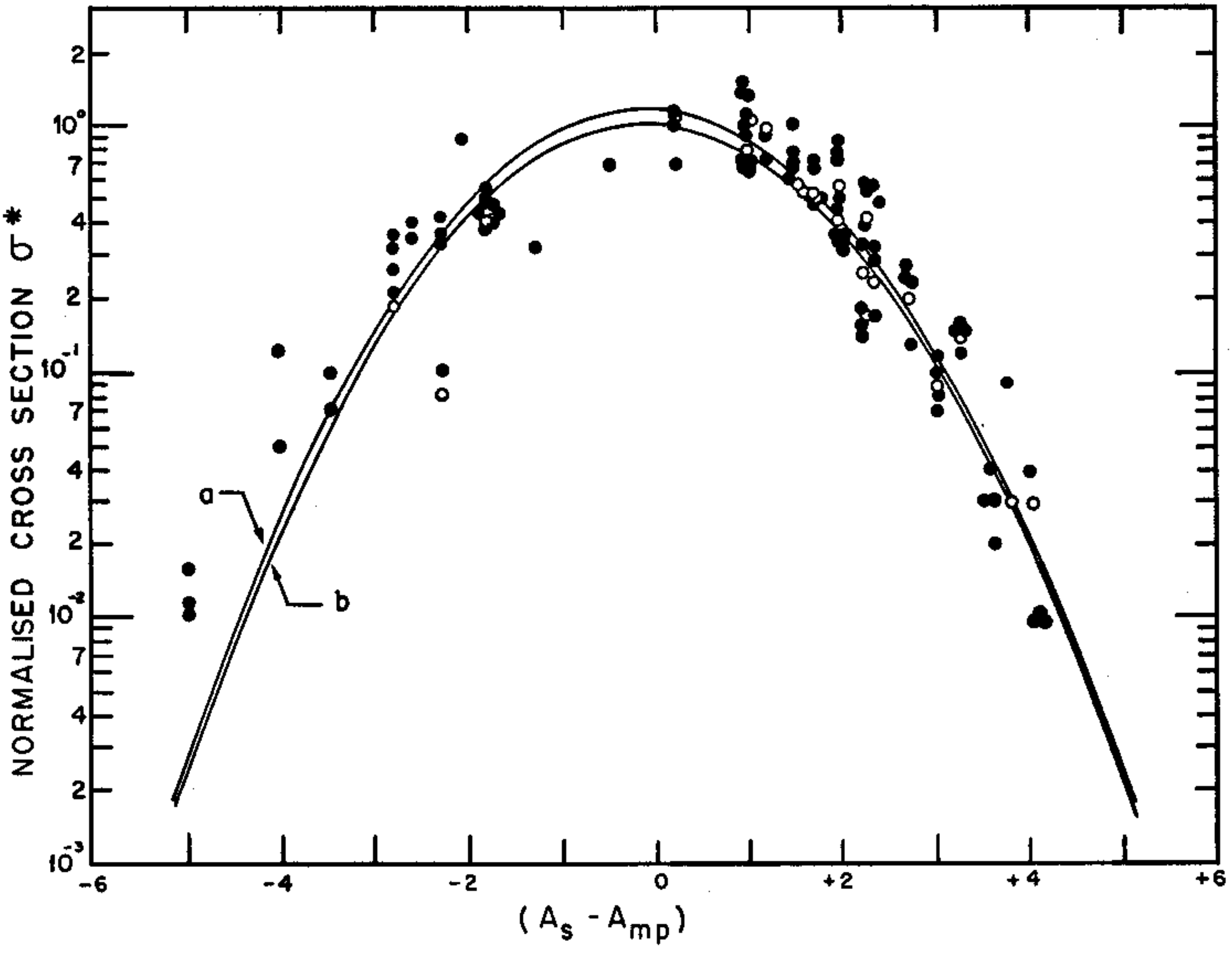


Fig. 2


NORMALISED CROSS SECTION σ^*
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10^0
7
4
2
10^{-1}
7
4
2
10^{-2}
7
4
2
10^{-3}
-6
-4
-2
0
+2
+4
+6
$(A_s - A_{mp})$
a
b

Fig. 3

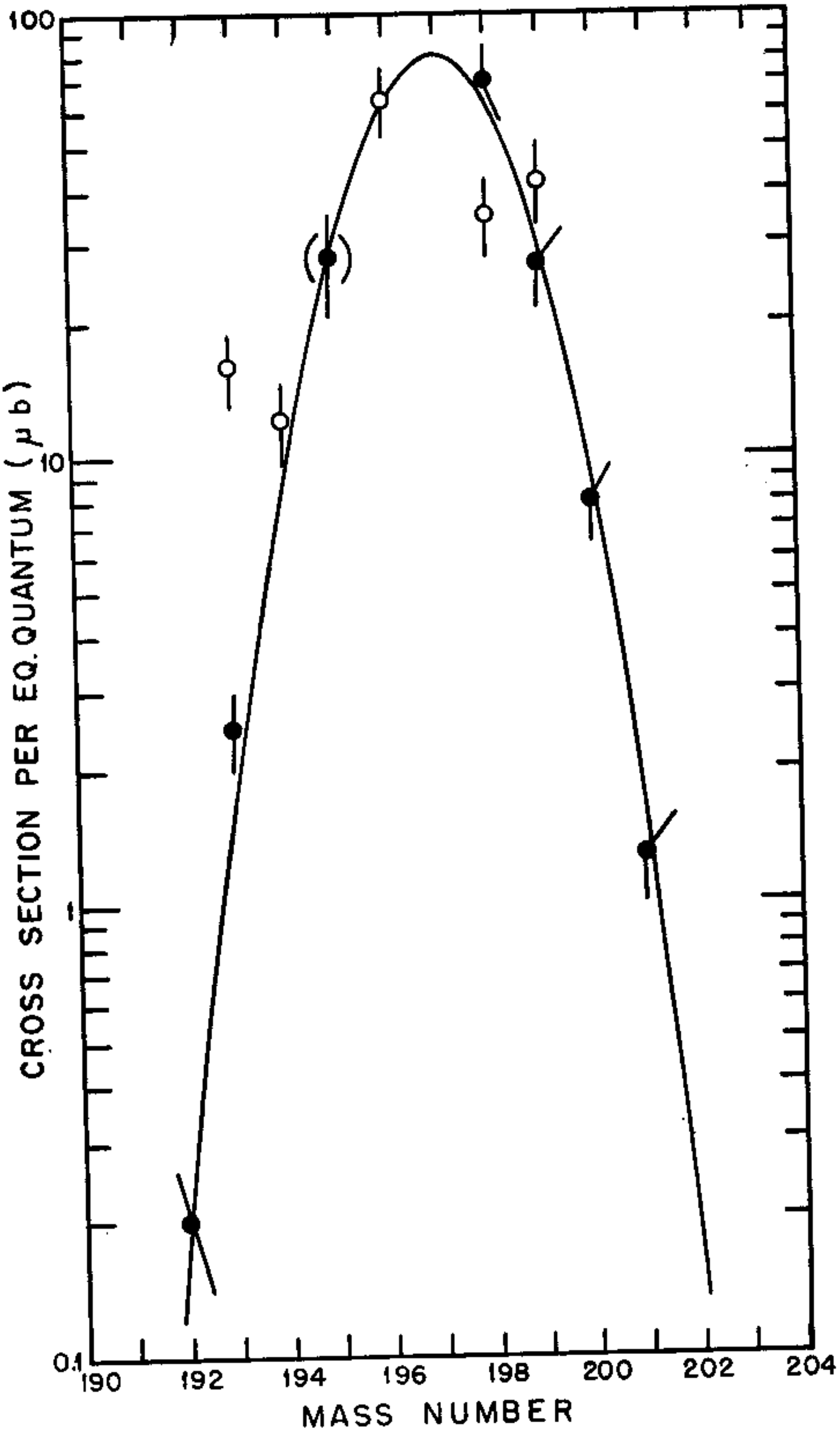
100
10
1
0.1
190
192
194
196
198
200
202
204
MASS NUMBER
CROSS SECTION PER EQ. QUANTUM (μ b)

Fig. 4

TABLE I. - Comparison between A_{sw} and A_{fit}.

ELEMENT	Z	A_{sw} Eq. (1)	A_{fit} Eq. (2)	$\Delta\%^{(*)}$	Spread (eq.6) $A_{fit,min}$	$A_{fit,max}$
C	6	12.01	11.47	+4.5	11.26	11.70
O	8	16.00	15.76	+1.5	15.43	16.09
Ne	10	20.18	20.16	+0.1	19.71	20.61
Al	13	27.00	26.92	+0.3	26.27	27.58
Cl	17	35.49	36.19	-2.0	35.26	37.15
Ar	18	39.99	38.54	+3.6	37.53	39.58
Sc	21	45.00	45.69	-1.3	44.44	46.97
V	23	51.00	50.51	+1.0	49.10	51.96
As	33	75.00	75.22	-0.3	72.94	77.57
I	53	127.00	126.85	+0.1	122.59	131.24
Sm	62	150.35	150.80	-0.3	145.59	156.20
Au	79	197.00	197.01	(-)0.0	189.87	204.40
Bi	83	209.00	208.04	+0.5	200.43	215.92
Th(**)	90	232.00	227.47	+2.0	219.03	236.23
U(**)	92	237.97	233.05	+2.1	224.37	242.06

(*) $\Delta\% = [(A_{sw}-A_{fit})/A_{sw}] \times 100$

(**) Out of systematics.

TABLE II. - R-values for different A_{mp} formulae.

Partial Z ranges					Cumulative Z ranges				References
6-20	21-40	41-60	62-83	62-92 (*)	6-40	6-60	6-83	6-92 (**)	
1.052	1.023	1.019	1.013	1.013	1.038	1.032	1.030	1.029	21, 4, 5
1.041	1.028	1.005	-	-	1.034	1.047	-	-	4, 5
1.051	1.025	1.057	-	-	1.038	1.045	-	-	9, 5
1.044	1.026	1.060	-	-	1.035	1.045	-	-	4,
1.040	1.037	1.063	-	-	1.038	1.048	-	-	12
1.044	1.034	1.061	-	-	1.038	1.047	-	-	12
1.059	1.032	1.049	-	-	1.045	1.047	-	-	12
1.035	1.031	1.044	-	-	1.033	1.037	-	-	12
1.038	1.023	1.017	1.006	1.008	1.031	1.027	1.023	1.023	Present work (eq.(2))

(*) Same Z values as in column 4, plus Z=90 and Z=92 (out of systematics).

(**) Same Z values as in column 8, plus Z=90 and Z=92 (out of systematics).

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