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QUANTUM OSCILLATORS IN THE CANONICAL COHERENT STATES

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Abstract

The main characteristics of the quantum oscillator coherent states including the two-particle Calogero interaction are investigated. We show that these Calogero coherent states are the eigenstates of the second-order differential annihilation operator which is deduced via Wigner-Heisenberg algebraic technique and correspond exactly to the pure uncharged-bosonic states. They possess the important properties of non-orthogonality and completeness. The minimum uncertainty relation for the Wigner oscillator coherent states are investigated. New sets of even and odd coherent states are point out.

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I. INTRODUCTION

In the beginning of the sixties, the coherent states were investigated via three definitions, viz., states of minimal uncertainty, eigenstates of the annihilation operator and as being the states obtained by application of the displacement operator on the ground state [1]. In reference [1] it has been shown that these three definitions are equivalent for the simple harmonic oscillator. Due to the fact that the energy spectrum of a particle in a potential with centripetal barrier

$$V(x) = \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}gx^{-2}, \quad g = \lambda(\lambda+1), \quad -\infty \leq x \leq \infty, \tag{1}$$

is equally spaced like that of the simple harmonic oscillator, this one-dimensional (1D) system is called an "isotonic oscillator" or two-particle Calogero interaction [2]. In 3-dimensional space this type of potential was first introduced by Davidson long ago [3].

Considered first by Weissman and Jortner [4] in the context of Gaussian wave functions, the dynamics and the energies of a coherent states for the 1D isotonic oscillator were studied. Elsewhere Nieto and Simmons, Jr. found the minimum-uncertainty coherent states (MUCS) and discussed various properties of this system [5].

A year later it was shown by Gutschick, Nieto and Simmons, Jr. [6] that the MUCS provide us with a better aproximation to the classical motion than do the Gaussians. In another work Nieto [7] has shown the mathematical and physical connection of the charged-boson coherent states [8]

with the MUCS. The canonical coherent states for the Wigner generalized oscillator in the Schrödinger representation were constructed by Sharma, Mehta and Sudarshan [9] and the representations and properties of para-bose oscillator operators were investigated in a Schrödinger description [10].

On the other hand, Leinaas and Myrhein [11] have investigated the relation bettween the fractal in 1+1 dimension and the Calagero problem.

In this work, we construct what we call canonical coherent states (CCS) [12], which are defined as the eigenstates of the annihilation operator $B^-(\lambda)$ of the Calogero interaction Hamiltonian.

Such annihilation operators are second-order differential ladder operators [13] and can be derived via Wigner-Heisenberg (WH) algebraic technique [14] which was recently super-realized for the SUSY isotonic oscillator [15]. The WH algebra has also been investigated for the three-dimensional non-canonical oscillator to generate a representation of the orthosympletic Lie superalgebra $osp(3/2)$ [16]. The coherent states of $SU(\ell, 1)$ groups have been explicitly constructed as orbits in some irreducible representations [17].

The motion of the peaks of the wavefunctions for the coherent states of the two-particle Calogero-Sutherland model were compared with the classical trajectory [20].

According to Calagero [2] the energy spectrum of the potential (1) and N bosons or fermions interacting are different by a energy shift proporcional to $\nu = -\lambda$. Using an operator formulation Brink et al found all N-particle eigenwave functions and extended the approach to the supersymmetric Calogero model [18] and Heisenberg algebra in the simple case of two particles [19].

The observables for the two-anyon problem, satisfy the same algebra as the observables in two-body Calogero problem.

Let us here point out the interesting connection between the mesoscopic effects and Calagero interaction for a Coulomb gas under a new universality in spectra of this chaotic system, which is descried by a random matrix theory [21].

Another approach is the application of the time-dependent parameters in the potential (1) in many quantum-mechanical effects. For instance, Pedrosa et al have used the Lewis-Riesenfeld invariant method and a unitary transformation to obtain the exact Schrödinger wave functions for a time-dependent harmonic oscillator with and without an inverse quadradtic potential [22].

Recently, Witten's supersymmetry formulation for Hamiltonian systems [23] has been extended to a system of annihilation operator eigenvalue equations associated with the supersymmetric unidimensional oscillator and supersymmetric isotonic oscillator, which define supersymmetric canonical coherent states containing mixtures of both pure bosonic and pure fermionic counterparts [24]. The breaking of supersymmetry due to singular potentials in supersymmetric quantum mechanics given by Eq. (1) has been recently investigated [25].

In this work we present some graphs of the behaviour of the minimum uncertainty for a particular set of Wigner oscillator CCS. This present work is organized in the following way. In Sec. II we start by summarizing the Wigner-Heisenberg algebraic technique for the Wigner isotonic oscillator [15]. In Sec III, we define and build without supersymmetry the two-particle Calogero interaction canonical coherent states as the eigenstates of the second-order differential annihilation operator (B^-). In Sec. IV, we construct the MUCS from Wigner first-order differential ladder operators. In this section, new even and odd canonical coherent states are pointed out. Sec. V contains the conclusion.

II. THE SUPER WIGNER-HEISENBERG ALGEBRA

For convenience we choose units so that $\hbar = \omega = m = 1$. Thus the system governed by the potential (1) becomes identical to the bosonic sector of the Wigner Hamiltonian. The 1D Wigner oscillator Hamiltonian in terms of the Pauli's matrices (σ_i, i=1,2,3) is given by

$$\begin{aligned} H(\lambda+1) &= \frac{1}{2}\left\{-\frac{d^2}{dx^2} + x^2 + \frac{1}{x^2}(\lambda+1)[(\lambda+1)\sigma_3 - 1]\sigma_3\right\} \\ &= \begin{pmatrix} H_-(\lambda) & 0 \\ 0 & H_+(\lambda) = H_-(\lambda+1) \end{pmatrix}, \end{aligned} \tag{2}$$

where the even and odd sector Hamiltonians are respectively given by

$$H_-(\lambda) = \frac{1}{2}\left\{-\frac{d^2}{dx^2} + x^2 + \frac{1}{x^2}\lambda(\lambda+1)\right\} \tag{3}$$

and

$$H_+(\lambda) = \frac{1}{2}\left\{-\frac{d^2}{dx^2} + x^2 + \frac{1}{x^2}(\lambda+1)(\lambda+2)\right\} = H_-(\lambda+1). \tag{4}$$

The even sector is the Hamiltonian of the oscillator with barrier.

From the super-realized first order ladder operators given by

$$a^{\pm}(\lambda+1) = \frac{1}{\sqrt{2}}\left\{\pm\frac{d}{dx} \pm \frac{(\lambda+1)}{x}\sigma_3 - x\right\}\sigma_1, \tag{5}$$

the Wigner Hamiltonian becomes

$$H(\lambda+1) = \frac{1}{2}\left[a^+(\lambda+1), a^-(\lambda+1)\right]_+ \tag{6}$$

and the WH algebra ladder relations are readily obtained as

$$\left[H(\lambda+1), a^{\pm}(\lambda+1)\right]_- = \pm a^{\pm}(\lambda+1). \tag{7}$$

Equations (6) and (7) together with the commutation relation

$$\left[a^-(\lambda+1), a^+(\lambda+1)\right]_- = 1 + 2(\lambda+1)\Sigma_3 \tag{8}$$

constitute the WH algebra. The Wigner eigenfunctions that generate the eigenspace associated with even(odd) Σ_3-parity for even(odd) quanta $n = 2m(n = 2m+1)$ are given by

$$\mid n = 2m, \lambda+1 >= \begin{pmatrix} \mid m, \lambda+1 > \\ 0 \end{pmatrix}, \quad \mid n = 2m+1, \lambda+1 >= \begin{pmatrix} 0 \\ \mid m, \lambda > \end{pmatrix} \tag{9}$$

and satisfy the following eigenvalue equation

$$H(\lambda+1) \mid n, \lambda+1 >= E^{(n)} \mid n, \lambda+1 >, \tag{10}$$

where the non-degenerate energy eigenvalues are obtained by the application of the raising operator on the ground eigenstate and are given by

$$E^{(n)} = \lambda + \frac{3}{2} + n, \quad n = 0, 1, 2, \ldots. \tag{11}$$

For the oscillator with barrier the energy eigenvectors satisfy the following equations

$$H_-(\lambda) \mid m, \lambda >= E_-^{(m)} \mid m, \lambda >, \tag{12}$$

where the eigenvalues are exactly constructed via WH algebra ladder relations and are given by

$$E_-^{(m)} = \lambda + \frac{3}{2} + 2m, \qquad m = 0, 1, 2, \ldots. \tag{13}$$

Also from the Wigner-Heisenberg algebra we obtain the second-order differential raising and lowering operators for the energy spectrum of the 1D oscillator with barrier,viz., on $\frac{1}{2}(1+\sigma_3)$ projection, the WH algebra representations decouple, $\left[H(\lambda+1), a^{\pm 2}(\lambda+1)\right]_- = \pm 2a^{\pm 2}(\lambda+1)$. Indeed, the left hand side leads us

$$\frac{1}{2}(1+\sigma_3)[H(\lambda+1), a^{\pm 2}(\lambda+1)]_- = \begin{pmatrix} [H_-(\lambda), B^{\pm}(\lambda)]_- & 0 \\ 0 & 0 \end{pmatrix}$$

and the right hand side becomes

$$\frac{1}{2}(1 \pm \sigma_3)a^{\pm 2}(\lambda+1) = \begin{pmatrix} B^{\pm}(\lambda) & 0 \\ 0 & 0 \end{pmatrix},$$

where

$$B^-(\lambda) = \frac{1}{2}\left\{\frac{d^2}{dx^2} + 2x\frac{d}{dx} + x^2 - \frac{\lambda(\lambda+1)}{x^2} + 1\right\} \tag{14}$$

and

$$B^+(\lambda) = \frac{1}{2}\left\{\frac{d^2}{dx^2} - 2x\frac{d}{dx} + x^2 - \frac{\lambda(\lambda+1)}{x^2} - 1\right\}. \tag{15}$$

Thus, these ladder operators obey the following commutation relations:

$$\begin{aligned} [B^-(\lambda), B^+(\lambda)]_- &= 4H_-(\lambda) \\ \left[H_-, B^\pm(\lambda)\right]_- &= \pm 2B^\pm(\lambda). \end{aligned} \tag{16}$$

Hence, the quadratic operators $B^\pm(\lambda)$ acting on the orthonormal basis of eigenstates of $H_-(\lambda)$, $\{\mid m, \lambda >\}$ where $m = 0, 1, 2, \cdots$ have the effect of raising or lowering the quanta by two units so that we can write

$$B^-(\lambda) \mid m, \lambda >= \sqrt{2m(2m + 2\lambda + 1)} \mid m - 1, \lambda > \tag{17}$$

and

$$B^+(\lambda) \mid m, \lambda >= \sqrt{2(m+1)(2m + 2\lambda + 3)} \mid m + 1, \lambda > \tag{18}$$

giving

$$\mid m, \lambda >= 2^{-m}\left\{\frac{\Gamma(\lambda + 3/2)}{m!\Gamma(\lambda + m + 3/2)}\right\}^{1/2} \left\{B^+(\lambda)\right\}^m \mid 0, \lambda >, \tag{19}$$

where $\Gamma(x)$ is the ordinary Gamma Function.

Note that $B^\pm(\lambda) \mid m, \lambda >$ are associated with the energy eigenvalues $E_-^{(m\pm1)} = \lambda + \frac{3}{2} + 2(m \pm 1), \quad m = 0, 1, 2, \ldots$

Let us to conclude this section presenting a very simple question: what is the structure generated by the new operators pointed out in this section from quantum oscillator? Note that the operators $\pm\frac{i}{2}\left(a^\pm(\lambda+1)\right)^2$ and $\frac{1}{2}H(\lambda+1)$ can be chosen as a basis for a realization of the $SO(2,1) \sim SU(1,1) \sim SL(2, \mathrm{R})$ Lie algebra, and therefore generate the generalized coherent states according to Perelomov [26,31].

When projected the $\frac{1}{2}\left(a^\pm\right)^2$ operators in the even sector we obtain that $\frac{1}{2}B^\pm$ and together with $\frac{1}{2}H_-$ generate once again the Lie algebra $SO(2,1)$.

III. CALOGERO INTERACTION CANONICAL COHERENT STATES

Now, we define the Calogero interaction canonical coherent states, $\mid \alpha, \lambda >$, as the eigenkets of the annihilation operator $B^{-}(\lambda)$,

$$B^{-}(\lambda) \mid \alpha, \lambda >= \alpha \mid \alpha, \lambda >, \tag{20}$$

where the eigenvalue α can be any complex number. Writing

$$\mid \alpha, \lambda >= \sum_{m=0}^{\infty} b_m \mid m, \lambda > \tag{21}$$

we obtain a recursion relation for the coeficients b_m

$$b_m = \frac{\alpha}{2} \left\{ m(m + \lambda + \frac{1}{2}) \right\}^{-\frac{1}{2}} b_{m-1} \tag{22}$$

which provides us with the normalized canonical coherent states for the Calogero interaction in the form

$$\mid \alpha, \lambda >= \{g(\mid \alpha \mid)\}^{-\frac{1}{2}} \sum_{m=0}^{\infty} \frac{(\frac{\alpha}{2})^m}{\{m!\Gamma(m + \lambda + \frac{3}{2})\}^{\frac{1}{2}}} \mid m, \lambda >, \tag{23}$$

where

$$g(\mid \alpha \mid) = \left\{ \frac{2}{\mid \alpha \mid} \right\}^{(\lambda + \frac{1}{2})} I_{\lambda + \frac{1}{2}}(\mid \alpha \mid) \tag{24}$$

and $I_{\nu}(\mid \alpha \mid)$ is the modified Bessel function of the first kind,

$$I_{\nu}(\mid \alpha \mid) = \sum_{m=0}^{\infty} \frac{\{\frac{|\alpha|}{2}\}^{(2m+\nu)}}{m!\Gamma(m + \nu + 1)}. \tag{25}$$

The Calogero interaction CCS are normalized however they are non-orthogonal since

$$< \xi, \lambda \mid \alpha, \lambda >= \{g(\mid \alpha \mid) g(\mid \xi \mid)\}^{-\frac{1}{2}} g\left((\xi^* \alpha)^{\frac{1}{2}} \right). \tag{26}$$

The resolution of unity is given by

$$\int \mid \alpha, \lambda >< \alpha, \lambda \mid \frac{1}{2\pi} K_{\lambda + \frac{1}{2}}(\mid \alpha \mid) I_{\lambda + \frac{1}{2}}(\mid \alpha \mid) d^2\alpha = \sum_{m=0}^{\infty} \mid m, \lambda >< m, \lambda \mid = \mathbf{1}, \tag{27}$$

where $x = \mid \alpha \mid, z = 1$ and $t = \sinh u$ with

$$\begin{aligned} K_{\nu}(\mid \alpha \mid) &= 2^{\nu} \frac{\Gamma(\nu + \frac{1}{2})}{\mid \alpha \mid^{\nu} \sqrt{\pi}} \int_0^{\infty} \frac{\cos(\mid \alpha \mid t)}{(t^2 + 1)^{\nu + \frac{1}{2}}} dt \\ &= \frac{2\Gamma(\nu + \frac{1}{2})}{\mid \alpha \mid \sqrt{\pi}} \int_0^{\infty} (\cosh u)^{-2\nu} \cosh(\mid \alpha \mid \sinh u) du, \end{aligned} \tag{28}$$

which is a particular form of $K_{\nu}(zx)$ is the modified Bessel function of the third kind [27].

Now let us consider a particular case of the Wigner oscillator coherent states.

IV. THE MINIMUM UNCERTAINTY CCS

Let us begin by making some remarks about the CCS and MUCS of ref. [5]. The MUCS $\mid \alpha, \lambda >$ are eigenstates of an annihilation operator A_n^-, where

$$\mid \alpha, \lambda >= \{g(\mid \alpha \mid)\}^{-\frac{1}{2}} \left\{\Gamma(\lambda + \frac{3}{2})\right\}^{\frac{1}{2}} \left\{\frac{\sqrt{\alpha B^+(\lambda)}}{2}\right\}^{-(\lambda+\frac{1}{2})} I_{\lambda+\frac{1}{2}} \left(\frac{\sqrt{\alpha B^+(\lambda)}}{2}\right) \mid 0, \lambda > . \tag{29}$$

The form of the operator acting on the ground state in above when compared with equation (2.72) of ref. [5], suggests that our α and $B^+(\lambda)$ correspond to 2β and $2A^+$ of ref. [5]. Besides, the completeness property here deduced is formally analogous to the resolution of the identity for the isotonic oscillator minimun-uncertainty coherent states [7]. However, one obtain this properties from our operators deduced via the WH algebra.

To complete our analysis, we trace below the construction of minimum uncertainty coherent states (MUCS) for the Wigner oscillator operators position $\hat{x}$ and momentum $\hat{p}$ defined by

$$\sqrt{2} a^{\mp} = (\mp i\hat{p} - \hat{x}) \tag{30}$$

$$\rightarrow \hat{x} = \Sigma_1 x, \quad \hat{p} = \Sigma_1 \left\{ -i\frac{d}{dx} + \frac{i}{x}(\lambda + 1)\Sigma_3 \right\} \tag{31}$$

which satisfy the super generalized quantum commutation relation

$$[\hat{x}, \hat{p}] = i \left\{ 1 + 2(\lambda + 1)\Sigma_3 \right\} \tag{32}$$

by virtue of (8). Developing the usual procedure for the construction of minimum uncertainty states $|\psi >_M$ with equal dispersions for position and momentum, one has

$$a^- |\psi >_M = \beta |\psi >_M, \quad \beta = \sqrt{\alpha} = -\frac{1}{\sqrt{2}} \left(\langle \hat{x} \rangle + i \langle \hat{p} \rangle \right), \tag{33}$$

which identifies that $|\psi >_M = |z >_{\mathrm{W}}$ is an eigenstate particular set of a^-.

Now, considering the spectation values of position and momentum on the Wigner oscillator coherent states associated to the even and odd quanta we obtain

$$
\begin{aligned}
<\hat{x}^2>_e &= 2[Re(\beta)]^2+\lambda+\frac{3}{2}\\
<\hat{x}^2>_o &= 2[Re(\beta)]^2-\lambda-\frac{1}{2}\\
<\hat{p}^2>_e &= -\left(2[Im(\beta)]^2+\lambda+\frac{3}{2}\right)\\
<\hat{p}^2>_o &= -2[Im(\beta)]^2+\lambda+\frac{1}{2}
\end{aligned}
\tag{34}
$$

where $<\hat{x}^2>_o$, for the odd quanta and $<\hat{x}^2>_e$, for the even quanta.

Therefore, the minimum uncertainty relation is given by

$$
\begin{aligned}
(\Delta\hat{x})^2 &= (\Delta\hat{p})^2=(\Delta\hat{x})(\Delta\hat{p})\\
(\Delta\hat{x})_e(\Delta\hat{p})_e &= \lambda+\frac{3}{2}, \text{for even quanta}\\
(\Delta\hat{x})_o(\Delta\hat{p})_o &= -\lambda-\frac{1}{2}, \text{for odd quanta.}
\end{aligned}
\tag{35}
$$

Note that when $\lambda=-1$, we obtain for both quanta the value for minimum uncertainty of the linear harmonic oscillator. In figures I, II and III we plot $<\hat{x}^2>_e$, $<\hat{p}^2>_e$ for the particular value of $\lambda=1$, and $(\Delta\hat{x})_e(\Delta\hat{p})_e$ as function of λ.

Note that a CCS $|z>_{\mathrm{W}}$ is an eigenstate of the Wigner annihilation operator according to Eq. (33), it is possible to show that the analogous of so called even and odd CCS $|z,\pm>$, which appear in coherent state for the usual harmonic oscillator and Quantum Optic for uncharged quanta [28] and charge quanta [29], are eigenstates of the operator $(a^-(\lambda+1))^2$ (but not of $a^-(\lambda+1)$), viz.,

$$
\left(a^-(\lambda+1)\right)^2|z,\pm>=z^2|z,\pm>,\quad |z,\pm>=c_\pm(|z>\pm|-z>),
$$

where $c_\pm$ are the normalization factors, $|z,->=|z,2m>$ (even CCS) and $|z,+>=|z,2m+1>$ (odd CCS). A detailed analysis of the generation of even and odd canonical coherent states via Wigner-Heisenberg algebra will be published elsewhere.

V. CONCLUDING REMARKS

We have presented the canonical coherent states (CCS) associated with the unidimensional harmonic oscillator plus a centripetal barrier (a Calogero interaction [2,26] with two particles or isotonic oscillator [13]), which preserve the property of non-orthogonality. These CCS were deduced via Wigner-Heisenberg (WH) algebra in non-relativistic quantum mechanics. Although we have mainly treated the Calogero interaction CCS, similar results can be adequately extracted for any physical D-dimensional radial oscillator system by the Hermitian replacement of $-i\frac{d}{dx} \rightarrow -i\left(\frac{d}{dr} + \frac{D-1}{2r}\right)$ and of the Wigner parameter $\lambda + 1 \rightarrow \ell_D + \frac{1}{2}(D-1)$ where $\ell_D(\ell_D = 0, 1, 2, ...)$ is the D-dimensional oscillator angular momentum. In tridimensional space this Hermitian replacement left us exactly to the potential first investigated by Davidson in the beginning of thirties [3].

Therefore we can construct new spherical coherent states for diatomic molecules with Davidson interaction [30], so that complete diatomic molecule energy spectra and eigenfunctions can be deduced algebraically via Wigner-Heisenberg factorization method [15].

Recently the coherent states for the isotonic oscillator has been considered in the coordinate representation by Bagchi and Bhaumik [32]. The correspondence between eigenvalue Eqs. (26) in Ref. [32] and our Eq. (20) provided us $z = \frac{\alpha}{\sqrt{2}}$, where $2z^2$ is the eigenvalue in Ref. [32].

Following [18,21,26] it is of interest to note that defining

$$A_j^{\pm} = \frac{1}{\sqrt{2}}\left\{p_j \pm i\left(\frac{\partial}{\partial x_j}W(x_j)\right)\right\}, \quad p_j = -i\frac{d}{dx_j}$$

we obtain in one dimension the factorized Hamiltonian of the Calogero interaction for a many particle system with Davidson interactions

$$H_- = \sum_j A_j^+ A_j^- = \frac{1}{2}\sum_j\left\{p_j^2 + \left(\frac{\partial}{\partial x_j}W(x_j)\right)^2\right\} + \frac{1}{2}\sum_{i,j}\frac{\partial^2 W(x_j)}{\partial x_i \partial x_j}.$$

Thus, following [21], under the context of mesocopic physics, in the case of N-boson or fermion Hamiltonian we can obtain the relation between the Brownian motion and Calogero model.

For instance in our case with two particles the choice $W(x) = \frac{1}{2}x^2 + (\lambda+1)\ell n(x)$ gives us H_- belonging to the even sector of $H(\lambda+1)$. These aspects will be considered elsewhere in construction of the supercoherent states [24] for diatomic molecules [30]. However, note that in this work the $A_j^{\pm}$ operators become

$$A_j^{\pm} \rightarrow \mathcal{A}^{\pm} = \frac{1}{\sqrt{2}}\left(-i\frac{d}{dx} \pm i\frac{(\lambda+1)}{x} \pm ix\right).$$

We also consider the construction of minimum uncertainty coherent states (MUCS) for the Wigner oscillator position $\hat{x}$ and momentum $\hat{p}$, for the even quanta and odd quanta.

Therefore, the present work opens a new route for future investigations on the Calogero interaction coherent states, for the instance let us point out that the WH algebra can be applied for a complete spectral resolution of the complex Calogero model with real energies [33], too. Finally, let us point out that one can consider an analysis of the Calogero interaction coherent states as reported in the work of Ref. [34].

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FIGURES

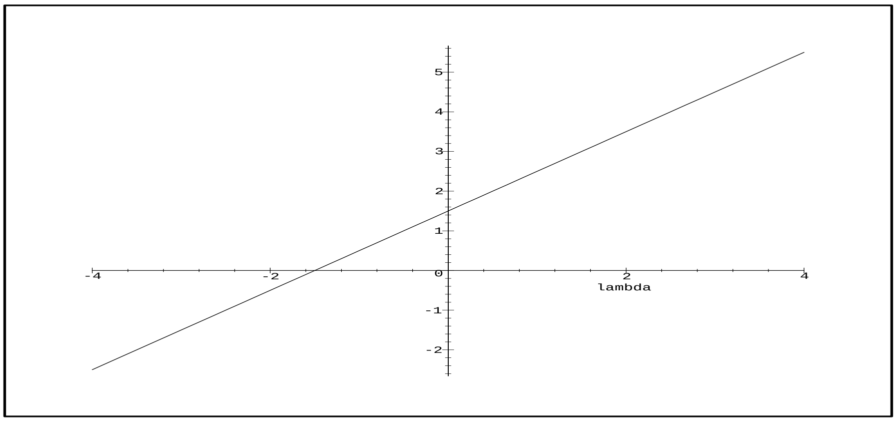

FIG. 1. The minimum uncertainty relation for Wigner oscillator CCS as function of λ, belonging to the even quanta.

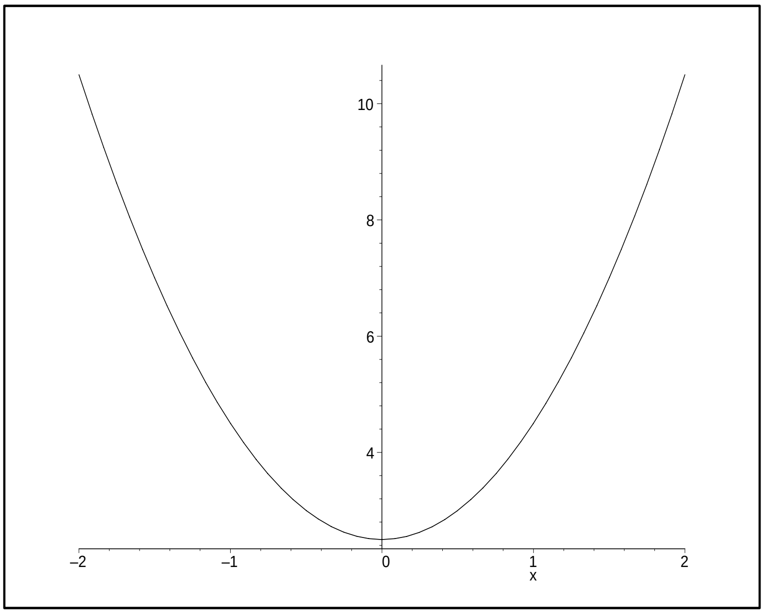

FIG. 2. The expectation value $< \hat{x}^2 >$ in the Wigner oscillator CCS, as a function of $Re(\alpha)$ for the particular case of $\lambda = 1$ associated to the even quanta.

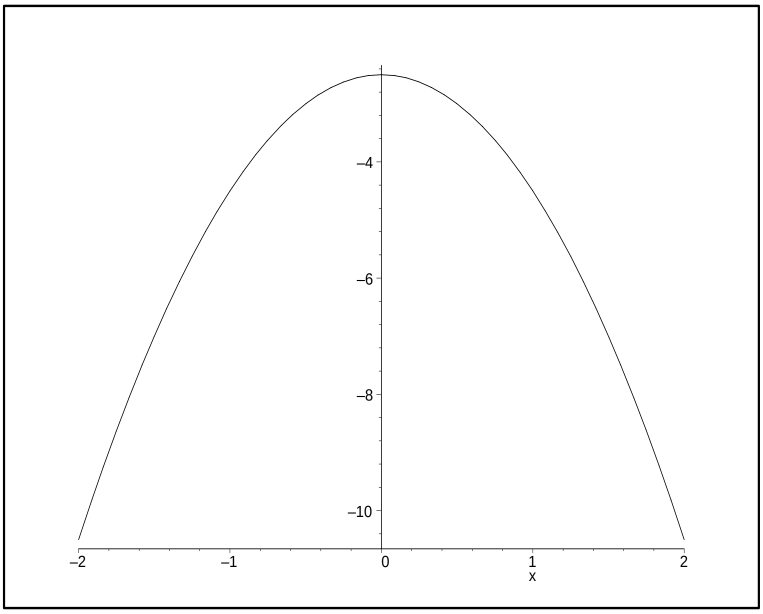

FIG. 3. The expectation value $<\hat{p}^2>$ in the Wigner oscillator CCS, as a function of $Im(\alpha)$ for the particular case of $\lambda = 1$ associated to the even quanta.

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