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Probing the Froissart Bound for Models with Charged Vector Fields in D=3

by

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Abstract

One discusses the tree-level unitarity and presents asymptotic behaviour of scattering amplitudes for 3-dimensional gauge-invariant models where complex Chern-Simons-Maxwell fields (with and without a Proca-like mass) are coupled to an Abelian gauge field.

The study of gauge field theories in 3 space-time dimensions has raised a great deal of interest since the early work of Deser, Jackiw and Templeton [1]. More recently, this line of investigation has been well-motivated in view of the possibilities of providing a gauge-theoretical foundation in the description of Condensed Matter phenomena, such as high-T_c superconductivity [2]. Also, the recent result on the Landau gauge finiteness of Chern-Simons theories is a remarkable property that makes 3-dimensional gauge theories so attractive [3].

In a recent work [4], some preliminary aspects of the dynamics concerning the interaction between Maxwell and complex Chern-Simons fields have been discussed. It was ascertained that the minimal coupling of the charged Chern-Simons-Maxwell (CSM*) and Chern-Simons-Maxwell-Proca (CSMP*) fields to a $U(1)$-gauge field breaks a local symmetry that is crucial for the decoupling of negative-norm 1-particle states associated to the charged massive vector field. As a follow up, our purpose in the present letter is to probe more deeply the tree-level unitarity of the CSM* and CSMP* models in the presence of a minimal $U(1)$ coupling [4]. This shall be achieved through the computation of tree-level amplitudes for the scattering between a CSM* (and CSMP*) particle and the photon in $D = 3$.

The minimal coupling of a CSM*-field, B_μ, to a $U(1)$-gauge field, A_μ, is described by the Lagrangian :

$$\begin{aligned}\mathcal{L}_{CSM} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}G^*_{\mu\nu}G^{\mu\nu} + \frac{m}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu} + im\omega\epsilon^{\alpha\mu\nu}B^*_\alpha A_\mu B_\nu + \\ &\quad -i\omega(G^*_{\mu\nu}A^\mu B^\nu - G_{\mu\nu}A^\mu B^{*\nu}) - \omega^2(A_\mu B_\nu - A_\nu B_\mu)A^\mu B^{*\nu} , \qquad (1)\end{aligned}$$

where $G_{\mu\nu}\equiv\partial_\mu B_\nu - \partial_\nu B_\mu$ and $F_{\mu\nu}\equiv\partial_\mu A_\nu - \partial_\nu A_\mu$ are the field-strengths, m is a real parameter with dimension of mass and ω a coupling constant with dimension of $(\text{mass})^{\frac{1}{2}}$.

The Lagrangian that describes the $U(1)$-invariant CSMP*-model reads :

$$\begin{aligned}\mathcal{L}_{CSMP} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}G^*_{\mu\nu}G^{\mu\nu} + \frac{m}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu} + \hat{\mu}^2 B^*_\mu B^\mu + \\ &\quad +im\omega\epsilon^{\alpha\mu\nu}B^*_\alpha A_\mu B_\nu - i\omega(G^*_{\mu\nu}A^\mu B^\nu - G_{\mu\nu}A^\mu B^{*\nu}) + \\ &\quad -\omega^2(A_\mu B_\nu - A_\nu B_\mu)A^\mu B^{*\nu} , \qquad (2)\end{aligned}$$

where $\hat{\mu}$ is a real parameter with dimension of mass.

Considering the Lagrangians above, we get the following free propagators for the B_μ-field :

$$\Delta^{\mu\nu}_{CSM}(k) = -i\frac{1}{k^2(k^2-m^2)}\left[im\,\epsilon^{\mu k\nu}k_k + k^2\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right] , \qquad (3.a)$$

$$\begin{aligned}\Delta^{\mu\nu}_{CSMP}(k) &= -i\frac{1}{(k^2-m_+^2)(k^2-m_-^2)}\left[im\epsilon^{\mu\kappa\nu}k_\kappa + (k^2-\hat{\mu}^2)\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right] + \\ &\quad +i\frac{1}{\hat{\mu}^2}\left(\frac{k^\mu k^\nu}{k^2}\right) , \qquad (3.b)\end{aligned}$$

where

$$m_+^2 = \frac{1}{2}[m^2 + 2\hat{\mu}^2 + \sqrt{m^2(m^2+4\hat{\mu}^2)}\,] \qquad (4.a)$$

and

$$m_-^2 = \frac{1}{2}[m^2 + 2\hat{\mu}^2 - \sqrt{m^2(m^2 + 4\hat{\mu}^2)}\,] . \tag{4.b}$$

From the interaction pieces of Lagrangians (1) and (2), we can read the 3- and 4-vertex Feynman rules required for our tree-level calculations (see Fig.1) :

$$(V_3)_{\alpha\mu\nu} = m\omega\epsilon_{\alpha\mu\nu} , \tag{5.a}$$
$$(\overline{V}_3)_{\alpha\mu\nu} = i\omega(\eta_{\nu\alpha}k_\mu - \eta_{\mu\alpha}k_\nu + \eta_{\nu\alpha}m_\mu - \eta_{\mu\nu}m_\alpha) \tag{5.b}$$

and

$$(V_4)_{\alpha\nu\beta\mu} = i2\omega^2(\eta_{\alpha\beta}\eta_{\mu\nu} - \eta_{\alpha\nu}\eta_{\beta\mu}) . \tag{5.c}$$

At tree-level, even if negative-norm states are shown to decouple from physical amplitudes, unitarity may be jeopardised by processes governed by the exchange of a charged massive quantum (i.e., B_μ-field exchange)[5, 6]. We then select the Compton-like scattering amplitude to be analysed (Fig.2). If unitarity bound is to be violated, the process of Fig.2 already signals such a violation. The total scattering amplitude, $\mathcal{M}$, may be written according to :

$$\mathcal{M} = \mathcal{M}_{V_4} + \sum_{\substack{v,v'=\text{all} \\ \text{3-vertices}}} \left(\mathcal{M}^{(1)}_{vv'} + \mathcal{M}^{(2)}_{vv'}\right) , \tag{6}$$

where v and v' denote all possible 3-vertices in the model and $\mathcal{M}_{V_4}$, $\mathcal{M}^{(1)}_{vv'}$ and $\mathcal{M}^{(2)}_{vv'}$ are the partial amplitudes for the "Compton" scattering graphs presented in Fig.2.

For the calculation of the individual graphs and for the evaluation of the product $\mathcal{M}^*\mathcal{M}$, use has been made of **FORM** software, since the number of terms generated is extremely high and algebraic manipulations become very cumbersome.

For the CSM*-model, we have found that in the limit of very high center-of-mass energies ($\sqrt{s} \to \infty$) the squared amplitude behaves according to :

$$\lim_{\sqrt{s}\to\infty} |\mathcal{M}_{CSM}|^2 = 90\,\omega^4 ; \tag{7}$$

this behaviour is not conflicting with unitarity. Nevertheless, for the CSMP*-model, $|\mathcal{M}|^2$ exhibits the following asymptotic behaviour :

$$\lim_{\sqrt{s}\to\infty} |\mathcal{M}_{CSMP}|^2 = \frac{\omega^4}{4\hat{\mu}^4}\, s^2 , \tag{8}$$

for both quanta, m_+^2 and m_-^2. Then, one concludes by (8) that unitarity is clearly violated, since the probability amplitude increases with the quartic power of the center-of-mass energy.

In order to justify such a conclusion, we refer to the works of Chaichian *et al.* [7, 8], where upper-limits are derived for the two-particle scattering cross-section in a $D(\geq 3)$-dimensional space-time [8, 9]

$$\lim_{\sqrt{s}\to\infty} \sigma_{tot} \leq c\, s^{\frac{(D-2)}{2}} (\ln s)^{D-2} . \tag{9}$$

Bearing in mind the optical theorem [10], we can make use of the limit (9) to read the asymptotic behaviour of $|\mathcal{M}|^2$ as dictated by unitarity. In $D=3$, it turns out to be

$$\lim_{\sqrt{s}\to\infty} |\mathcal{M}|^2 \leq c\, s^{\frac{3}{2}}\, \ln s\,. \tag{10}$$

Therefore, the asymptotic behaviour given in eq.(8) is in conflict with the unitarity bound typical of 3 space-time dimensions.

It would be however interesting to check if it is possible, as it happens in $D=4$ [6], to introduce a gauge-invariant non-minimal coupling that might restore Froissart bound at tree-level.

The most general gauge-invariant non-minimal coupling that respects renormalisability in $D=3$ reads :

$$\mathcal{L}_g = ig\, F^{\mu\nu} B^*_\mu B_\nu\,, \tag{11}$$

where g is a real coupling constant with dimension of $(\text{mass})^{\frac{1}{2}}$. This interacting Lagrangian (11) yields the following 3-vertex Feynman rule (see Fig.1):

$$(\widetilde{V}_3)_{\alpha\mu\nu} = ig(l_\alpha \eta_{\nu\mu} - l_\nu \eta_{\alpha\mu})\,. \tag{12}$$

Reconsidering the Lagrangian (2) and adding to it the Lagrangian $\mathcal{L}_g$ of eq.(11), one obtains, by using the equation (3.b), (6) and all vertex Feynman rules, the following high-energy behaviour for the squared amplitude, $|\mathcal{M}_{CSMP_g}|^2$, associated to "Compton" scattering :

$$\lim_{\sqrt{s}\to\infty} |\mathcal{M}_{CSMP_g}|^2 = \frac{1}{4\hat{\mu}^4}\left(\omega^4 - \frac{3}{2}\omega^2 g^2 + g^4\right) s^2\,. \tag{13}$$

As already mentionned, the purpose is to eliminate the s^2-dependence, since it destroys the unitarity. This would be achieved whenever the condition

$$\omega^4 - \frac{3}{2}\omega^2 g^2 + g^4 = 0\,, \tag{14}$$

is fulfilled. Nevertheless, it can be quickly seen that

$$\omega^4 - \frac{3}{2}\omega^2 g^2 + g^4 > 0\,, \tag{15}$$

for all non-vanishing real values of ω and g. Then, equation (15) reveals the impossibility of restoring the Froissart bound of the CSMP*-model (2) by the addition of a non-minimal coupling (11) in order to cancel the s^2-dependence of $|\mathcal{M}_{CSMP}|^2$ (8).

Our final conclusion is that the gauge-invariant CSM*-model discussed in ref.[4] exhibits a well-behaved asymptotic limit for the "Compton" scattering amplitudes at tree-level, even if ghosts do not decouple. On the other hand, the case of the CSMP*-field coupled to an Abelian gauge-field violates Froissart bound in $D=3$ and, contrary to what happens in the case of 4-dimensional massive charged vector fields coupled to the Maxwell-field, Froissart bound cannot be restored at the expenses of a gauge-invariant non-minimal coupling.

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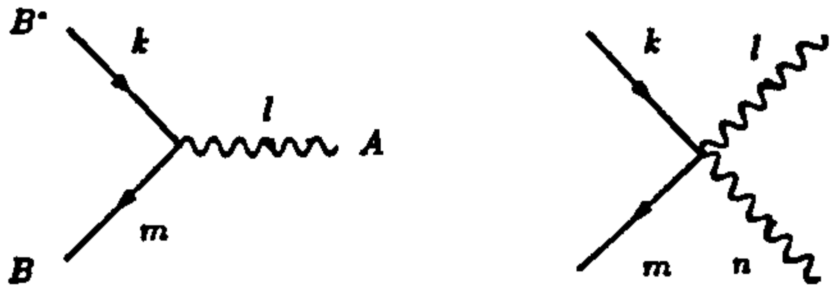

Figure 1: Interaction 3- and 4-vertices.

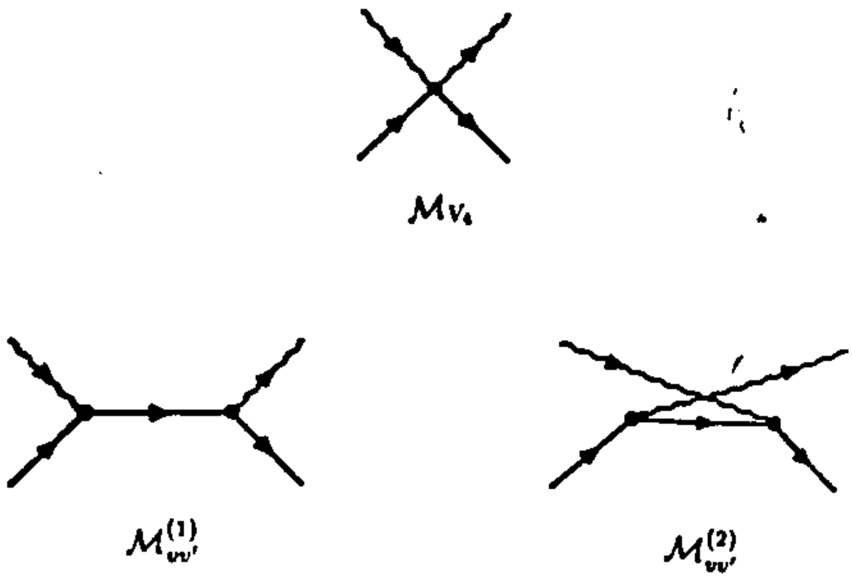

Figure 2: Compton scattering diagrams $\mathcal{M}_{V_4}$, $\mathcal{M}^{(1)}_{vv'}$ and $\mathcal{M}^{(2)}_{vv'}$.

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