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DYNAMICAL SUPERSYMMETRY BREAKING IN A TWO-DIMENSIONAL FIELD THEORY

by

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Abstract

We show that a supersymmetric two dimensional field theory with quartic fermion interactions can be dynamically broken. As a consequence the fermion field acquires mass.

Key-words: Dynamical supersymmetry breaking.

For some classes of massless fermion field theories in two dimensions (Gross and Neveu (1974); Pohlmeyer (1976); D'Adda et al. (1978)), it has been shown that they are asymptotically free, renormalizable and present dynamical mass generation among other properties.

Recently the supersymmetric generalization of the Gross-Neveu model for four fermion interactions in two dimensions has been exhibited (Mahdavi (1984)). It has then been shown that it is finite up to one loop and that the ultraviolet divergences for the self-energy graphs up to two loops cancel each other. Here we want to show that one can also apply a dynamical symmetry-breaking mechanism to generate mass for the fermion field.

The supersymmetric Gross-Neveu Lagrangian (Mahdavi (1984)) is given by

$$L = \bar{\psi} i \not{\partial} \psi + \partial_\mu A^* \partial^\mu A + FF^* - \frac{1}{2} g [(A^*F + AF^*)^2 +$$

$$+ (\bar{\psi}\psi)^2 + (A \overleftrightarrow{\partial}_\mu A^*)^2 + (AA^* \bar{\psi} i \overleftrightarrow{\not{\partial}} \psi + \bar{\psi} A i \overleftrightarrow{\not{\partial}} A^* \psi + \text{h.c.})] \qquad (1)$$

where g is a dimensionless parameter and $\gamma_0 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$, $\gamma_1 = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$.

This Lagrangian is invariant under the following infinitesimal supersymmetry transformations

$$\delta A = i \bar{\xi} \psi \qquad (2.a)$$

$$\delta \psi = (F - \not{\partial} A) \xi \qquad (2.b)$$

$$\delta F = -i \bar{\xi} \not{\partial} \psi \qquad (2.c)$$

where ξ is a constant Majorana spinor parameter.

In order to obtain dynamical mass generation let us consider the generating functional for the Green's function in the path integral formulation.

$$Z(\eta,\bar{\eta},J,J^*,J,J^*) = \int d\psi d\bar{\psi} dA dA^* dF dF^* \exp\{i \,.$$

$$\cdot \int d^4x (L + \bar{\eta}\psi + \bar{\psi}\eta + JA + J^*A^* + JF + J^*F^*)\} \qquad (3)$$

with L given in eq. (1).

In order to functionally integrate in the fermion field, let us introduce a scalar field σ (Nambu and Jona-Lasinio (1961)). Then, we can write

$$Z = \int d\psi d\bar{\psi} dA dA^* dF dF^* d\sigma \exp\{i \int d^4x [\bar{\psi} i \not{\partial} \psi + \partial_\mu A^* \partial^\mu A + FF^*] -$$

$$- [\frac{g}{2}(A^*F + AF^*)^2 + \sigma^2 + i\sqrt{2g}\ \bar{\psi}\psi\sigma + \frac{g}{2}(A\overleftrightarrow{\partial}_\mu A^*)^2 +$$

$$+ (\frac{g}{2}(AA^*\bar{\psi} i \overleftrightarrow{\partial} \psi) + \frac{g}{2}\ \bar{\psi} A i \overleftrightarrow{\not{\partial}} A^* \psi + h.c.)]\} \qquad (4)$$

or

$$Z = \int d\psi d\bar{\psi} d\sigma dA dA^* dF dF^* \exp i \int d^4x \{\bar{\psi}[i\not{\partial} - i\sqrt{2g}\,\sigma -$$

$$- (igAA^*\not{\partial} + igA\not{\partial}A^* + h.c.)]\ \psi + \bar{\eta}\psi + \bar{\psi}\eta + \ldots\} \qquad (5)$$

So after integrating over ψ and $\bar{\psi}$, we obtain

$$Z = \int d\sigma dA dA^* dF dF^* \exp i\,\{[-i \mathrm{Tr}\, \ell n\, \Delta_{\alpha\beta}] +$$

$$+ \int d^4x [-(\bar{\eta}\Delta^{-1})_\alpha \Delta^{\alpha\beta} (\Delta^{-1}\eta)_\beta - \sigma^2 + \ldots]\} \qquad (6)$$

where

$$\Delta_{\alpha\beta} = i\not{\partial}\delta_{\alpha\beta} - i\sqrt{2g}\ \delta_{\alpha\beta}\sigma - (igAA^*\not{\partial}\delta_{\alpha\beta} + ig\,\delta_{\alpha\beta}A\not{\partial}A^* + h.c.) \qquad (7)$$

In order to avoid infrared divergences we would like the scalar field σ to acquire a non-vanishing vacuum expectation value. We can show that this is indeed the case by investigating the σ effective potential in the one-loop approximation.

The one-loop effective potential for the field σ with an ultraviolet cutoff Λ is given by

$$V_{eff}(\sigma) = [\sigma^2 + \frac{g}{2\pi}\sigma^2\,(\ell n\,\frac{2g\sigma^2}{\Lambda^2} - 1)] \qquad (8)$$

As one can see from the figure for $V_{eff}(\sigma)$ the minimum occurs for $<\sigma(x)>=\sigma_0$. Then we can perform the shifting

$$\sigma'(x) = \sigma(x) - \sigma_0 \qquad (9)$$

In this way, after the substitution of (8) into (6) we see that the ψ field acquires a mass term given by

$$m_\psi = \sqrt{2g}\ \sigma_0 \qquad (10)$$

and so supersymmetry is dynamically broken.

Let us foccus our attention now in expression (7). It contributes to the effective action as

$$S_{eff} = -i\,\mathrm{Tr}\,\ell n\,\{1 - \frac{1}{i\not{\partial} - i\sqrt{2g}\,\sigma_0}\,g[(iAA^*\not{\partial} + iA\partial A^* + h.c.) + i\sqrt{\frac{2}{g}}\,\sigma']\} + \ldots \qquad (11)$$

Let us expand (11) into a power series in g.

$$- i \,\mathrm{Tr}\, \ell n \{1 - \frac{1}{i\not{\partial} - i\sqrt{2g}\,\sigma_0} g[(iAA^*\not{\partial} + iA\not{\partial}A^* + h.c.) + i\sqrt{\frac{2}{g}}\,\sigma')]\} + --- =$$

$$= \sum_{n=1}^{\infty} \frac{i}{n} \mathrm{Tr}\{\frac{1}{i\not{\partial} - i\sqrt{2g}\,\sigma_0} g[(iAA^*\not{\partial} + iA\not{\partial}A^* + h.c.) + i\sqrt{\frac{2}{g}}\,\sigma')]\}^n + --- \tag{12}$$

Now considering those terms containing only the scalar field σ' one sees that by requiring the vanishing of the linear term in σ' we can obtain the value of σ_0 (in euclidean momentum):

$$4g\sigma_0 \int \frac{d^2p}{(2\pi)^2} \frac{\sigma'(0)}{p^2 + 2g\sigma_0^2} = 2\sigma_0\sigma'(0) \tag{13}$$

or

$$2g \int \frac{d^2p}{(2\pi)^2} \frac{1}{p^2 + 2g\sigma_0^2} = 1 \tag{14}$$

The quadratic term in σ' ($\equiv \Gamma^{\sigma'\sigma'}$) is given by:

$$\Gamma^{\sigma'\sigma'} = \frac{i}{2} \mathrm{Tr}\left[\frac{\sigma'}{i\not{\partial} - i\sqrt{2g}\,\sigma_0}\right]^2 + i\int d^2x \,\frac{\sigma'^2}{g} \tag{15}$$

where

$$\frac{i}{2} \mathrm{Tr}\left[\frac{\sigma'}{i\not{\partial} - i\sqrt{2g}\,\sigma_0}\right]^2 = -2i \int \frac{d^2p\,d^2k}{(2\pi)^2} \frac{\tilde{\sigma}'(p)\,\tilde{\sigma}'(-p)\,[k^2 + kp - 2g\,\sigma_0^2]}{[k^2 + 2g\,\sigma_0^2]\;[(k+p)^2 + 2g\,\sigma_0^2]} \tag{16}$$

If we use the Feynman-Schwinger integral we can write (16) as

$$\frac{i}{2} \mathrm{Tr}\left[\frac{\sigma'}{i\not{\partial} - i\sqrt{2g}\,\sigma_0}\right]^2 = -2i \int_0^1 dx \int \frac{d^2p\,d^2k}{(2\pi)^2} \frac{\tilde{\sigma}'(p)\,\tilde{\sigma}'(-p)\,[k^2 - p^2x(1-x) - 2g\sigma_0^2]}{[k^2 + p^2x(1-x) + 2g\,\sigma_0^2]^2} \tag{17}$$

So now from (17) we easily see that the field σ' acquires kinetic and mass terms, with

$$m^2_{\sigma'} = 4g\,\sigma^2_0 \tag{18}$$

We can note that the number of degrees of freedom is the same for the bosonic and fermionic modes

$$2(m_\psi)^2 = m^2_\sigma \tag{19}$$

as one should wait from a theory with broken supersymmetry.

Then we have seen that supersymmetry has been dynamically broken and as a consequence the fermion field has acquired mass. So the physical spectrum of the theory is given by a massive scalar field (σ), a massive fermion field (ψ) and a massless scalar field (A).

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FIGURE CAPTION