ISSN 0029 - 3865

CBPF-NF-063/84

A UNIFIED MODEL FOR GRAVITY-ELECTRO-WEAK INTERACTIONS

by

M. Novello and Ligia M.C.S. Rodrigues

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

ABSTRACT

A model is presented in which gravity and electro-weak interactions are described in a unique framework. As a consequence we are led to predict very massive particles (of masses ≅ 369 and 520,3 GeV) which intermediate W_μ - W_μ and W_μ - Z_μ interactions. This new interaction is the short-range counterpart of gravitational interaction.

Key-words: Unification; Gravity-electro-weak interactions.

In the last years many attempts have been made to unify gravitation and electro-weak interactions. One serious difficulty of this program is the unadequacy of the formalisms to either geometrize electro-weak interaction (Einstein's unified program revisited) or to put gravity in the scheme of standard gauge theories. In the present letter we propose a new model in which this difficulty is overcome in a natural way.

The starting point of our model is Jordan's[1] formulation of gravitation theory. In his approach, Bianchi identities are treated as the true dynamical equations for the gravitational field, yielding equation

$$W^{\alpha\beta\mu\nu}{}_{;\,\nu} = 0 \tag{1}$$

for the Weyl conformal tensor $W^{\alpha\beta\mu\nu}$.

It is extremely important to note that Jordan's formulation is completely equivalent to Einstein's theory of gravitation, as was proved by Lichnerowicz[2], in a theorem that can be stated as follows[3]: If $R_{\mu\nu}(\Sigma) = 0$, in which Σ is a space oriented hypersurface in a four-dimensional Riemannian manifold M_4, then equation (1) guarantees that $R_{\mu\nu} = 0$ in every point of M_4.

The next step in the construction of our model is to make appeal to a result by Lanczos[4].

In a 1962 article, this author showed that Weyl's tensor $W_{\alpha\beta\mu\nu}$ can be written as first order derivatives of a three--index tensor $H_{\mu\nu\lambda}$ in the following way:

(2) $$-W_{\alpha\beta\mu\nu} = H_{\alpha\beta\mu;\nu} - H_{\alpha\beta\nu;\mu} + H_{\mu\nu\alpha;\beta} - H_{\mu\nu\beta;\alpha} + \frac{1}{2}(H_{\nu\alpha}+H_{\alpha\nu})g_{\beta\mu} + \frac{1}{2}(H_{\beta\mu}+H_{\mu\beta})g_{\alpha\nu} - \frac{1}{2}(H_{\alpha\mu}+H_{\mu\alpha})g_{\beta\nu} - \frac{1}{2}(H_{\beta\nu}+H_{\nu\beta})g_{\alpha\mu} + \frac{2}{3}H^{\sigma\lambda}{}_{\sigma;\lambda}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})$$

in which

$$H_{\alpha\mu} \equiv H_{\alpha}{}^{\sigma}{}_{\mu;\sigma} - H_{\alpha}{}^{\lambda}{}_{\lambda;\mu} \quad .$$

This tensor $H_{\mu\nu\lambda}$, called Lanczos potential, has the symmetries

(3a) $$H_{\mu\nu\lambda} = -H_{\nu\mu\lambda}$$

(3b) $$H_{\mu\nu\lambda} + H_{\nu\lambda\mu} + H_{\lambda\mu\nu} = 0$$

Expression (2) is invariant under the gauge transformation

(4) $$H_{\mu\nu\lambda} \rightarrow H'_{\mu\nu\lambda} = H_{\mu\nu\lambda} + \chi_{\mu}g_{\nu\lambda} - \chi_{\nu}g_{\mu\lambda}$$

for an arbitrary vector χ_{μ}[5].

As it was pointed out by Lanczos[4], $H_{\mu\nu\lambda}$ "is reducible to the metric tensor $g_{\mu\nu}$ only by an integral operation, i.e., the value of $H_{\mu\nu\lambda}$ depends globally on the geometry of the manifold, and yet the tensor $H_{\mu\nu\lambda}$ participates locally in the formation of the field equations".

However, an explicit local dependence can be exhibited in the case of weak gravity, that is, when $g_{\mu\nu} \cong \eta_{\mu\nu} + \varepsilon\,\psi_{\mu\nu}$, where $\varepsilon^2 << \varepsilon$. Lanczos potential can then be written as

$$(5) \qquad H_{\mu\nu\lambda} = \frac{\varepsilon}{4}\,[\psi_{\mu\lambda,\nu} - \psi_{\nu\lambda,\mu} + \frac{1}{6}\,\psi_{,\mu}\eta_{\nu\lambda} - \frac{1}{6}\,\psi_{,\nu}\eta_{\mu\lambda}]$$

in which $\psi \equiv \psi_{\mu\nu}\eta^{\mu\nu}$ and $\psi_{\mu\lambda,\nu} \equiv \frac{\partial}{\partial x^{\nu}}\,\psi_{\mu\lambda}$. In this case, equation (1) reduces to the Pauli-Fierz equation for a spin-2 particle[6].

Lanczos' result remained almost forgotten until 1982 when Bampi and Caviglia[7] proved the existence of $H_{\mu\nu\lambda}$ in <u>any</u> Riemannian manifold.

When Weyl's tensor is written as in (2), equation (1), is the dynamical equation for Lanczos' field. This Jordan-Lanczos description of gravitational phenomena is equivalent to Einstein's gravitation due to Lichnerowicz's theorem mentioned above. It will be seen to be an adequate framework to unify gravitation and electro-weak interactions. Salam[8] and Weinberg[9] have independently succeeded to show that electromagnetic and weak forces can be unified by making appeal to a gauge structure. The essential idea in their treatment is that photons and intermediate vector mesons are the gauge fields of a $SU(2) \times U(1)$ structure. This model is presently largely accepted due to the experimental confirmation of its predictions.

In Salam-Weinberg's model, long and short range interactions are distinguished due to the behavior of a spin-zero doublet ϕ in vacuum. Indeed, it is the existence of a non-null expectation value in vacuum for ϕ which generates the masses of the vector bosons which intermediate weak interactions.

We will follow this road and propose here a more general model which accomodates also gravity in the unified description, without the introduction of any new parameter and keeping the same gauge structure.

In order to do this we start by considering a multiplet of Lanczos potential $A^{(i)}_{\mu\nu\lambda}$ and $B_{\mu\nu\lambda}$ – in which (i) is an SU(2) index.

We decompose these tensors in their irreducible parts

$$A^{(i)}_{\mu\nu\lambda} = a^{(i)}_{\mu\nu\lambda} + \frac{g}{3} A_{[\mu}{}^{(i)} g_{\nu]\lambda} \equiv a^{(i)}_{\mu\nu\lambda} + \frac{g}{3} (A_{\mu}{}^{(i)} g_{\nu\lambda} - A_{\nu}{}^{(i)} g_{\mu\lambda})$$

$$B_{\mu\nu\lambda} = b_{\mu\nu\lambda} + \frac{g'}{3} B_{[\mu} g_{\nu]\lambda}$$

in which $g\ A^{(i)}_{\mu} \equiv A^{(i)}_{\mu\nu}{}^{\nu}$ and $g' B_{\mu} \equiv B_{\mu\nu}{}^{\nu}$ are identified to the gauge fields of Salam-Weinberg's model; g and g' are introduced in order to have the right dimensions, $\dim[A_{\alpha\mu\lambda}] = \text{length}^{-1}$. $a^{(i)}_{\mu\nu\lambda}$, the trace-free part of $A^{(i)}_{\mu\nu\lambda}$, transforms as a vector under SU-2 and $A^{(i)}_{\mu}$ transforms as an internal connection. The total covariant derivative for an object which is a tensor under a general coordinate transformation and a vector under the internal group is given by

$$T^{(i)}_{\mu\|\lambda} = T^{(i)}_{\mu;\lambda} + g[A_{\lambda}, T_{\mu}]^{i}$$

where (;) stands for the space-time covariant derivative.

The fundamental invariant Lagrangian of our model is the sum of three terms:

$$\mathcal{L}_1 = \sqrt{-\det g_{\mu\nu}} \left\{ - \frac{1}{8M_0^2 k} (\vec{W}^{\alpha\beta\mu\nu} . \vec{W}_{\alpha\beta\mu\nu} + C^{\alpha\beta\mu\nu} C_{\alpha\beta\mu\nu}) + \right.$$
$$- \left(\frac{3}{2g}\right)^2 \left(A^{\mu\nu\lambda}_{(i);\lambda} + g[A_{\lambda}, A^{\mu\nu\lambda}_{(i)}]\right)^2 +$$
$$\left. - \left(\frac{3}{2g'}\right)^2 \left(B^{\mu\nu\lambda}{}_{;\lambda}\right)^2 \right\}$$

$$\mathcal{L}_2 = \sqrt{-\det g_{\mu\nu}}\left\{\frac{i}{6}\bar{R}\Sigma^{\mu\nu}\gamma^{\lambda}(g_{\nu\lambda}\nabla_\mu - g_{\mu\lambda}\nabla_\nu + 3B_{\mu\nu\lambda})R + \right.$$

$$\left. + \frac{i}{6}\bar{L}\Sigma^{\mu\nu}\gamma^{\lambda}\left(g_{\nu\lambda}\nabla_\mu - g_{\mu\lambda}\nabla_\nu + 3\vec{\tau}.\vec{A}_{\mu\nu\lambda} + \frac{3}{2}B_{\mu\nu\lambda}\right)L\right\}$$

$$\mathcal{L}_3 = \sqrt{-\det g_{\mu\nu}}\left\{\frac{1}{6}\left[(\nabla_\mu\phi)g_{\nu\lambda} - (\nabla_\nu\phi)g_{\mu\lambda} - 3i\vec{\tau}.\vec{A}_{\mu\nu\lambda}\phi + 3iB_{\mu\nu\lambda}\phi\right]^2 + \right.$$

$$\left. + \sigma(\phi^+\phi)^2 - m^2\phi^+\phi - g_e(\bar{L}\phi R + \bar{R}\phi^+ L)\right\}$$

$\vec{W}_{\mu\nu\lambda\rho}$ and $C_{\mu\nu\lambda\rho}$ are Weyl's tensors constructed in a totally covariant way, respectively with $a^{(i)}_{\mu\nu\lambda}$ and $b_{\mu\nu\lambda}$. Therefore the dynamics of the pure tensorial parts of our gauge fields are obtained when we independently vary $a^{(i)}_{\mu\nu\lambda}$ and $b_{\mu\nu\lambda}$ and are written in the form of Jordan-like equations, that is

(6a) $\vec{W}^{\mu\nu\lambda\rho}{}_{||\rho} = \vec{J}^{\mu\nu\lambda}$

(6b) $C^{\mu\nu\lambda\rho}{}_{;\rho} = j^{\mu\nu\lambda}$

in which the currents $\vec{J}^{\mu\nu\lambda}$ and $j^{\mu\nu\lambda}$ are constructed with the terms coupled to $a^{(i)}_{\mu\nu\lambda}$ and $b_{\mu\nu\lambda}$, respectively. $k = \frac{8\pi G}{c^4}$, G is Newton's constant and M_0, as we will see, is the mass of the heavy tensor bosons.

The last two terms in $\mathcal{L}_1$ contain the standard ones $-\frac{1}{4}f^{(i)}_{\mu\nu}f^{\mu\nu}_{(i)}$ and $-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}$ where

$$f^{(i)}_{\mu\nu} \equiv A^{(i)}_{\mu,\nu} - A^{(i)}_{\nu,\mu} - g[A_\mu, A_\nu]^{(i)}$$

and

$$B_{\mu\nu} \equiv B_{\mu,\nu} - B_{\nu,\mu} ,$$

as can be directly checked. These terms are constructed in a similar way as in Lanczos gauge-fixing procedure. The independent variation of $A^{(i)}_\mu$ and B_μ yields the usual equations.

L is by definition the left-handed doublet $L = (\frac{1+\gamma^5}{2}) \begin{pmatrix} \nu \\ e \end{pmatrix}$, and R is the right-handed singlet $R = (\frac{1-\gamma^5}{2}) e$; the scalar ϕ is an iso-doublet under SU-2 and $\Sigma_{\mu\nu} \equiv \frac{\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu}{2}$. We have taken into account the soforth experimentally confirmed maximal parity violation of weak interactions.

Due to (3b), $\mathcal{L}_2$ can be reduced to

$$\mathcal{L}_2 = \sqrt{-\det g_{\mu\nu}} \left\{ i\bar{R}\gamma^\mu \nabla_\mu R + i\bar{L}\gamma^\mu \nabla_\mu L + g\bar{L}\gamma^\mu \vec{\tau} L . \vec{A}_\mu + g' (\tfrac{1}{2}\bar{L}\gamma^\mu L + \bar{R}\gamma^\mu R) B_\mu \right\}$$

from which we can conclude that leptons do not interact directly with the tensors $a^{(i)}_{\mu\nu\lambda}$ and $b_{\mu\nu\lambda}$.

Following the standard procedure Lagrangian $\mathcal{L}_3$ is introduced in order to provide a mechanism to give mass to the intermediate vector and tensor mesons. This mechanism relies on the fact that our iso-scalar doublet can have a non-null vacuum expectation value, that is, we can set $\phi = \begin{pmatrix} 0 \\ (\lambda + \chi)/\sqrt{2} \end{pmatrix}$ with $\langle 0|\chi|0\rangle = 0$.

With a proper re-arrangement of our tensor and vector fields, we will see that the existence of this non-null value for ϕ_0 will give mass to three tensors and to three vectors. The remaining massless tensor and vector bosons will be identified to the graviton and the photon, respectively.

In order to have the correct angle θ_W, that is, to have for the vector mesons the standard Salam-Weinberg's result, the tensor fields $a^{(3)}_{\mu\nu\lambda}$ and $b^{\mu\nu\lambda}$ have to be rotated of 45^o - which is just another way of stating the fact that there is not any dimensional constant associated to the tensorial interaction in the present model.

Indeed, if we define

$$W^{\pm}_{\mu\nu\lambda} = \frac{a^{(1)}_{\mu\nu\lambda} \pm i a^{(2)}_{\mu\nu\lambda}}{\sqrt{2}}$$

$$Z_{\mu\nu\lambda} = \frac{a^{(3)}_{\mu\nu\lambda} + b_{\mu\nu\lambda}}{\sqrt{2}}$$

$$a_{\mu\nu\lambda} = \frac{-a^{(3)}_{\mu\nu\lambda} + b_{\mu\nu\lambda}}{\sqrt{2}}$$

and the vector mesons are rotated in the standard way[9], we will get

$$m(W^{\pm}_{\mu\nu\lambda}) = \frac{3}{2}\lambda$$

$$m(Z_{\mu\nu\lambda}) = \frac{3}{2}\sqrt{2}\lambda$$

$$m(a_{\mu\nu\lambda}) = 0.$$

Knowing that $m(W_\mu) = g\lambda/2$ we obtain the values $m(W_{\mu\nu\lambda}) \cong 369$ GeV and $m(Z_{\mu\nu\lambda}) \cong 520,3$ GeV. Our Lagrangian can now be rewritten in terms of the new tensor and vector fields $W^{\pm}_{\mu\nu\lambda}$, $Z_{\mu\nu\lambda}$, $a_{\mu\nu\lambda}$, $W^{\pm}_\mu$, Z_μ, A_μ ; A_μ is the photon field. $a_{\mu\nu\lambda}$ is then associated to the usual long-range gravitation and its dynamics is obtained from the variational principle yielding[11]

$$(7) \qquad \frac{1}{k} W^{\alpha\beta\mu\nu}{}_{;\nu} = -\frac{1}{2} T^{\mu[\alpha;\beta]} + \frac{1}{6} g^{\mu[\alpha}T^{,\beta]}$$

where $W^{\alpha\beta\mu\nu}$ is now the Weyl tensor constructed with $a_{\mu\nu\lambda}$. As we have already stressed, equation (7) is equivalent to Einstein's equation. $T^{\mu\nu}$ is the energy-momentum tensor of all the fields involved.

It is important to remark that although $\delta g_{\mu\nu}$ and $\delta a_{\mu\nu\lambda}$

should in principle be treated as independent quantities, we have found out a relation between these variations which is necessary to obtain equation (7) from a variational principle that is,

$$(8) \qquad \delta g_{\mu\nu} = \frac{1}{2M_0^2} [\delta a_{(\mu \ \nu);\alpha}^{\ \ \alpha} - \frac{2}{3} g_{\mu\nu} \ \delta a_{\alpha\beta\mu;\sigma} \ g^{\beta\mu} \ g^{\alpha\sigma}$$

The constant $M_o = m(W_{\mu\nu\lambda})$ which appears both in $\mathcal{L}_1$ and in relation (8) due to dimensional considerations, disappears from equation (7), as it should be.

Going back to $\mathcal{L}_1$ we see that tensor bosons can intermediate $W_\mu - W_\mu$ and $W_\mu - Z_\mu$ interactions through the appearance of terms like $W^{+\mu\nu\rho} \ W_\mu^+ \ W_\nu^- \ W_\rho^+$, $W^{\mu\nu\rho} \ Z_\nu \ Z_\rho \ W_\mu^+$ and so on[10]. Therefore we have here a model that starts to answer the very pertinent question of the nature of the interaction of vector mesons with themselves.

The results above suggest that it exists a new interaction that can be thought as the short-range counterpart of gravitation, as weak interactions are the short-range counterpart of electromagnetism.

We are aware that this model leaves many opened questions but we nonetheless consider it an interesting contribution to a few important problems concerning the search for a unified theory of fundamental interactions. The fact that we can predict values (which are within the present accelerators limits) for the masses of the tensor bosons allows this model to be experimentally probed.