hep - ph / 9508414

CBPF-NF-061/95

Landau - Pomeranchuk - Migdal Effect for Nuclear Matter in QCD

Eugene Levin$^{*)}$

LAFEX, Centro Brasileiro de Pesquisas Físicas (CNPq)

Rua Dr. Xavier Sigaud 150, 22290 - 180 Rio de Janeiro, RJ, BRASIL

and

Theory Department, Petersburg Nuclear Physics Institute

188350, Gatchina, St. Petersburg, RUSSIA

Abstract

Soft photon and gluon radiation off a fast quark propagating through nuclear matter is discussed. The close anology between the Landau - Pomeranchuk - Migdal (LPM) effect in QED and the emission of soft gluons, suggested in ref. [1] for "hot" plasma, is confirmed and the relation between Mueller's approach and traditional calculations is established. It is shown that perturbative QCD can be applied to take into account the LPM coherent suppression both for photon and gluon induced radiation. The formulae for the photon and gluon radiation densities are presented.

Key-words: QCD; Landau-Pomeranchuk-Migdal effect; Coherence; Nuclear matter

---

* Email: levin@lafex.cbpf.br

1 Introduction

The goal of this paper to study the Landau - Pomeranchuk - Migdal (LPM) effect [2] for the emission of photons and gluons in nuclear matter. We show that the typical distances which characterize the succesive rescaterring of quarks and gluons in nuclear matter are small enough to justify the QCD approach to this case. We shall generalized the formalism suggested in refs. [3][1] to the case of nuclear matter using the QCD approach for the interaction of quarks and gluons.

We consider the emission of soft gluons (photons) with energies $\omega \ll E$, where E is the energy of the projectile. The assumption that the mean free path λ of the projectile is much larger than the screening radius in the nuclear matter, $\lambda \gg \mu^{-1}$, allows one to treat successive rescatterings as independent (see ref. [2]) and simplifies all formulae reducing the problem to an eikonal picture of classical propagation of a relativistic particle with $E \gg \mu$ through a medium.

As is well known, the QED emission amplitude for a single scattering can be written in terms of a transverse velocity $\vec{u}_\perp = \frac{\vec{k}_\perp}{\omega}$ where $\vec{k}_\perp$ is the transverse momentum of the photon with respect to the direction of the fast (massless) charged particle. For the emission of a photon from the scattering center i it reads:

$$\vec{J}_i \;=\; \frac{\vec{u}_{i\perp}}{u^2_{i\perp}} \;-\; \frac{\vec{u}_{i-1\perp}}{u^2_{i-1,\perp}} \;;\;\; \vec{u}_{i\perp} \;=\; \vec{u}_{(i-1)\perp} \;-\; \frac{\vec{q}_{i\perp}}{E} \;; \tag{1}$$

where $\vec{q_{i\perp}}$ is the momentum transfer to the scattering center i.

The LPM effect comes from the intenference between the amplitudes (1) in the calculation of the inclusive cross section of photon production. The relative eikonal phase of the two amplitudes due to centers i and j in relativistic kinematics is equal to:

$$\phi_{ij} \;=\; k^\mu (x_i - x_j)_\mu \;=\; \sum_{m=i}^{j-1} \frac{z_m - z_{m+1}}{\tau_m(\omega)} \;;\;\; \tau_m \;=\; \frac{2\omega}{(\vec{k}_{m\perp})^2} \;=\; \frac{2}{\omega u^2_{m\perp}} \;; \tag{2}$$

where τ_m is the radiation formation time and z_m is the longitudinal coordinate of the m-th centre.

One can easily estimate the average phase [2], assuming that all $z_m - z_{m+1}$ are equal to the mean free path λ, and obtain:

$$\phi(n = j - i) \;\approx\; \frac{\lambda\omega}{2} \sum_{m=0}^{j-i-1} u^2_{i+m} \;\approx\; \frac{\lambda\omega}{2} \,\{\, n u^2_{i\perp} \;+\; \frac{n(n-1)}{2}\,\frac{\mu^2}{E^2}\,\} \tag{3}$$

Here we have used the assumption that the transverse momentum of the projectile, after m collisions with a typical momentum transfer $\langle q^2_\perp \rangle = \mu^2$, can be treated as a random walk in the transverse plane which gives : $u^2_{i+m,\perp} = u^2_{i\perp} + m\,\frac{\mu^2}{E^2}$. Since the amplitude of Eq.(1) is small at $u^2_{i\perp} > \frac{\mu^2}{E^2}$ the second term in Eq. (3) dominates at large $N \gg 1$. The cross section vanishes if $\phi > 1$. It means that the number (n) of scattering centres that act coherently and can be treated as a single radiator, is restricted from above by the value $N_{coh}(\omega)$:

$$n \;\leq\; N_{coh}(\omega) \;=\; 2\sqrt{\frac{E^2}{\lambda\mu^2\omega}} \tag{4}$$

Considering a group of centeres with $i-j < N_{coh}$ as a single radiator, we can estimate the radiation density (see refs. [2] [1] [4] for details):

$$\omega \frac{d^2I}{d\omega\, dz} = \frac{1}{\lambda}\{\omega\frac{dI}{d\omega}\}_{one\ centre} \cdot \frac{1}{N_{coh}(\omega)} \propto \frac{\alpha_{em}}{\lambda N_{coh}} \tag{5}$$

One can see three kinematic regions in which Eq. (5) gives different answers for the radiation density:

1. $N_{coh}(\omega) > 1$ or $\omega < \omega_{BH} = \frac{2E^2}{\lambda\mu^2} = E\cdot\frac{E}{E_{LPM}}$ where $E_{LPM} = \frac{1}{2}\mu^2\lambda$. Here

$$\omega \frac{d^2I}{d\omega\, dz} \propto \alpha_{em}\sqrt{\frac{\mu^2}{\lambda E^2}\,\omega} = \frac{\alpha_{em}}{\lambda}\sqrt{\frac{\omega}{\omega_{BH}}} \tag{6}$$

This equation gives the Migdal result [2].

2. $N_{coh}(\omega) \approx 1$ or $\omega \geq \omega_{BH}$. In this kinematic region each scattering centre radiates separately and we derive the well known Bethe - Heitler limit:

$$\omega \frac{d^2I}{d\omega\, dz} = \frac{1}{\lambda}\{\,\omega\frac{dI}{d\omega}\}_{one\ centre} \propto \frac{\alpha_{em}}{\lambda} \tag{7}$$

3. In a medium with a final longitudinal size L, we have to assume that L is large enough so as to have N_{coh} successive interactions. It means:

$$\lambda N_{coh} < L \ \ or \ \ \omega > \omega_{fact} = \frac{\lambda E^2}{L^2\mu^2} \tag{8}$$

For $\omega < \omega_{fact}$ one recovers the factorization limit in which the whole medium radiates as one centre with an amplitude

$$\sum_{i=1}^{n}\vec{J}_i = \frac{\vec{u}_{n\perp}}{u^2_{n\perp}} - \frac{\vec{u}_{1\perp}}{u^2_{1\perp}}\ ; \tag{9}$$

At first sight, the factorization region is irrelevant for the emission in a nuclear matter. However, it is not so, since each produced quark or gluon lives only a finite time ($\tau_{max} \propto \frac{E}{\langle p^2_\perp\rangle}$ where $p_\perp$ is the transverse momentum of the produced charged particle which embodies successive rescattering) inside a nuclear matter. Substituting $L = \tau_{max}$ in Eq. (8) we obtain $\omega_{fact} = \frac{\lambda\langle p^2_\perp\rangle}{\mu^2}$. Therefore we expect the LPM effect for the photon energy range

$$\frac{\lambda\langle p^2_\perp\rangle}{\mu^2} = \omega_{fact} < \omega < \omega_{BH} = \frac{E^2}{E_{LPM}} \tag{10}$$

The situation becomes even more interesting in the case of gluon emission. As was discussed in ref.[1], the produced gluon itself embodies the successive rescatterings in a medium. The life time of the gluon is $\tau_{max} \propto \frac{\omega}{k^2_\perp} \approx \frac{1}{\omega}$. To discuss the contribution of gluon rescattering to the LPM effect we have to demand that

$$\lambda N_{coh}(\omega) < \tau_{max}(gluon) \approx \frac{1}{\omega} \ \ or \ \ \omega < \frac{1}{\lambda^2\omega_{BH}} \tag{11}$$

Therefore we anticipate for gluons the classic LPM suppression only for $\omega < min\{\,\omega_{BH},\frac{1}{\lambda^2\omega_{BH}}\,\}$ and some new behaviour of the radiation density for $min\{\,\omega_{BH},\frac{1}{\lambda^2\omega_{BH}}\,\} < \omega < max\{\,\omega_{BH},\frac{1}{\lambda^2\omega_{BH}}\,\}$.

From the above sketch of the physics of the LPM effect it is clear that there are a number of questions that we have to answer: (i) what is the value of the mean free path λ in QCD; (ii) what is the correct procedure of averaging over transverse momentum and what is the value of μ; and, (iii) what kind of LPM supression do we expect for gluon emission. We will answer these questions in this paper, starting with the photon emission from a fast quark in a nuclear matter in the second section. In the third section we shall consider the interconnection between the LPM effect and the Mueller technique [5], which was widely used to calculate the inclusive spectra of produced particles. In the fourth section we discuss the LPM like supression for gluon emission in a nuclear matter. A summary and final discussion are presented in the conclusion.

In general, we attempt to adjust our notation and normalization to agree with those of Baier et al [1], using E and p for the energy and momentum of a projectile which propagates through a nuclear matter; ω and k for the energy and momentum of a radiated photon (gluon) and $q_{i\perp}$ for the momentum transfer to the i-th scattering centre.

2 The LPM effect for photon radiation

The inclusive soft photon spectrum from a fast quark with energy E can be written in the form (see for example [1]):

$$\omega \frac{dI}{d\omega\, d^2u} = \frac{\alpha_{em}}{\pi^2} \left\langle \sum_{i=1}^{N} |\vec{J}_i e^{i\, k_\mu x^\mu}|^2 \right\rangle = \tag{12}$$

$$\frac{\alpha_{em}}{\pi^2} \left\langle 2\,\mathrm{Re} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \vec{J}_i \vec{J}_j \left[e^{ik_\mu (x_i - x_j)^\mu} - 1 \right] + |\sum_{i=1}^{N} \vec{J}_i|^2 \right\rangle$$

The brackets $\langle\ \rangle$ indicate the averaging procedure discussed in [3] [1], and we shall discuss it below in detail. The differential energy distribution of photons radiated *per unit length* is given by [1]

$$\omega \frac{dI}{d\omega\, dz} = \frac{\alpha_{em}}{\pi} \int \frac{d^2 U_0}{\pi} \left\langle 2\,\mathrm{Re} \sum_{n=0}^{\infty} \vec{J}_1 \vec{J}_{n+2} \left[\exp\left\{ i \frac{1}{\tau} \sum_{l=1}^{n+1} U_l^2\, (\, z_{l+1} - z_l\,) \right\} - 1 \right] \right\rangle. \tag{13}$$

Here we have expressed the relative phases in terms of the rescaled transverse velocity,

$$\vec{U}_l \equiv \vec{u}_l \cdot E = \frac{\vec{k}_\perp}{x_F} - \sum_{i=1}^{l} \vec{q}_{i\,\perp}\ ; \quad \vec{U}_0 = \frac{\vec{k}_\perp}{x_F} \tag{14}$$

where $x_F = \frac{\omega}{E}$ is the fraction of quark energy carried by the photon. We also introduced the characteristic parameter

$$\frac{1}{\tau} = \frac{\omega}{2\, E^2} \tag{15}$$

Now we shall define the averaging procedure that has to be implemented in Eq. (13), starting with the simplest problem, namely, with the propagation of the quark through a nuclear matter. This problem was solved a long time ago [6]. Indeed, let us consider the inclusive spectrum of a fast quark produced at a point z, with respect to its transverse momentum ($p_\perp$) after n rescatterings with nucleons at points $z_1, z_2, ...z_n$. This spectrum can be written as [6] (see Fig.1)

$$\frac{d\sigma_n}{d^2p_\perp} = e^{-\sigma_{tot}\,(z_1 - z)}\, dz_1\, \rho\, \frac{d\sigma}{d^2q^2_{1\perp}} \cdot e^{-\sigma_{tot}\,(z_2 - z_1)}\, dz_1\, \rho\, \frac{d\sigma}{d^2q^2_{2\perp}} ... e^{-\sigma_{tot}\,(z_1 + L - z_n)}\, dz_n\, \rho\, \frac{d\sigma}{d^2q^2_{n\perp}}$$

$$\delta^{(2)}\left(\sum_{m=1}^{n} \vec{\mathbf{q}}\mathbf{m}_\perp - \vec{\mathbf{p}}_\perp\right) \prod_m d^2q_{m\perp} \quad (z_1 < z_2 < ... < z_n < z_1 + (L = \tau_{max})) \tag{16}$$

Here, ρ is the nucleon density, $\frac{d\sigma}{d^2q_{m\perp}}$ is the spectrum of the quark due to rescattering with momentum transfer $q_{m\perp}$ which can be written through the unintegrated parton density ϕ [6] (see also relevant papers [7] [8]):

$$\frac{d\sigma}{d^2q_{m\perp}} = \frac{8\pi^2\alpha^2_{\rm S}(q^2_{m\perp})}{9\,q^4_{m\perp}} \cdot (x\phi_N(x, q^2_{n\perp})) \tag{17}$$

where $x\phi_N(x, q^2_\perp)$ is the nucleon parton density with $x = \frac{1}{\tau_{max}}$. The relation between the unintegrated parton density ϕ and the nucleon's gluon structure function $xG_N(x, q^2_\perp)$ can be calculated using the following equation:

$$\alpha_{\rm S}(Q^2) \cdot xG(x, Q^2) = \int^{Q^2} d\,q^2_\perp\, \alpha_{\rm S}(q^2_\perp)\, \phi(x, q^2_\perp)$$

The factor $exp(-\sigma_{tot} \ (z_m - z_{m-1}))$ in Eq. (16) describes the fact that the quark has no inelastic collisions between the centres z_{m-1} and z_m. The total cross section is equal to $\sigma_{tot} = \int d^2q_{m\perp} \frac{d\sigma}{d^2q_{m\perp}}$ and it is formally divergent at $q_{m\perp} \to 0$. However, this divergency cancels against the divergency of the inelastic cross section. To see this fact, it is convenient to describe the process in the transverse coordinate space $r_\perp$ using the well know representation of the δ function:

$$\int \frac{d^2r_\perp}{(2\pi)^2} \exp[i \sum_m (\vec{\mathbf{q}}\mathbf{m}_\perp - \vec{\mathbf{p}}_\perp) \cdot \vec{\mathbf{r}}_\perp] = \delta^{(2)}\left(\sum_{m=1}^{n} \vec{\mathbf{q}}\mathbf{m}_\perp - \vec{\mathbf{p}}_\perp\right)$$

Summing over n we obtain

$$\frac{d\sigma}{d^2p_\perp} = \int \frac{dr^2_\perp}{4\,\pi^2}\, J_0(p_\perp\, r_\perp)\, \exp\left(-\,\sigma(r^2_\perp)\,\rho\, L\right) \tag{18}$$

with

$$\sigma(r^2_\perp) = \int dq^2_\perp\, \{1 - J_0(q_\perp\, r_\perp\, \}\, \frac{d\sigma}{dq^2_\perp} = \frac{\alpha_{\rm S}(\frac{4}{r^2_\perp})}{3}\, \pi^2\, r^2_\perp \left(xG_N(\frac{4}{r^2_\perp}, x) \right) \tag{19}$$

Let us investigate $\frac{d\sigma}{d^2p_\perp}$, adopting the following method of integration over $r_\perp$ in Eq. (18):(i) we consider $xG_N(\frac{4}{r^2_\perp}, x)$ as a smooth function of $r^2_\perp$ since it depends on $r^2_\perp$ only logarithmically; (ii) we integrate over $r_\perp$ taking only into account $\sigma \propto r^2_\perp$; and, (iii) we put in $xG_N(\frac{4}{r^2_\perp}, x)\, r^2_\perp = r^2_{0\perp}$, where $r^2_{0\perp}$ is the typical value of $r^2_\perp$ in the integral. The result of the integration is obvious, namely

$$\frac{d\sigma}{d^2p_\perp} = \frac{1}{4\,\rho\, L\, \pi^2\, [\, \frac{\alpha_{\rm S}\pi^2}{3} x\, G_N(\frac{4}{r^2_{0\perp}}, x)\,]}\, e^{-\frac{p^2_\perp}{4\,\rho\, L\, \frac{\alpha_{\rm S}\pi^2}{3} x\, G_N(\frac{4}{r^2_{0\perp}}, x)}} \tag{20}$$

One can find the value of $r^2_{0\perp}$ by calculating the saddle point in $r_\perp$ integration, which gives:

$$r^2_{0\perp} = \frac{p^2_\perp}{4\,(\,\rho\, L\, \frac{\alpha_{\rm S}\pi^2}{3} x\, G_N(\frac{4}{r^2_{0\perp}}, x)\,)^2} \quad for \ \ p^2_\perp\, r^2_{0\perp} \gg 1\,; \tag{21}$$

$$r^2_{0\perp} \;=\; \frac{1}{2\,(\,\rho\,L\,\frac{\alpha_S \pi^2}{3} x\,G_N(\frac{4}{r^2_{0\perp}},x)\,)} \quad for \;\; p^2_\perp\, r^2_{0\perp} \;\ll\; 1\;;$$

We can trust our calculation in pQCD only if $r_\perp \;\ll\; r_{soft}$, where r_{soft} is the scale at which the nonperturbative QCD corrections become essential. It means that the value of $p_\perp$ should be smaller than $2\,(\,\rho\,L\,\frac{\alpha_S \pi^2}{3} x\,G_N(\frac{4}{r^2_{0\perp}},x)\,)$. For bigger $p_\perp$, in Eq. (18), we take $r_\perp \;\propto\; \frac{1}{p_\perp}$ which gives $\frac{d\sigma}{d^2 p_\perp} \;\propto\; \frac{1}{p^2_\perp}\,\sigma(\frac{1}{p^2_\perp})\,\rho\,L$. The main message from this calculation is that the typical distances that work in the rescattering inside the nuclear matter turns out to be small. This justifies the use of pQCD in this case. Therefore, we can make a guess that the parameter μ that has been discussed in the introduction is equal to $\mu^2 \;=\; \frac{1}{r^2_{0\perp}}$. However, we have to be very carefull because the $p_\perp$ - distribution has a tail at large values of $p_\perp$. We will show that we can safely use Eq. (20) to study the LPM effect.

The value of the mean free path λ can also be estimated from Eq. (18) and it is equal $\lambda \;=\; \frac{1}{\sigma(r^2_{0\perp})\,\rho}$. This is clear if we rewrite Eq. (18) as

$$\frac{d\sigma(\Delta z)}{d^2 p_\perp\,dz} \;=\; \int \frac{d r^2_\perp}{2}\,\sigma(r^2_\perp)\,\rho\,J_0(\,p_\perp\,r_\perp\,)\,\exp\left(-\,\sigma(\,r^2_\perp\,)\,\rho\,\Delta z\,\right) \tag{22}$$

where Δz is the longitudinal distance that the quark passes in a nuclear matter $(\,\sigma(\,r^2_\perp\,)\,\rho\,\Delta z \;\;>\;1\,)$.

Actually, Eq. (22) sets the averaging procedure for Eq. (13). Indeed

$$\langle\;\rangle \;=\; \prod_{l=1}^{n+2} \frac{d^2 U_{l\perp}}{\pi} \prod_{l=1}^{n+2} dz_l\;\frac{d\sigma(z_l - z_1)}{d^2 p_{l\,\perp}\,dz}\;;\;\; z_1 \;<\; z_2\,...\;<\;z_{l-1}\;<\;z_l\;<\;...\;<\;z_{n+2}\;<\;z_1\;+\;L \tag{23}$$

Using the explicit expression for $\vec{J}_i$ of eq.(1), one can see, after integration over the azimuthal angle, that only big transfer momenta $q_{i\perp} \;\gg\; \frac{k_\perp}{x_F}$ contribute to Eq. (13). It means that $p_{l\perp} \;\rightarrow\; U_l$. This fact allows us to make the integration over U_l in Eq. (23) explicitly which leads to

$$\left\langle |\frac{dI_l}{dz_l}| \right\rangle \;\equiv\; \int_{U^2_0}^{\infty} d\,U^2_{l\perp}\;\frac{d\sigma(z_l - z_1)}{d^2 U_l\;d\,z_l}\;\exp\left\{ i\frac{1}{\tau} U^2_l\,(\,z_{l+1} - z_l\,)\right\} \tag{24}$$

Introducing a new variable $\tilde{z}_l \;=\; \rho\,z_l\,\mathbf{D} \;=\; \rho\;z_l\;\frac{\alpha_S \pi^2}{3} x\,G_N(\frac{4}{r^2_{0\perp}},x)$ one can take the integral over U_l and obtain:

$$\left\langle |\frac{dI_l}{dz_l}| \right\rangle \;=\; \mathbf{D}\,\rho\,\frac{d}{d\tilde{z}_l}\,\{\,\frac{1}{i\,\kappa\,\tilde{z}_l\,\Delta\tilde{z}_l\;-\;\frac{1}{4}}\;\cdot\,\exp[-\,\frac{U^2_0}{4\tilde{z}_l}\;+\;i\,\kappa\,U^2_0\,\Delta\tilde{z}_l\,]\,\}\;= \tag{25}$$

$$=\;\{\,16\,i\,\kappa\,\Delta\,\tilde{z}_l\;-\;\frac{U^2_0}{\tilde{z}^2_l}\,\}\;\cdot\;\exp[-\,\frac{U^2_0}{4\tilde{z}_l}\;+\;i\,\kappa\,U^2_0\,\Delta\tilde{z}_l\,]$$

where $\kappa \;=\; \frac{1}{\tau\,\cdot\,\mathbf{D}\,\rho}$ and $\Delta\tilde{z}_l \;=\; \tilde{z}_{l+1} - \tilde{z}_l$. We anticipate that $\kappa\tilde{z}_l\Delta\tilde{z}_l \;\ll\; 1$ and have expanded the answer with respect to this parameter.

The above equation allows us to obtain the functional equation for

$$\Phi_n(U^2_0, z) \;=\; \prod^{n} \left\langle |\frac{dI_l}{dz_l}| \right\rangle \tag{26}$$

Namely,

$$\Phi_{n-1}(U^2_0, z - \Delta z)\,\{\,16 i\kappa\Delta z\;-\;\frac{U^2_0}{z^2_l}\,\}\;\cdot\;\exp\,[-\,\frac{U^2_0}{4z}\;+\;i\,\kappa\,\Delta z\,]\;=\;\Phi_n(U^2_0, z) \tag{27}$$

Considering $\Delta z \ll z$, we can solve Eq. (27) and obtain

$$\Phi_n(\kappa, U_0^2, z) \;=\; \left(-\frac{U_0^2}{\tilde{z}^2}\right)^{n+1} \exp\left[-\frac{U_0^2}{4\tilde{z}}\,(\,n\,+\,1\,)\;+\;i\,\kappa U_0^2\,\tilde{z}\;+\;\frac{16 i\kappa\,\tilde{z}^3}{3\,U_0^2}\right] \tag{28}$$

Substituting Eq. (28) in Eq. (13), summing over n and substructing the value of the integral at $\kappa = 0$, which corresponds to 1 in Eq. (13), we obtain the answer:

$$\omega\,\frac{dI}{d\omega dz} \;=\; \frac{\alpha_{em}}{\pi}\,4\,\mathbf{D}\,\rho \int \frac{dU_0^2}{U_0^2} \int d\tilde{z}\;\frac{sin^2\{\frac{\kappa}{2}\,[\,\frac{16\tilde{z}^3}{3\,U_0^2}\;+\;U_0^2\,\tilde{z}\,]\,\}\,(\,-\frac{U_0^2}{\tilde{z}^2}\,)^2\,e^{-\frac{U_0^2}{2\tilde{z}}}}{1\;+\;\frac{U_0^2}{\tilde{z}^2}\,e^{-\frac{U_0^2}{4\tilde{z}}}} \tag{29}$$

Introducing a new variable $\xi \;=\; \frac{U_0^2}{\tilde{z}}$ one can rewrite Eq. (29) in the form:

$$\omega\,\frac{dI}{d\omega dz} \;=\; \frac{\alpha_{em}}{\pi}\,4\,\mathbf{D}\,\rho \int \frac{dU_0^2}{U_0^2} \int_{\xi_0}^{\infty} \xi^2\,d\,\xi\;\frac{sin^2(\frac{\kappa\,U_0^2}{2}\,[\frac{16}{3\xi^3}\;+\;\frac{1}{\xi}]\,e^{-\frac{\xi}{2}}}{U_0^2\;+\;\xi^2\,e^{-\frac{\xi}{4}}} \tag{30}$$

The lower limit in the ξ integral is $\xi_0 \;=\; \frac{U_0^2}{\rho\,\mathbf{D}\,L}$. For $\kappa\,U_0^4 \;\gg\; 1$ $\xi \;\approx\; 1$ contributes to the integral and we recover the BH limit. For $\kappa\,U_0^4 \;\ll\; 1$ the main contribution comes from the region of small ξ, but $\xi \;>\; \xi_{min} \;=\; (\frac{8\kappa U_0^4}{3})^{\frac{1}{3}}$. In this region the argument of the sin in Eq. (30) turns out to be small. Expanding sin and doing all integrations we get

$$\omega\,\frac{dI}{d\omega dz} \;=\; \frac{\alpha_{em}}{\pi}\,4\,\mathbf{D}\,\rho\,\frac{8}{9}\,\sqrt{\kappa} \;=\; \frac{32\alpha_{em}}{9\,\pi}\,\sqrt{\frac{\omega\,\mathbf{D}\,\rho}{2\,E^2}} \tag{31}$$

To obtain the final answer we need to specify the argument of the gluon structure function in $\mathbf{D}$. It turns out that the value of the typical $r_{0\perp}^2 \;\propto\; \sqrt{\kappa}$ and it is small for small κ for which we have derived Eq. (44). Therefore we are justified in our approach and the final answer is

$$\omega\,\frac{dI}{d\omega dz} \;=\; \frac{32\alpha_{em}}{9\,\pi}\,\sqrt{\frac{\omega\,\frac{\alpha_S\pi^2}{3}\;\;(x\,G(x, r_{soft}^2\frac{1}{\sqrt{\kappa}})\,\rho}{2\,E^2}} \tag{32}$$

The region of applicability of Eq. (32) can be found from the condition that the typical distances in our calculations are small so as to use perturbative calculation. In other words, it can be found from the integral of Eq. (30) in the region $\xi \;\approx\; 1, \kappa\,U_0^4 \;\gg\; 1$. It gives

$$\omega_{BH} \;=\; \sigma(r_{soft}^2)\,\rho\,E^2 \tag{33}$$

where r_{soft}^2 is the scale from where we can use perturbative QCD ($r_\perp^2 \;<\; r_{soft}^2$), and which should be calculated in a nonperturbative QCD approach.

The factorization limit comes from the contstraint that $\xi_{min} \;\leq\; \xi_0$ and gives

$$\omega_{fact} \;=\; \frac{\langle p_\perp^2 \rangle}{\rho\sigma(\frac{1}{\langle p_\perp^2 \rangle})} \tag{34}$$

Summarizing this section we want to mention that we did not obtain the
$sqrtkappa\,ln\kappa$ behaviour of the radiation density in a nuclear matter as it was done for "hot" deconfined plasma in ref.[1]. The difference comes from the averaging over momentum transfers $\vec{q}_{l\perp}$ which turns out to be quite different from the screened Coulomb potential applied in ref.[1] for a "hot" plasma.

3 The LPM effect and Mueller technique.

In this section we are going to discuss the interrelation between our approach and the Mueller's technique [5], which, together with the AGK cutting rules [9], remarkably simplify the calculation of the inclusive spectra of produced particles. It is widely used in all Reggeon inspired calculations. The Mueller diagrams for the inclusive spectra of emitted photons are shown in Fig.2. There are two main assumptions that make the Mueller technique simple and attractive: (i) the spectrum of a produced particle does not depend on the kinematics of the incoming one; and, (ii) the longitudital part of the momentum transfer turns out to be so small at high energy that it can be neglected.

The first assumption holds for the emission of sufficiently soft particles and it corresponds to the factorization properties of the production of particles from sufficiently small distances in QCD [10]. At first sight one can prove that the diagrams of Fig.2b do not contribute to the inclusive cross section using the AGK cutting rules and factorization. However, we are dealing with the emission from the fastest particle and factorization does not work in this case. As a result the sum of the diagrams of Fig.2 gives the factorization limit, or in other words the second term in Eq. (12). We plan to discuss this term in a separate paper where we shall consider the inclusive cross section for the emitted photon or gluon. However, even the sum of all Mueller diagrams cannot reproduce the LPM effect.

The LPM effect comes from a more carefull consideration of the dependance of the production amplitude on the longitudinal part of the momentum transfer q_l. From Fig.3 we can see that the interaction with the centre 'l' in the first term of Eq. (12) has the longitudinal component of the momentum transfer q_z (z is the direction of the incoming quark) which is equal:

$$q_z \;=\; \frac{1}{2E} \cdot \{\,(\,p\,+\,k\,)^2\;-\;(\,p'\,+\,k\,)^2\,\} \;=\; \frac{1}{2E} \cdot \{\,2\,k_\mu\,q_{l\mu}\,\} \;=\; \frac{\omega}{2E^2}\,\{\,U^2_{l+1}\,-\,U^2_l\,\} \tag{35}$$

where we have neglected the change in the energy of the fast quark due to the photon emission, as well as due to rescattering. On the other hand, the longitudinal part of the momentum q'_l is small. At high energy we are doing all calculations for quark - nucleon interactions in the leading log(1/x) approximation (see for example ref.[11]), in which we neglect the longitudinal momentum dependance for all gluon and quark propagators. The only dependance on the longitudinal momentum comes from the fact that the total momentum transfer for the scattering off the l-th centre ($Q_l\;=q_l+q'_l$) differs from zero and it is equal $Q_{z\,l}\;=\;q_{z\,l}$. Considering nucleons, in a nuclear matter, as nonrelativistic particles, we have neglected the change of the energy component of Q_l. Finally, we have for each rescattering

$$\Psi^*_{initial}(l)\,\cdot\,\Psi_{final}(l) \;=\; \rho\,e^{i\,Q_{z l}\,z_l} \;=\; \rho\,e^{i\frac{1}{\tau}\,(\,U^2_{l+1}\,-\,U^2_l\,)}\;, \tag{36}$$

where $\Psi(l)$ is the wave function of the l-th nucleon in a nuclear matter. One recognizes the phase that we have taken into account in Eq. (13).

Therefore, the LPM effect can be derived from Mueller diagrams if one takes into account the dependance on the longitudinal part of the momentum transfer. We will show that the Mueller diagrams will be very useful to reach simple and transparent understanding of the LPM - like effect in the case of gluon emission.

4 The LPM effect for gluon emission.

In this section we consider the gluon radiation in a nuclear matter using the Mueller diagrams of Fig.3. To write down the diagrams of Fig.3 we have to specify the expression for $\vec{J}_i$ for gluon emission and calculate the longitudinal part of the momentum transfer (see Eq. (35)).

The first ingredient in the soft limit ($\omega \ll E$) was calculated many years ago by Lipatov and collaborators [12] and has been confirmed using quite different technique (see ref. [3] in which such a calculation was done just for the case of induced gluon radiation in a medium. It is very relelevant to our approach). The answer is (see Fig.4):

$$\vec{J}_i \;=\; \frac{\vec{k}_\perp}{k^2_\perp} \;-\; \frac{\vec{k}_\perp \;-\; \vec{q}_{l\,\perp}}{(\,\vec{k}_\perp \;-\; \vec{q}_{l\,\perp}\,)^2} \tag{37}$$

with the colour factor which is equal to the colour factor of the Feynman diagram of Fig.4(2). To simplify the calculation of the colour factor we perform them for QCD with a large number of colours N_c neglecting all terms of the order of $1/N_c$.

For a gluon we have two rescatterings which are shown in Fig.4b and Fig.4c. The value of q_{lz}, for the quark rescattering, has been calculated (see Eq. (35)) and it is small in the soft region where $E \gg \omega$. For the gluon rescattering q_{lz} is equal to:

$$q_z \;=\; \frac{1}{2E}\cdot\{\,(\,p\,+\,k\,)^2 \;-\; (\,p\,+\,k'\,)^2\,\} \;=\; \frac{1}{2E}\cdot\{\,2\,p_\mu\,q_{l\mu}\,\} \;=\; \frac{\omega}{2\omega}\{\,U^2_{G,l+1} \,-\, U^2_{G,l}\,\} \tag{38}$$

where

$$\vec{U}_{G,l} \;=\; \vec{k}_\perp \;-\; \sum_{i=1}^{l} \vec{q}_{i\,\perp}\;;\quad \vec{U}_{G,0} \;=\; \vec{k}_\perp\;. \tag{39}$$

Comparing Eq. (35) and Eq. (38) one can see that q_{lx} due to quark rescattering is much smaller than q_{lz} for a gluon collision. Therefore, we can neglect the quark rescattering in the first approximation to the problem.

Therefore, we can write an equivalent expression to the one in Eq. (13) for gluon radiation density, using the following substitutes:

$$\vec{U}_l \;\rightarrow\; \vec{U}_{G,l}\;;\;\; \kappa \;\rightarrow\; \kappa_G \;=\; \frac{1}{2\,\omega\,\mathbf{D}\,\rho}\;. \tag{40}$$

We have to make two comments:

1 . Inspite of the fact that we are dealing with gluon rescattering in the averanging procedure over the transverse momentum of the produced gluon $k_\perp$ given by Eq. (23), we should use the same quark cross section given by Eq. (19). Indeed, the gluon - quark pair scatters in the medium and the transverse

separation between them due to many rescatterings off nucleons in the limit $q_{lz} \rightarrow 0$, as we will show below, is of the order $r_{\perp}^2 \propto \sqrt{\kappa_G}$. The average momentum transfer in a single collision $\langle q_{l\perp} \rangle$ is of the order of $\langle q_{l\perp}^2 \rangle \approx \frac{1}{r_{\perp}^2 N_{col}} \ll \frac{1}{r_{\perp}^2}$, where N_{col} is the number of collisions which is big enough $\propto \frac{1}{\sqrt{\kappa_G}}$. Therefore, each gluon in the nucleon cross section carries a transverse momentum $q_{l\perp}$ which is much smaller than $\frac{1}{r_{\perp}}$. It means that such a gluon interacts with the total colour charge of the gluon - quark pair, which is equal to the charge of the quark at $N_c \gg 1$. The typical time that a gluon lives is $\tau_G = \frac{\omega}{U_0^2}$ which is much smaller than the life-time of the quark $\tau_Q = \frac{E}{\langle p_{\perp}^2 \rangle}$. It means that for each emitted gluon the quark plays the role of a spectator that neutralizes half of the gluon colour charge.

2. It was shown in ref.[3] (see also ref. [1]) that quark rescattering contributes to the radiation density of gluons, if one takes into account the $1/N_c$ corrections. The origin of this contribution is the dynamic gluon correlations that have been studied in ref. [13]. Such correlations change the Glauber-type formula of Eq. (18) that was used for the averaging in the nuclear matter.

The final answer is Eq. (30) with the substitutions defined in Eq. (40). The value of ξ_0 is equal to $\xi_0 = \frac{U_0^2}{\rho \mathbf{D} L} = 2\kappa_G U_0^2$. For $\kappa_G U_0^4 \ll 1$, we derive the answer:

$$\omega \frac{dI_G}{d\omega dz} = \frac{N_C \alpha_S}{2\pi} 4 \mathbf{D} \rho \frac{8}{9} \sqrt{\kappa} = \frac{16 N_c \alpha_S}{9\pi} \sqrt{\frac{\frac{\alpha_S \pi^2}{3} \, (x\, G(x, \frac{1}{r_{soft}^2 \sqrt{\kappa}})\, \rho}{2\, \omega}} \tag{41}$$

r_{soft}^2 in Eq. (41) is a scale of the "soft" interaction. We can trust a perturbative calculation for the nucleon gluon distribution only for $r_{\perp} < r_{soft}$. The value of $1/x$ in Eq. (41) is equal to $\frac{\tau_G}{\tau_{soft}}$ where $\tau_G = \frac{\omega}{U_0^2} = \sqrt{\kappa_G}\, \omega$ and τ_{soft} is a typical time for the "soft" processes which we cannot specify in the leading log (1/x) approximation (LL(1/x)A) which we have used to obtain the answer. To trust the LL(1/x)A we have to assume that thr emitted gluon energy is so big that $\alpha_S ln \frac{\tau_{soft}}{\tau_G} \geq 1$. It means that $\sqrt{\kappa_G}\, \omega \gg \tau_{soft}$.

The value of ω_{BH} can be obtained from the equation $\sqrt{\kappa} = r_{soft}^2$ which gives

$$\omega_{BH}^G = \frac{1}{r_{soft}^2 \; \sigma(r_{soft}^2) \; \rho} \tag{42}$$

It should be stressed that unlike the QED case we cannot find the factorization limit for induced gluon emission. The physical reason for this is obvious since the phase of the propagating gluon cannot be small due to its rescattering in a nuclear matter, at least in the kinematic region where κ_G is small. We would like to recall that we cannot trust our formulae for big valus of κ_G as has been discussed above.

The radiation density of Eq. (41) is very close to the result of ref. [1] and quite different from other attemps to estimate the LPM effect for gluon radiation [3] [14]. The main difference between Eq. (41) and the result of ref. [1] is the fact that the gluon nucleon density enters our answer while the radiation density of ref. [1] is proportional to $\frac{1}{\sqrt{\omega}} \ln \omega$. Since HERA data [15] shows sufficiently steep energy behaviour of the gluon structure function [15] which could be parametrized as $\frac{1}{x^{\omega_0}}$ with the value of $\omega_0 \sim 0.3 - 0.4$, the ω behaviour of the gluon radiation density can be evaluated as $\omega \frac{dI_G}{d\omega dz} \propto \omega^{-\frac{1}{2} + \frac{\omega_0}{4}}$. The energy losses of the fast quark due to gluon emission can be estimated integrating Eq. (41) over ω

up to E :

$$- \frac{dE}{dz} \propto \frac{N_c \alpha_S}{2\pi} \cdot \frac{1}{\frac{1}{2} + \frac{\omega_0}{4}} \cdot E^{\frac{1}{2} + \frac{\omega_0}{4}} . \tag{43}$$

However, we shall be very careful with such kind of estimates since the value of ω_0 depends crussially on the value of the gluon virtuality which depends on ω in its turn (see Eq. (41)). The integral over ω in $\frac{dE}{dz}$ concentrates at $\omega \approx E$ where the soft energy approximation is not valid. Because of this we can consider Eq. (43) only as a rough estimate which we are going to improve later.

5 Conclusions

In this letter we have considered the LPM effect for photon and gluon radiation off a fast quark propagating in a nuclear matter. The close analogy betwen photon and gluon emission suggested in ref. [1] has been confirmed and the relation between the Mueller approach [5] and traditional calculations has been established. The main result reads:

$$\omega \frac{dI}{d\omega dz} = \frac{32\alpha}{9\,\pi} \sqrt{\frac{\frac{\alpha_S \pi^2}{3} \, (x\,G(x, \frac{1}{r^2_{soft}\,\sqrt{\kappa}})\,\rho}{\tau}} \tag{44}$$

where κ is defined in Eq. (25) and for QED: $\alpha = \alpha_{em}\,; \frac{1}{\tau} = \frac{\omega}{2E^2}\,; x = \frac{\langle p^2_\perp \rangle}{mE}$. For QCD: $\alpha = \frac{N_c \alpha_S}{2}\,; \frac{1}{\tau} = \frac{1}{2\omega}\,; x = \frac{r^2_{soft}}{\omega\sqrt{\kappa}}$.

The answer does not depend on the nonperturbative QCD scale. It depends on the ratio $\frac{\mu^2}{\lambda} = \rho \cdot \{ \frac{\alpha_S \pi^2}{3} \, (x\,G(x, \frac{1}{r^2_{soft}\,\sqrt{\kappa}} \}$ which depends on the QCD scale only in the argument of the gluon structure function. However, the value of ω for which we can apply the above formula comes from $\kappa \ll r^2_{soft}$ and crucially depends on the value of r^2_{soft} (see Eq. (42)) . r_{soft} establishes the scale of distances ($r_\perp < r_{soft}$) where we can trust the perturbative QCD approach for nucleon interactions. This scale has clear nonperturbative origin, however, it becames large and grows with x in the small x region where the saturation of gluon density in a nucleon should be reached [11]. Taking $r^2_{soft} = 0.5 GeV^{-2}$ and $\rho = 0.17 Fm^{-3}$ we derive that our formula for gluon induced radiation can be justified for $\omega > \omega_{BH} = 100 GeV$. We are going to present more reliable estimates, as well as the application to the gluon inclusive cross section, in further publications.

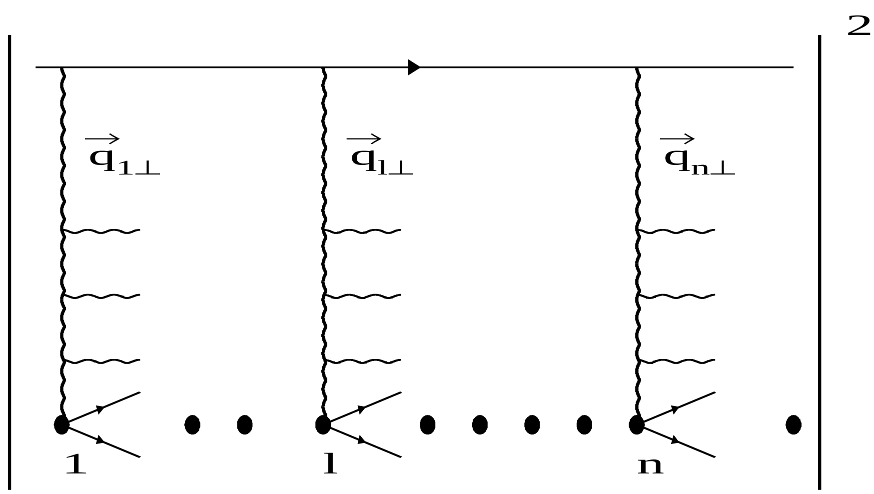

Figure 1: Propagation of the quark through a nuclear matter.

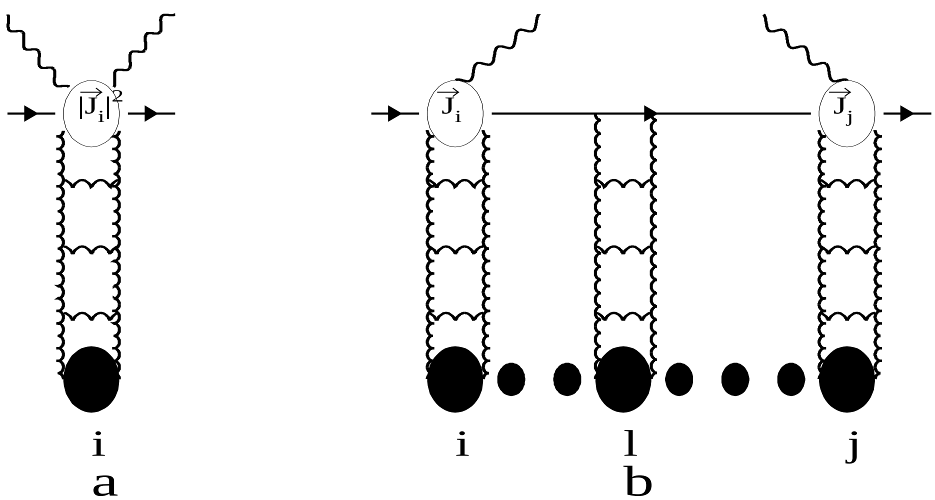

Figure 2: Mueller diagrams for the photon emission.

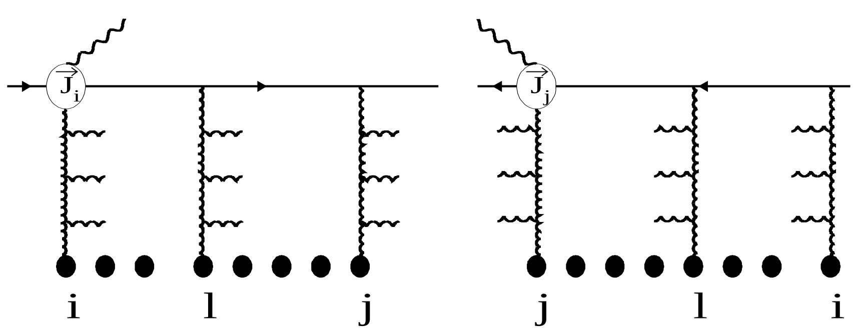

Figure 3: Diagrams responsible for the LPM effect.

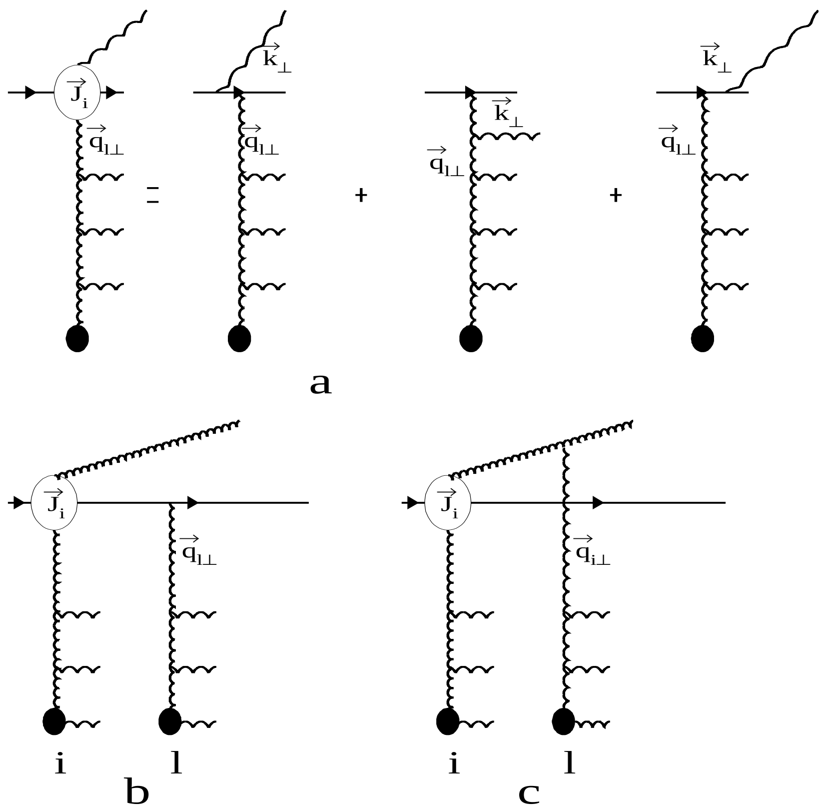

Figure 4: Mueller diagrams for the gluon emission.

References

[1] R.Baier, Yu. L. Dokshitzer, S. Peigne and D. Schiff: LPTHE-Orsay 94/98.

[2] L. D. Landau and I. Ya. Pomeranchuk: *Dokl. Akad. Nauk SSSR* **92** (1953) 535, 735; A. B. Migdal: *Phys. Rev.* **103** (1956) 1811.

[3] M. Gyulassy and X.-N. Wong: *Nucl. Phys.* **B420** (1994) 583; M. Gyulassy, X.-N. Wong and M. Plumer: LBL-35980, August 1994.

[4] E. L. Feinberg and I. Ya. Pomeranchuk: *Nuovo Cimento Suppl.* **III** (1956) 602; V.M.Galitsky and I.L.Gurevich:*Nuovo Cimento* **XXXII** (1964) 396; M.L. Ter-Mikaelian: High Energy Electromagnetic Processes in Condensed Media, John Wiley & Sons, 1972; J.D. Bjorken: Fermilab Report PUB-82/59-THY,1982; A.I.Akhiezer and N.F. Shul'ga: *Sov. Phys. Usp.* **30** (1987) 197; M.Gyulassy et al.: *Nucl.Phys.***A538** (1992) 37c; A.H.Sorensen: *Z. Phys.* **C53** (1992) 595 and references therein.

[5] A.H.Mueller: *Phys. Rev.* **D23** (1970) 2963.

[6] E.M.Levin and M.G.Ryskin: *Sov. J. Nucl. Phys.* **33** (1981) 901.

[7] E.M. Levin and M.G.Ryskin: *Sov. J. Nucl. Phys.* **45** (1987) 150.

[8] A.H. Mueller: *Nucl. Phys.* **B335** (1990) 115.

[9] V.A.Abramovski,V.N. Gribov and O.V. Kancheli: *Sov. J. Nucl. Phys.* **18** (1973) 308.

[10] J.C. Collins, D.E. Soper and G. Sterman : *Nucl. Phys.* **B308** (1988) 833.

[11] L. V. Gribov, E. M. Levin and M. G. Ryskin: *Phys.Rep.* **100** (1983) 1.

[12] E.A. Kuraev, L.N. Lipatov and V.S. Fadin: *Sov. Phys. JETP* **45** (1977) 199 ; Ya.Ya. Balitskii and L.V. Lipatov:*Sov. J. Nucl. Phys.* **28** (1978) 822.

[13] J. Bartels: *Z. Phys.* **C60** (1993) 471, *Phys. Lett.* **B298** (1993) 204; E.M. Levin, M.G. Ryskin, A.G. Shuvaev: *Nucl. Phys.* **B387** (1992) 589.

[14] M.G. Ryskin: *Sov. J. Nucl. Phys.* **52** (1990) 139.

[15] ZEUS collaboration, M. Derrick et al.: *Z. Phys.* **C65** (1995) 379; H1 collaboration, T. Ahmed et al.: DESY 95-086 (1995);DESY 95-086 (1995).