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Abstract

We calculate the only-loop effective potential for N=1, d=4 su pergravity theory coupled to the Volkov-Akulov field. Then it is shown that after an ajustment of some parameters the local supersymmetry is dynamically broken and as a consequence the gravitino acquires mass.

Key-words: One-loop effective potential for simple D=4 supergravity.

The Volkov-Akulov field coupled to the N=1, d=4 supergravity gives rise to the Super-Higgs effect (Deser, Zumino 1977). In this way the gravitino absorbes the degrees of freedom of the Volkov-Akulov field so that we end up with the N=1, d=4 supergravity lagrangian plus a (negative) cosmological constant. At this stage supersymmetry is not broken.

On the other hand, taking into account quantum processes, it can be shown (Jasinschi, Smith 1983) that the gravitino acquires a dynamically generated mass-like term in addition to a (positive) cosmological constant. The effective cosmological constant (sum of the two contributions), being different from zero, does not permit to interpret the gravitino mass term as physical, unless it is fine tuned to zero (thus turning space-time Minkowskian). That is why we called the gravitino mass term "mass-like". (Townsend 1978). If we make the effective cosmological constant zero the gravitino mass term becomes a physical parameter and supersymmetry is broken. This gravitino mass gives to the vacuum energy a positive contribution so that it turns into a supersymmetry breaking parameter.

We will show that the one-loop effective potential for the gravitino scalar condensate exibits gravitino mass generation and supersymmetry breaking. The N=1, d=4 supergravity Lagrangian coupled to the Volkov-Akulov field λ is given by

$$\mathcal{L} = -\frac{e}{2a^2} - i\frac{e}{2}\bar{\lambda}\not{D}\lambda + i\frac{\kappa}{2a^2}e\,\bar{\lambda}\not{\psi} - \frac{e}{2\kappa^2}R(e,\psi) - \frac{1}{2}\varepsilon^{\mu\nu\lambda\rho}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\lambda\psi_\rho + \frac{1}{3}(A_\mu^2 - S^2 - P^2) \qquad (1)$$

where a describes a dimensionfull constant $[(\mathrm{mass})^{-2}]$, $\kappa = (16\pi G_N)^{1/2}$

with G_N the Newtonian gravitational coupling constant and the set (A_μ, S, P) the minimal supergravity algebra closure auxiliary fields.

Using the Super-Higgs effect and the gauge

$$\gamma^\mu \psi_\mu = 0 \tag{2}$$

leads us to the following lagrangian

$$\begin{aligned}\mathcal{L} = &-\frac{e}{2a^2} - \frac{e}{2\kappa^2} R(e) + \frac{e}{4}\{\frac{1}{e}\partial_\mu[e^\mu_a e^\nu_b e][\bar{\psi}_\nu \gamma^a \psi^b - \bar{\psi}_\nu \gamma^b \psi^a + \\ &+ \bar{\psi}^a \gamma_\nu \psi^b]\} - \frac{1}{2}\varepsilon^{\mu\nu\lambda\rho}\bar{\psi}_\mu \gamma_5 \gamma_\nu (\partial_\lambda + \frac{1}{2}\omega_\lambda^{ab}(e)\,\sigma_{ab})\,\psi_\rho - \\ &- \frac{11}{16}\kappa^2 e[(\bar{\psi}_\mu \psi^\mu)^2 - (\bar{\psi}_\nu \gamma_5 \psi^\mu)^2] + \frac{33}{64}\kappa^2 e(\bar{\psi}_\mu \gamma_5 \gamma_\nu \psi^\mu)^2 + \frac{1}{3}(A_\mu^2 - S^2 - P^2)\end{aligned} \tag{3}$$

where R(e) represents the scalar curvature without torsion.

The quartic gravitino interaction terms, appearing in (3), can be eliminated in favor of a set of auxiliary fields (Eguchi 1976) (σ, π, λ_a). In this sense lagrangian (3) can be re-writen as:

$$\begin{aligned}\mathcal{L} = &-\frac{e}{2a^2} - \frac{e}{2\kappa^2} R(e) + \frac{e}{4}\{\frac{1}{e}\partial_\mu[e^\mu_a e^\nu_b e][\bar{\psi}_\nu \gamma^a \psi^b - \bar{\psi}_\nu \gamma^b \psi^a + \\ &+ \bar{\psi}^b \gamma_\nu \psi^a]\} - \frac{1}{2}\varepsilon^{\mu\nu\lambda\rho}\bar{\psi}_\mu \gamma_5 \gamma_\nu (\partial_\lambda + \frac{1}{2}\omega_\lambda^{ab}(e))\,\psi_\rho + \sigma^2 + \frac{\sqrt{11}}{2}\kappa(\bar{\psi}_\mu \psi^\mu)\sigma + \\ &+ \pi^2 + \frac{\sqrt{11}}{2} i\kappa(\bar{\psi}_\mu \gamma_5 \psi^\mu)\,\pi + \frac{\sqrt{33}}{2}\kappa i(\bar{\psi}_\mu \gamma_5 \gamma_\nu \psi^\mu)\,\lambda^\nu + \frac{1}{3}(A_\mu^2 - S^2 - P^2)\end{aligned} \tag{4}$$

Lagrangian (4) is equivalent to (3) if we use the auxiliary fields equation of motion (in fact they have no kinetic terms at the classical level and consist of condensates of the gravitino field in the scalar-σ-, pseudo-scalar-π- and axial-λ_μ-channels).

The one-loop effective potential for the σ field is given by

$$V_{eff} = e\{\lim_{v\to\infty}[\frac{i}{2}\frac{1}{v}\sum_{n=1}^{\infty}\frac{[\sqrt{11}\kappa]^n}{n}(-1)^{n+1}\,Tr[P_{ab}\sigma]^n + \frac{1}{2a^2} - \sigma^2\} \qquad (5)$$

where v is the space-time four volume and P_{ab} the gravitino propagator (using gauge (2)), which in momentum space is represented as

$$\tilde{P}_{ab}(p) = -\frac{i}{2}\frac{\gamma_b \not{p} \gamma_a}{p^2} \qquad (6)$$

The trace operator Tr acts on all kinds of variables. It can be easily checked that the trace (Tr) of an arbitrary odd product of P_{ab} is zero so that only even products of them contribute to V_{eff}. As a result of this we obtain:

$$V_{eff} = e\{\lim_{v\to\infty}[\frac{i}{2}\frac{1}{v}\sum_{n=1}^{\infty}\frac{[\sqrt{11}\kappa]^{2n}}{2n}\,Tr[P_{ab}\sigma]^{2n}] + \frac{1}{2a^2} - \sigma^2\} \qquad (7)$$

Summing up the infinite series and operating with the trace leads us (in euclidean momentum representation) to the following effective potential:

$$V_{eff} = \frac{4e}{(2\pi)^4}\int d^4p\; \ell n(1+\frac{11\kappa^2\sigma^2}{p^2}) + \frac{1}{2a^2} - \sigma^2 \qquad (8)$$

As it can be argued, using perturbative power counting argument, the N=1, d=4 supergravity model is unrenormalizable. Then we will introduce an ultraviolet cut-off Λ, as an independent parameter, which will cut off the divergent integrals at scale Λ. One assumes that these divergences at Λ are cut off by a complete theory above Λ. As it has been pointed out in reference (Hall 1984) this is a possible way of making progress since for an unrenormalizable theory one does not know how to make it meaninful at the loop level. So the effective potential (8) becomes regularized and is represented as

$$V_{eff} = e\{\frac{1}{4\pi^3}\{\frac{121\kappa^4\sigma^4}{2}[\ell n\, 11\kappa^2\sigma^2/\Lambda^2 - \frac{1}{2}] + 11\kappa^2\sigma^2\Lambda^2\} + \frac{1}{2a^2} - \sigma^2\} \qquad (9)$$

One can check that with the values

$$\Lambda = \kappa^{-1} \qquad (10a)$$

and

$$a = \frac{\kappa^2}{\sqrt{2x0.310}} \qquad (10b)$$

the effective potential is non-negative and has a minimum at $\sigma=\sigma_0\neq 0$ which occurs at zero potential. In this way we have imposed the absence of the cosmological constant.

Rescaling $\sigma^2 \to \kappa^{-4}\sigma^2$ in (9), with (10a) and (10b) gives us that

$$V_{eff} = e\kappa^{-4}\{\frac{1}{4\pi^3}\{\frac{121\sigma^4}{2}[\ell n\; 11\sigma - \frac{1}{2}] + 11\sigma^2\} - \sigma^2 + 0.310\} \qquad (11)$$

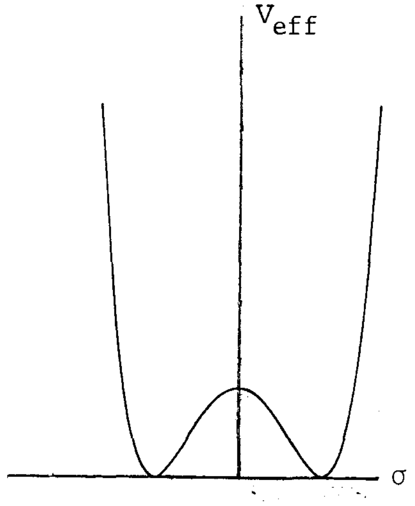

Fig. 1 - Effective potential specified in equation (11).

As it can be depicted from the figure corresponding to the effective potential (11) supersymmetry is broken and the gravitino becomes massive after the shifting $\sigma'(x)=\sigma(x)-\sigma_0$. Simple numerical procedures show us that the minimum of this potential is described by the coordinates $(\sigma, V_{eff})_{min} = (\pm 0.726,\ 0)$.

The gravitino mass term takes the following form:

$$-M_{3/2}\bar{\psi}_\mu \Gamma^{\mu\nu} \psi_\nu \qquad (12)$$

or using gauge (2)

$$\frac{M_{3/2}}{2} (\bar{\psi}_\mu \psi^\mu) \ , \qquad (13)$$

so that in our particular case it is represented by

$$M_{3/2} = \sqrt{11}\ \kappa^{-1} \sigma_{min} \qquad (14)$$

or

$$M_{3/2} = 2.408\ \kappa^{-1} \qquad (15)$$

This leads us to conclude that this gravitino mass is proportional to the Planck mass M_p $(M_p=\sqrt{8\pi}/\kappa)$, that is $M_{3/2}=0.480\ M_p$, which serves as a supersymmetry breaking parameter. Although this results depend on the assumptions (10a) and (10b) they acquire importance due to the fact that we succeeded in showing that N=1, d=4 supergravity can have a supersymmetry breaking process without using any additional matter fields (Ferrara,...1979). In fact we have only to couple it to the Volkov-Akulov field (which is thereafter eliminated in favor of a massive-like gravitino via the Super-Higgs effect) and quantize the gravitino, at the one-loop level.

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