ISSN 0029 - 3865

CBPF-NF-060/83

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ABSTRACT. Critical temperatures for SU(N)-singlet bags are determined and a deconfinement phase transition is exhibited.

Abstract

Critical temperatures for SU(N)-singlet bags are determined and a deconfinement phase transition is exhibited.

Confinement of quarks and gluons is one of the most important problems yet unsolved in particle physics. The doubt persists whether that property is already contained in QCD, as it is formulated today (including its peculiar vacuum state), or it would be necessary to still modify the formulation, in order that a complete theoretical proof of confinement could be given. Nevertheless, it is largely accepted that an absolute confinement is excluded [1] and that at very high temperatures quarks and gluons behave as free particles. This is in fact a result from the renormalization group equations which show that QCD is asymptotically free at very high temperatures and densities [2,3]. The problem is how to describe quantitatively a phase transition from the (perturbative) quark-gluon plasma phase to a (non-perturbative) hadronic phase. Some results have been obtained, such as an equation of state for the quark-gluon plasma [4,5], and the establishment that at least two phase transitions may occur when the temperature is lowered, associated to confinement and chiral symmetry breaking [6,7]. The confinement phase transition is believed to be mainly connected with gluons, but as already known, non-perturbative effects, as instantons, caused by the gluon self-interaction, make its description very complicated. Only in a few cases, non-perturbative effects are inessential and the results obtained agree relatively well with the expectations [8].

Another aproach to the problem is the phenomenological one, in particular that making use of bag models [9]. In this model, all the complicated effects of gluon interactions inside the hadron are globally simulated by a "pression" B exerted on the bag (the "hadron") by the external vacuum. In its simplest version, which is adopted here, B is a constant. In this case a hadron may be viewed as a representative of an infinite set of bags submitted to a constant external pressure B, that is to say, in the language of Statistical Mechanics, it will be a member of an isobaric ensemble.

In a recent paper [10], it has been obtained the partition function for an ideal gas of Boltzmann or Bose-Einstein massless particles, with the requirement that only singlet states with respect to a given internal symmetry group (SU(2) and SU (3)) be allowed. In an earlier paper [11], Gorenstein examined the deconfinement phase transition in a bag of ideal partons obeying Boltzmann statistics using the isobaric ensemble formalism. In this note we extend the results of ref. [11] to the singlet ("colourless") bag of ref. [10], investigating the possibility of deconfinement phase transitions, in both cases of Boltzmann and Bose-Einstein statistics, and of SU(2) and SU (3) internal symmetry ("colour").

Let us start by examining the critical temperature for a SU(2)-singlet Boltzmann bag, with Hamiltonian $H=\sum_{i=1}^{N}|\vec{p}_i| + BV$, where N is the number of particles inside the bag of volume V, and B is the constant vacuum external pressure. In this case, the partition function for the isobaric ensemble is [10,11]

$$Z(\beta,B) = \frac{1}{V_0}\int_0^\infty dV \exp\left[-(\beta B - \frac{d}{\pi^2\beta^3})V\right]\left\{I_0(\frac{2dV}{\pi^2\beta^3}) - I_1(\frac{2dV}{\pi^2\beta^3})\right\} \quad (1)$$

where V_0 is an arbitrary volume introduced to make Z dimension less, d is the number of internal degrees of freedom of the constituent particles, and β is, as usual, the inverse temperature.

The integral

$$\int_0^\infty dx \exp(-\alpha x)\, I_\nu(\gamma x)$$

converges for Re $\nu > -1$ and Re $\alpha > |\text{Re}\,\gamma|$ [12], and this implies immediately that (1) will be defined for temperatures T satisfying

$$T < \left(\frac{\pi^2 B}{3d}\right)^{1/4} \equiv T_c \ , \tag{2}$$

that is to say, the partition function will exist only for temperatures below the critical temperature T_c given by (2). At this value of T the partition function is singular and we expect the occurrence of a phase transition at this point.

If the particles in the bag obey Bose-Einstein statistics, we can obtain, from formula (21) of ref. [10],

$$Z_{BE}(\beta,B) = \frac{1}{V_o}\int_0^{\frac{1}{2}} dx(1 - \cos 2\pi x)\int_0^\infty dV \exp[-g(x)V] \ , \tag{3}$$

where

$$g(x) = \beta B - \frac{3d\zeta(4)}{\pi^2\beta^3} + \frac{2d\pi^2}{3\beta^3}\, x^2(1-x)^2 \tag{4}$$

and $\zeta(4) = \frac{\pi^4}{90}$ is the Riemann zeta function. As $x^2(1-x)^2$ has only one minimum in the interval $[0,1/2]$ at the point $x=0$, a sufficient condition for the existence of the integral (3) is $g(x) > 0$, $\forall\, x \in [0,1/2]$, what implies that Z_{BE} will be defined for temperatures below the critical temperature

$$T_c^{BE} = \left[\frac{\pi^2 B}{3d\zeta(4)} \right]^{1/4} . \tag{5}$$

As we change from SU(2) to SU(3) things are more involved and an analytical treatment becomes rather complicated. To proceed for a SU(3) singlet bag of Boltzmann "gluons" we analyse the partition function [10]

$$Z'(\beta,B) = \frac{1}{V_0} \int d\mu(G) \int_0^\infty dV \exp\left[-(\beta B - \chi_c \frac{d}{\pi^2 \beta^3})V\right] \tag{6}$$

where $d\mu(G)$ is the SU(3) invariant measure and χ_c is the character of the adjoint ("gluon") representation. The convergence condition of integral (6) gives the critical temperature

$$T'_c = \operatorname*{Inf}_{\chi_c} \left(\frac{\pi^2 B}{\chi_c d}\right)^{\frac{1}{4}} . \tag{7}$$

The determination of T'_c requires the minimization of $1/\chi_c$ where,

$$\chi_c = \sum_{\nu,\nu'=0}^{3} a_\nu a_{\nu'} \cos(\delta_\nu - \delta_{\nu'}) - 1, \tag{8}$$

the a's and δ's being obtained from the eight SU(3) parameters.

Using standard minimization computer techniques, we obtained $\underset{\chi_c}{\mathrm{Min}} \left\{ \frac{1}{\chi_c} \right\} = \frac{1}{8}$, which gives

$$T'_c = \left(\frac{\pi^2 B}{8d} \right)^{\frac{1}{4}} \quad . \tag{9}$$

If we look at eqs. (2) and (9) we note that the critical temperatures for bags with SU(2) and SU(3) internal symmetry contain respectively the factors 3 and 8 multiplying the number of internal degrees of freedom of the particles. These factors are just the number of generators of those groups. With respect to the critical temperature corresponding to the simplest version of this model [11], $T_c^{(0)} = \left(\frac{\pi^2 B}{d} \right)^{1/4}$, we see that the introduction of an internal symmetry group SU(N), together with the requirement that the bag be a singlet state relative to this group, gives critical temperatures lower than $T_c^{(0)}$ by a factor $(N^2 - 1)^{-1/4}$.

To go beyond the critical temperature one must regularize the integrals by introducing an upper volume cutoff V*. In ref. [11] this method, was applied in the case of a simple bag of Boltzmann particles, to show that the critical temperature corresponds to the deconfinement phase transition. We will show here, as an example, that the same follows for a SU(2)-singlet bag of Bose-Einstein particles. In this case, the partition function becomes a function of the cutoff V*, it is regular anywhere, and can be expressed as

$$Q_{BE}(\beta, B, V^*) = \frac{1}{V_0} \int_0^{\frac{1}{2}} dx \, [g(x)]^{-1} (1 - \cos 2\pi x) \left\{ 1 - \exp[-g(x) V^*] \right\} \quad . \tag{10}$$

We investigate the behaviour of the average bag volume

$$\bar{V} = -(1/\beta)\frac{\partial}{\partial B}[\ell n Q_{BE}(\beta, B, V^*)] \tag{11}$$

for $T > T_c^{BE}$ and $T < T_c^{BE}$ when we take the thermodynamic limit $V^* \to \infty$.

Computer analysis gives the results displayed in fig. 1, where we have used $d = 2$ and $B^{1/4} = 190$ MeV [13]. These values lead to a phase transition at $T_c^{BE} = 211$ MeV. We have analysed the ratio $\bar{V}/V^*$ as a function of T for several values of V^*: for temperatures larger than T_c^{BE} our results indicate that deconfinement exists for this example since the average bag volume $\bar{V}$ grows with the space volume V^*. On the other hand, for temperatures below T_c^{BE} the ratio $\bar{V}/V^*$ goes to zero as V^* grows indicating that confinement is preserved.

We feel that a similar dependence of $\bar{V}$ with V^* exists for a SU(3) singlet bag of Bose particles. We conjecture that it is possible to treat SU(N) singlet bags in the framework of the isobaric ensemble formalism and that in view of our results it should be possible to demonstrate deconfinement and to obtain the critical parameters as a function of the external vacuum pressure.

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Figure Caption:

Fig. 1 - A plot of $\overline{V}/V^*$, for an SU(2) singlet bag of bosons, as a function of temperature, for two values of the control volume V^*, exhibiting the deconfinement phase transition.

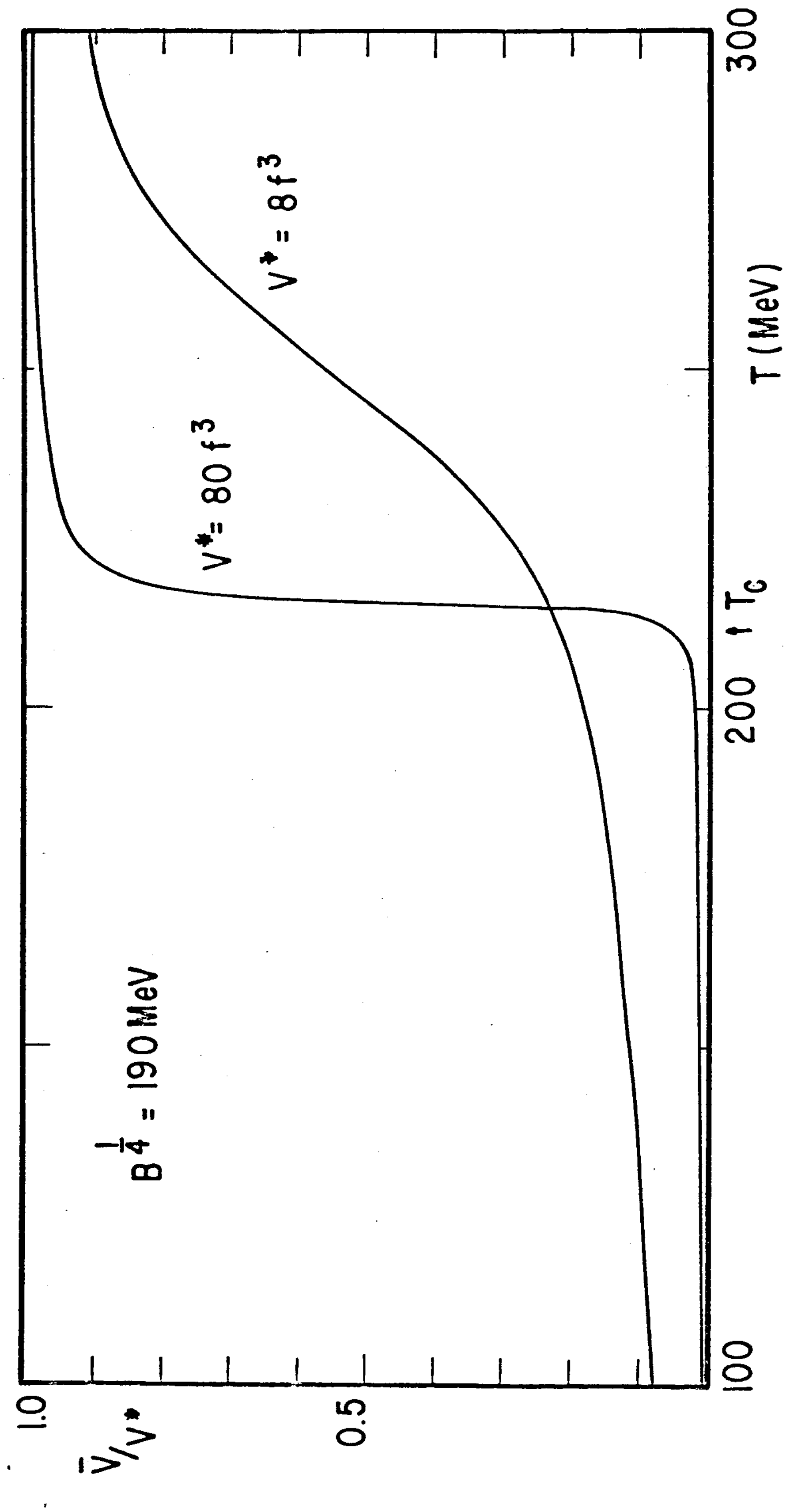

Fig. 1