CBPF-NF-059/94

Unitarity Screening Corrections in High Energy Hadron Reactions

Uri Maor

LAFEX, Centro Brasileiro de Pesquisas Físicas (CNPq)
Rua Dr. Xavier Sigaud 150, 22290 - 180, Rio de Janiero, RJ, BRASIL

and

School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv 69978, ISRAEL *

Invited talk presented at the XV Encontro Nacional de Fisica de Particulas e Campos
Angra dos Reis, RJ, October 5-8,1994

Abstract

The role of s-channel unitarity screening corrections, calculated in the eikonal approximation, is investigated for elastic and diffractive hadron-hadron and photon-hadron scattering in the high energy limit. We examine the differences between our results and those obtained from the supercritical Pomeron-Reggeon model with no such corrections. It is argued that the saturation of cross sections is attained at different scales for different channels. In particular, we point out that wheras the saturation scale for elastic scattering is apperently above the Tevatron energy range, the appropriate diffraction scale is considerably lower and can be assessed with presently available data. A review of the relevant data and its implications is presented.

*Permanent address

Over the past few years phenomenological investigations of Pomeron exchange reactions have been confined mostly to the study of forward elastic scattering and total cross sections. In particular, Donnachie and Landshoff (DL) have promoted [1] an appealing and very simple Regge picture in which

$$\sigma_{tot} = X(\frac{s}{s_0})^{\Delta} + Y(\frac{s}{s_0})^{-\eta} \tag{1}$$

with universal $\Delta = 0.0808$ and $\eta = 0.4525$. This corresponds to a supercritical Pomeron trajectory $\alpha_P(t) = 1+\Delta+\alpha'_P t$ and a Regge trajectory $\alpha_R(t) \simeq 0.5+t$. Indeed,the analysis of Block, Kang and White [2], who examin the nuclear slope of high energy pp and $\bar{p}p$ elastic scattering, yields an excelent reproduction of the data with $B = b_0 + 2\alpha' ln(\frac{s}{s_0})$ where $\alpha'_P = 0.25\ GeV^{-2}$. Eventough DL offer a global fit to all available hadron-hadron and photon-hadron total csoss sections, it should be noted that in reality only $\bar{p}p$ and γp reactions have attained high enough energies in, which the Pomeron parameters can be unambigiously tested, provided experimental errors are small enough. Clearly, such a simple model is bound, eventually, to violate s-channel unitarity. Nevertheless, as we shall see, DL have introduced a P-P cut correction which extends appreciably the domain of applicability of their model.

Elastic scattering and diffraction dissociation are similar processes which have predominantly forward imaginary amplitudes corresponding to the exchange of vacum quantum numbers in the t-channel. As such, both are dominated in the high energy domain by Pomeron exchange and are expected, in a simple Regge model, to exhibit rather similar dependences on the kinematic variables. In this review I wish to address some of the apperent differences between these processes and their interpretation. In particular, I wish to examine the role of screening corrections in the calculation of these reactions and show that they saturate the above cross sections at very different scales. I shall add a few comments on the new data coming from HERA and its relrvance to this discussion. Both H1 and ZEUS report [3,4)] that in the HERA energy domain $(\sigma_{el} + \sigma_{diff})$ is about 35-40% of the total cross section. This is similar to the ratio observed in $\bar{p}p$ scattering in the ISR-Tevatron energy range, where the need for some unitary corrections is apperent [1,5].

In order to catalog the differences between an uncorrected supercritical Pomeron model and a similar model which includes screening corrections, let us specify their features side by side. I wish to present a calculation which is reasonably realistic but also as simple as possible. To this end I present the Pomeron amplitude in an exponential form and calculate the screening corrections utilizing the eikonal approximation. This approximation accounts for elastic rescatterings which are the leading contribution to the screening process.

Our amplitude is normalised so that

$$\frac{d\sigma}{dt} = \pi|f(s,t)|^2 \tag{2}$$

$$\sigma_{tot} = 4\pi Imf(s,0) \tag{3}$$

The scattering amplitude in b-space is defined as

$$a(s,b) = \frac{1}{2\pi}\int d\mathbf{q} \;\; e^{-i\mathbf{q}\cdot\mathbf{b}} f(s,t) \tag{4}$$

where $t = -q^2$.
In this representation

$$\sigma_{tot} = 2\int d\mathbf{b} \;\; Ima(s,b) \tag{5}$$

$$\sigma_{el} = \int d\mathbf{b} \;\; |a(s,b)|^2 \tag{6}$$

The introduction of screening rescattering corrections is greatly simplified in the eikonal approximation where at high energy $a(s,b)$ is assumed to be pure imaginary, and can be written in the simple form

$$a(s,b) = i(1 - e^{-\Omega(s,b)}) \tag{7}$$

where the opacity $\Omega(s,b)$ is a real function. As we shall utilize Regge parametrizations, analyticity and crossing symmetry are easily restored by substituting $s^{\alpha} \rightarrow s^{\alpha}e^{-i\pi\alpha/2}$, where α denotes the exchanged Regge trajectory.

In previous publications [6,7] we have shown that the eikonal approximation can be summed analytically for a Gaussian input

$$\Omega(s,b) = \nu(s)e^{-\frac{b^2}{R^2(s)}} \tag{8}$$

which corresponds to an exponential representation in t space. This is a good approximation [5,6] for Regge type amplitudes, where

$$Imf(s,t) = Ce^{R_0^2 t}(\frac{s}{s_0})^{\alpha(t)-1} sin[\frac{\pi\alpha(t)}{2}] \simeq C(\frac{s}{s_0})^{\Delta} e^{\frac{1}{4}R^2(s)t} \tag{9}$$

With this amplitude we obtain

$$\sigma_{tot} = 4\pi C(\frac{s}{s_0})^{\Delta} \tag{10}$$

$$\sigma_{el} = \frac{\pi C^2(\frac{s}{s_0})^{2\Delta}}{B_{el}} \tag{11}$$

$$\sigma_{in} = \sigma_{tot} - \sigma_{el} \tag{12}$$

On transforming to b-space we get

$$\nu(s) = \frac{\sigma_0}{2\pi R^2(s)}(\frac{s}{s_0})^{\Delta} \simeq \frac{\sigma_{tot}}{4\pi B_{el}} \tag{13}$$

$$R^2(s) = 4[R_0^2 + \alpha' ln(\frac{s}{s_0})] \tag{14}$$

where $\sigma_0 = \sigma(s_0)$ and $B_{el} = \frac{1}{2}R^2(s)$. With this input, we obtain in the eikonal approximation

$$\sigma_{tot} = 2\pi R^2(s)[ln\nu(s) + C - Ei(-\nu(s))] \tag{15}$$

$$\sigma_{in} = \pi R^2(s)[ln2\nu(s) + C - Ei(-2\nu(s))] \tag{16}$$

$$\sigma_{el} = \sigma_{tot} - \sigma_{in} \tag{17}$$

where $Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$, and C = 0.5773 is the Euler constant.

The above parametrization also allows one to obtain a closed expression for single diffraction dissociation , where in the triple Regge limit with no screening corrections we have

$$\frac{M^2 d\sigma_{sd}}{dM^2 dt} = \sigma_0^2(\frac{s}{M^2})^{2\Delta+2\alpha' t}[G_{PPP}(\frac{M^2}{s_0})^{\Delta} + G_{PPR}(\frac{M^2}{s_0})^{-\frac{1}{2}}] \tag{18}$$

With the introduction of screening corrections [7] we obtain

$$\frac{M^2 d\sigma_{sd}}{dM^2} = \frac{\sigma_0^2}{2\pi \bar{R}_1^2(\frac{s}{M^2})}(\frac{s}{M^2})^{2\Delta} \cdot [G_{PPP}(\frac{M^2}{s_0})^{\Delta} a_1 \frac{1}{(2\nu(s))^{a_1}}\gamma(a_1, 2\nu(s))$$

$$+ G_{PPR}(\frac{M^2}{s_0})^{-\frac{1}{2}} a_2 \frac{1}{(2\nu(s))^{a_2}}\gamma(a_2, 2\nu(s))] \tag{19}$$

where G_{PPP} and G_{PPR} are the triple Regge couplings corresponding to single diffraction dissociation,

$$\bar{R}_i^2(\frac{s}{M^2}) = 2R_{0i}^2 + r_{0i}^2 + 4\alpha' ln(\frac{s}{M^2}) \tag{20}$$

$r_{0i} \leq 1 GeV^{-2}$ denotes the radius of the triple vertex and can safely be neglected.

$$a_i = \frac{2R^2(s)}{\bar{R}_1^2(\frac{s}{M^2}) + 2\bar{R}_i^2(\frac{M^2}{s_0})} \tag{21}$$

The indices i =1,2 corresponds to P (Pomeron) and R (Reggeon) exchanges. $\gamma(a,2\nu)$ denotes the incomplete Euler gamma function $\gamma(a,2\nu) = \int_0^{2\nu} z^{a-1} e^{-z} dz$. In the high energy limit the above expression simplifies to

$$\frac{M^2 d\sigma_{sd}}{dM^2} = \pi R^2(s)[G_{PPP}(\frac{M^2}{s_0})^{\Delta} + G_{PPR}(\frac{M^2}{s_0})^{-\frac{1}{2}}] \tag{22}$$

Table I. Asymptotic predictions of the Supercritical Pomeron and Eikonal models

	Supercritical Pomeron	Eikonal model
σ_{tot}	s^{Δ}	$ln^2(\frac{s}{s_0})$
σ_{el}	$\frac{s^{2\Delta}}{ln(\frac{s}{s_0})}$	$\ln^2(\frac{s}{s_0})$
σ_{sd}	$\frac{s^{2\Delta}}{ln(\frac{s}{<M^2>})}$	$ln(\frac{s}{s_0})$
$\frac{\sigma_{el}}{\sigma_{tot}}$	$\frac{s^{\Delta}}{ln(\frac{s}{s_0})}$	$\frac{1}{2}$
$\frac{\sigma_{sd}}{\sigma_{tot}}$	$\frac{s^{\Delta}}{ln(\frac{s}{<M^2>})}$	$\frac{1}{ln(\frac{s}{s_0})}$

The remarkable differences between the non eikonalized and eikonalized versions of the supercritical Pomeron model are best illustrated in the asymptotic region and are summarized in Table I. As can be readily seen, the most dramatic change takes place for single diffraction where a $s^{2\Delta}$ divided by a ln(s) term is replaced by $R^2(s)$ which behaves as ln(s). Problem is that the information summarized in Table I is of no practical use as long as we do not specify the appropriate energy scales at which the screening corrections become appreciable. To this end we need to asign some numerical values to the free parameters of our equations. It is an easy excersise to see that if we insert the DL parameters into Eqs. (9) and (13) our amplitude will violate s-channel unitarity for small b at c.m. energies which are as low as 2.5 TeV, just above the Tevatron range. Indeed, the CDF group [8] finds that a(s,b=0) = 0.96 at 1.8 TeV. This problem is avoided in the DL model by their introduction of a weak P·P cut, its strength is fixed so as to reproduce the high t dip observed in elasic differential cross sections. This correction extends considerably the energy range at which the DL model can be applied provided we consider Δ as an effective parameter which should go down eventually. The net practical result is that by confining our study only to total cross sections in the available energy range, we are in no position to assess the importance of the screening corrections and the scales at which they become appreciable. To attain this wider scope of investigation let us turn to a discussion of some elastic and diffractive data and its interpratation.

Much of the information we are trying to get can be deduced from a close examination of available data on $\frac{\sigma_{el}}{\sigma_{tot}}$ and $\frac{\sigma_{sd}}{\sigma_{tot}}$. Without screening corrections we expect both ratioes to grow indefenitely like s^{Δ} devided by the appropriate slopes which behave as ln(s). This is, obviously, not a physical picture. In a rather general formulation, Pumplin [9] has suggested that $\frac{\sigma_{el}+\sigma_{diff}}{\sigma_{tot}}$ is bounded by $\frac{1}{2}$. Indeed, once we eikonalize we get $\frac{\sigma_{el}}{\sigma_{tot}} \rightarrow \frac{1}{2}$ wheras $\frac{\sigma_{sd}}{\sigma_{tot}} \rightarrow 0$. The emerging picture is that we expact $\frac{\sigma_{el}}{\sigma_{tot}}$ to start its rise behaving approximately as s^{Δ} and at the energy scale where screening becomes important to temper its rise approaching $\frac{1}{2}$ from below at exeedingly high energies. $\frac{\sigma_{sd}}{\sigma_{tot}}$, on the other hand, is supposed to reach a maximum at its screening scale after which it will start going down becoming diminishingly small in the high energy limit. The relevant data is shown in Fig. 1 with the highest data points at c.m. energy of 1.8 TeV [8,10]. The remarkable conclusion we make, after inspecting these data, is that wheras we have not reached, as yet, the screening scale corresponding to elasic $\bar{p}p$ scattering, the single diffraction scale is considerably smaller at c.m. energies of about 50-100 GeV.

An additional source of interesting information is available by inspecting the ratio between the forward single diffraction cross section at fixed M^2 and the corresponding forward elastic cross section $\frac{(\frac{d\sigma_{sd}}{dM^2dt})_{t=0}}{(\frac{d\sigma_{el}}{dt})_{t=0}}$. By virtue of the factorization theorem we expact in a simple Regge model that this ratio, being just the ratio of the relevant couplings to aproach rather quickly a constant value. This constant behaviour should change into a moderate increase if we chose to integrate over the diffractive mass. The introduction of screening corrections diminishes the forward single diffraction cross section and as a result we expact once again that the above ratio should reach a maximum at about the diffraction screening scale after which it should go down rather rapidely. The data which is shown in Fig. 2 supports this point of view. Moreover, the ratioes presented in Fig. 1 and Fig. 2 peak at the same energy of 50-100 GeV as expacted.

The CDF experiment [8], which was carried out at c.m. energies of 546 and 1800 GeV, conviniently enables us to examine the ratioes of the total, elastic and single diffraction cross sections. These are presented in Table II together with the theoretical expectations. By using the data of a single experiment we avoid the need to compare the relative normalisations and other algorithms used in the analysis of different experiments. The data, as compared with a simple supercritical Pomeron model, showes again the inability of the simple model to reproduce the energy dependence of the diffractive cross sections. On the other hand, once screening corrections are included (through the eikonal sum) a very reasonable reproduction of the experimental ratioes is attained. Some caution should be applied when regarding these data points. There is a clear discrapency between the 1800 GeV total cross section reported by the CDF [8] and the one reported by E710 [10] which is considerably lower. We have tried to overcome this difficulty by examining the ratioes rather than the magnitudes of the cross sections of interest. Nevertheless, we see that a $\Delta = 0.11$ value is preffered on the DL value of $\Delta = 0.0808$. Regardless of this difficulty, I consider the diffractive cross section ratioes and magnitudes to be a very serious problem for supercritical Pomeron models without a severe unitary correction. DL do not discuss diffraction dissociation in their publications. However, considering the fact that their unitarity correction is very weak and thus their Δ is universal and fixed, I do not beleive that in a model like theirs it is possible to ammend the problems we have just encountered.

Table II. CDF cross section ratioes $[\frac{\sigma(1800)}{\sigma(546)}]$ against the theoretical predictions

	CDF	Supercritical Pomeron $\Delta = 0.0808$	Supercritical Pomeron $\Delta = 0.11$	Eikonal model $\Delta = 0.0808$	Eikonal model $\Delta = 0.11$
σ_{tot}	1.31 ± 0.04	1.21	1.30	1.18	1.26
σ_{el}	1.53 ± 0.075	1.36	1.57	1.30	1.48
σ_{sd}	1.20 ± 0.075	1.43	1.65	1.21	1.33

A very exiting new window of information, through which we can re-assess our ideas on Pomeron physics, has been opened recently when the second generation of HERA results became available. Actually, we have two sources of new knowledge. To begin with, and as we have already noted, real photoproduction is the only channel, in addition to $\bar{p}p$, where our c.m. energies exceed 100 GeV. On a more profound level, a carefull study of DIS and its Q^2 dependence reveals new information on the Pomeron and in particular the dependence of $\alpha(0)$ on Q^2.

Clearly, a very important element in the DL model is the universality of Δ. As is evident from Ref. [1], this is perfectly compatible with the available data on πp and Kp total cross sections. Problem is that this data is confined to relatively low energies where the Pomeron parameters can be checked for consistency but not be determined independently. The analysis of the global data on $\sigma_{tot}(\gamma p)$, including the HERA points [3,4], are presently consistent with a wide range of choices. In particular, we specify the DL value of $\Delta = 0.0808$, as well as the excelent ALLM [11] reconstruction of the potoproduction and DIS data which has $\Delta = 0.045$. Obviously, this is just a reflection of the fact that at present the systematic error on $\sigma_{tot}(\gamma p)$ in the HERA measurements is too big. This experimental deficiency should be ammended within the next year or so and finally enable us yet another independent look at the behaviour of total cross sections.

Our discussion, thus far, was confined to soft processes, within the domains of conventional Regge physics. In perturbative QCD a hard Pomeron can be percieved [12] through the summation of gluon ladder diagrams. As a result one obtains a series of poles in the complex j plane above unity. These poles sum to an effective $\Delta = \frac{12}{\pi}\alpha_s ln2$. We shall refer to this as the BFKL Pomeron. One of the more exciting observations established by the HERA groups is that the proton structure function $F_2(x, Q^2)$ behaves in the exceedingly small limit of x like x^{-J} with $J \simeq 0.30$. This is remarkably close to the BFKL Pomeron and very different from the $Q^2 = 0$ photoproduction results which are reproduced with the soft Pomeron parameters. The transition from $Q^2 = 0$ to the high Q^2 domain has been carefully studied by ALLM [11] in a dual model. Their results, which were subsequently corroborated also by other studies, indicate a fairly rapid transition from the soft to the hard domain taking place at Q^2 values of a few GeV^2. This is shown in Fig. 3. To further study the exact nature of this transition we need better data at moderate Q^2 which is not available as yet. Nevertheless, the above observation, as such, invites some speculations. We shall mention here just two possible explenations to this behaviour:

1) We have actually two Pomerons, a soft one and a hard one. Wheras the small Q^2 data is dominated by the soft Pomeron the higher Q^2 data is dominated by the BFKL Pomeron. The net effect of the two contributions is shown in Fig. 3.

2) We have only a hard BFKL Pomeron which, due to its high Δ, receives very strong unitarity corrections. These result in an effective low Δ observed in hadron-hadron and photon-hadron reactions as we have seen above. However, for highly virtual photons those corrections start to diminish at some moderate Q^2 values. Only preliminary attempts in developing such a model have been undertaken thus far [5] and the idea seems to me

intriguing enough to be persued furthermore.

Acknowledgements: I wish to thank Prof. A. Santoro and my colleagues at LAFEX (CBPF) for their kind hospitality and help, and the CNPq for financial support. Special thanks to my friends E. Gotsman and E.M. Levin for a very constractive and enjoyable collaboration.

References

[1] A. Donnachie and P.V. Landshoff, *Nucl. Phys.* **B231**, (1984) 189: *Phys. Lett.* **B296** (1992) 227.

[2] M.M. Block, K. Kang and A.R. White, Northwestern University Preprint No. 172 (1992).

[3] H1 Collaboration, *Phys. Lett.* **B299**, (1993) 376; DESY Red Report 93-077; presentations at the Eilat Conference, January 1994.

[4] ZEUS Collaboration, *Phys. Lett.* **B293**,(1992) 465; DESY Red Report 94-032; presentations at the Eilat Conference, January 1994.

[5] E. Gotsman, E.M. Levin and U. Maor, *Zeit. für Phys.* **C57** (1993) 667.

[6] E. Gotsman, E.M. Levin and U. Maor, *Phys. Lett* **B309** (1993) 109.

[7] E. Gotsman, E.M. Levin and U. Maor, *Phys. Rev.* **D49** (1994) R4321.

[8] F. Abe et. al., CDF Preprint July 1993; S. Belforte et. al., CDF Preprint July 1993.

[9] J.D. Pumplin, *Phys. Rev.* **D8** (1973) 2849.

[10] E710 collaboration,N.A. Amos et. al., *Phys. Lett.* **B243** (1990) 158; *Phys. Rev. Lett.* **68** (1992) 2443; *Phys. Lett.* **B301** (1993) 313.

[11] H. Abramowicz, E.M. Levin, A. Levy and U. Maor, *Phys. Lett.* **B269** (1991) 465.

[12] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Sov. Phys. JETP* **44** (1976) 443; ibid **45** (1977) 199; Ya.Ya. Balitskii and L.N. Lipatov, *Sov. J. Nucl. Phys.* **28** (1978) 822.

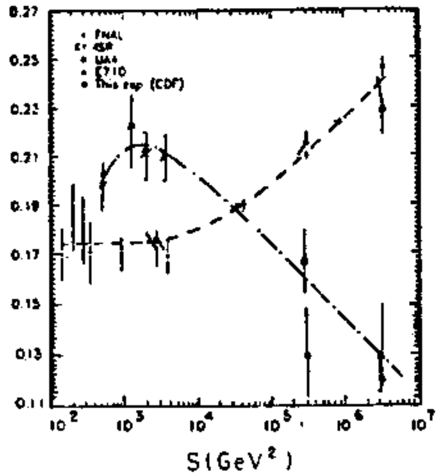

Fig. 1: s dependence of $\frac{\sigma_{el}}{\sigma_{tot}}$ (dashed line) and $\frac{\sigma_{sd}}{\sigma_{tot}}$ (dashed dotted line)

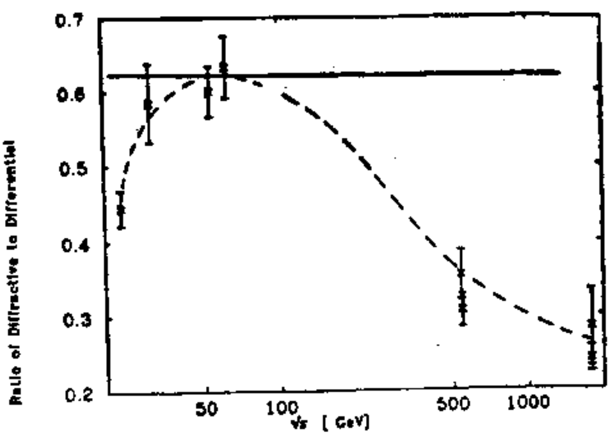

Fig. 2: $\sqrt{s}$ dependence of the ratio between $(\frac{d\sigma_{sd}}{dt})_{t=0}$ and $(\frac{d\sigma_{el}}{dt})_{t=0}$

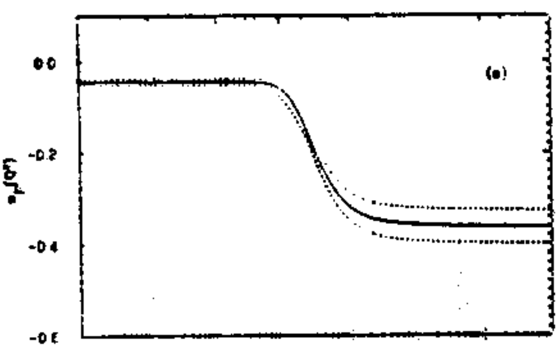

Fig. 3: Q^2 dependence of $a_P = \alpha_P(0) - 1$