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Effective Lagrangian for full one-loop QED and its influence on the primordial universe

M. Novello, J.M. Salim, V.A. De Lorenci and R. Klippert
Centro Brasileiro de Pesquisas Físicas,
Rua Dr. Xavier Sigaud, 150, Urca
22290-180 — Rio de Janeiro, RJ — Brazil
E-mail: novello@lafex.cbpf.br

Abstract

We show that the full one-loop QED, regarded as an effective classical field theory, generates a non-singular (homogeneous and isotropic) FRW universe. The main difficulties of standard cosmology are thus overcome by considering the quantum properties of matter, while the gravitational field is described by Einstein classical general relativity.

Key-words: Cosmology; Quantum electrodynamics.

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The effective Lagrangian, which describes the properties of magnetic fields [1] when quantum effects are taken into account, was numerically evaluated[1] in the entire range of magnetic fields by many authors (see the review [2] and references therein). It is a well known result [3] that the one-loop Euler-Heisenberg correction $L^{(1)}[H]$ is non-negative, monotonically increasing, and does not exhibit a minimum as a function of the magnetic field. Nevertheless, the total one-loop Lagrangian ($L = L^{(0)} + L^{(1)}$), as it represents the energy density for arbitrary values of the magnetic field, breaks down the above monotonic behavior, with remarkable consequences to cosmology.

The effective Lagrangian [2], in units of H_{cr}^2 (where $H_{cr} = m_e^2 c^3/e\hbar \approx 4.4\times 10^{13} Gauss$), is given by

$$\begin{aligned} L = -\tfrac{1}{2}H^2 + \tfrac{\alpha}{2\pi}\Big\{ \tfrac{3}{4} + [1-\ln(2\pi)]\,H - \tfrac{4}{3}H^2 - 4H^2\textstyle\int_0^1 dx\ \ln\left[\Gamma_1(1+x)\right] + \\ + \left[\tfrac{1}{2} + H + \tfrac{1}{3}H^2\right]\ln(2H) + 4H^2 \textstyle\int_1^{1+1/2H} dx\ \ln\left[\Gamma_1(1+x)\right]\Big\}. \end{aligned} \tag{1}$$

In the standard cosmological scenario the geometry of the universe is given by FRW line element[2]

$$ds^2 = c^2 dt^2 - A^2(dx^2 + dy^2 + dz^2). \tag{2}$$

The Hubble expansion parameter is $\theta = 3\dot{A}/A$, where $A = A(t)$ is the dimensionless scale-factor.

The associated energy-momentum tensor of this theory, obtained through a spatial average procedure [4], admits a simple interpretation in terms of a perfect fluid configuration with energy density ρ and pressure p. The resulting expressions are

$$\rho = -L, \tag{3}$$

$$p = -\rho - \frac{2}{3}\frac{H}{}\frac{\partial L}{\partial H}. \tag{4}$$

Energy conservation law yields that the magnetic field turns out to be a function of the scale-factor $A(t)$ as

$$H = \frac{H_o}{A^2}, \tag{5}$$

where H_o is an arbitrary constant.

Energy density ρ and pressure p can be numerically evaluated as functions of the magnetic field. The result is plotted in figures 1 and 2.

Einstein field equations for this model reduce to a single ordinary first order differential equation for the scale-factor, namely

$$\theta^2 = 3\kappa\rho, \tag{6}$$

where $\kappa = 8\pi G/c^4$ is the Einstein gravitational constant. Whence it follows that (as $H \leq H_{max}$) the scale-factor is bounded from bellow at a finite value[3], A_{min}. Therefore, this

[1] Special limiting cases of interest, in which the effective Lagrangian had been analytically evaluated, deals with low magnetic fields ($e\hbar H/m_e^2 c^3 \ll 1$) and strong magnetic fields ($e\hbar H/m_e^2 c^3 \gg 1$).

[2] We will restrict ourselves to the Euclidean section case.

[3] The actual minimum value of the scale-factor depends on the constant H_o, and is linked to spatial units.

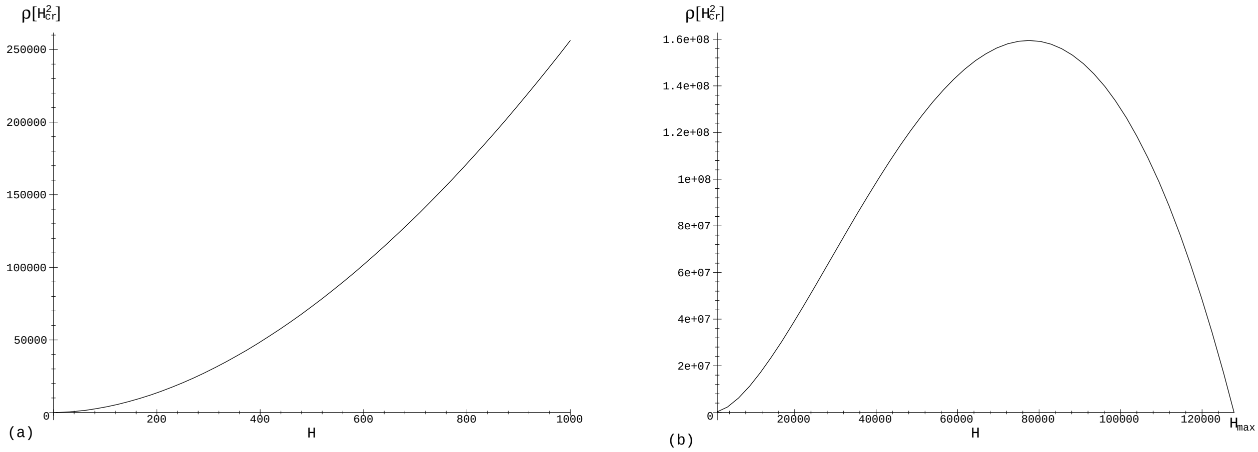

Figure 1: (a) Energy density matches the Maxwell counterpart for small values of H, and (b) presents a maximum value $\rho_{max} \approx 1.6 \times 10^8 H_{cr}^2$. The maximum value of the magnetic field is $H_{max} \approx 5.7 \times 10^{18} Gauss$, and occurs at the most condensed phase (corresponding to the minimum value of the scale-factor).

cosmological model turns out to be of a non-singular type. The applicability of standard singularity theorems are circumvented by the presence of a high (negative) pressure — figure 2.

Thus, one can conclude that the so called cosmological singularity of FRW models is a distinguished feature of *classical* electrodynamics, and does not occur at all when one-loop quantum corrections are considered.

The above model can thus overcome two of the nowadays most important difficulties of cosmology:

- the horizon problem – being a non-singular universe there is no absolute physical horizon, thus allowing cosmic microwave background radiation to be globally at thermal equilibrium;

- the singularity problem – a non-singular universe provides a consistent framework of classical gravitation, and no breakdown of physical laws are required.

It is worth to remark that this simple toy model is in complete agreement with all cosmological observations, that is to say, up to nucleosynthesis. This led us to extrapolate such model until the point of the minimum value of the scale-factor in order to exhibit its non singular behavior. To go beyond this point would provoke a series of new questions, the analysis of which is out of the scope of this letter.

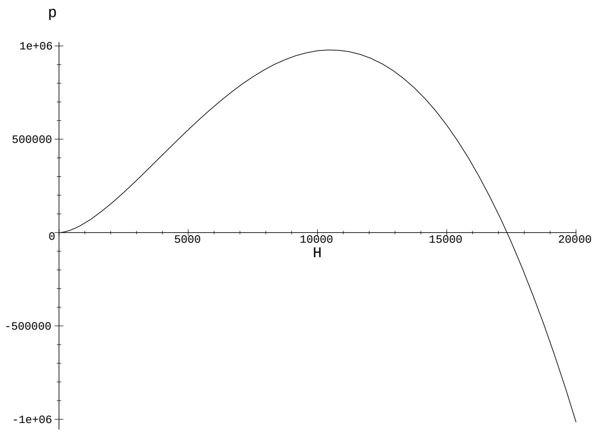

Figure 2: Pressure becomes highly (but finitely, due to the existence of an upper bound for the magnetic field) negative near the maximum condensation point A_{min}.

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References

[1] We assume that the material content of the early universe (viz. at the nucleosynthesis era) is roughly dominated by an electromagnetic field with a highly ionized ponderable mass (modeled as a plasma). Hence it immediately follows the absence of a primordial electric field. Moreover, first order calculations show that the plasma itself does not modify the qualitative behavior of the corresponding solution.

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