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IS THE NUMBER OF PHOTONS CONSERVED IN AN EXPANDING UNIVERSE?

by

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ABSTRACT

We describe a Friedman- like scenario in which - due to gravitational interaction - the total number of photons existing in the Universe changes as the Universe expands. The photon number distribution function, in thermal equilibrium, exhibits an explicit dependence on a non-vanishing chemical potential term. This non-null value for the chemical potential of the photon is due to the presence of tidal effects, arising from the direct coupling of the photons - which, in a gravitational field, should not be treated as idealized point-like objects - to the curvature of space-time. As a result, the ratio n_γ/n_B between the numbers of photons and baryons in the Universe is shown to be not a constant, but rather a function of cosmic time, vanishing when the singularity of the standard FRW model is reached. In consequence, some of the supposed "fundamental problems" of standard Cosmology (e.g., the explanation of the constancy of the ratio n_γ/n_B, and thus of the origin of the total ammount of entropy observed today) are but apparent ones in the present scenario, and may be <u>ipso facto</u> solved.

Key-words: Photon creation; Cosmology.

1 - INTRODUCTION

The fundamental role assigned to cosmic photons in current attempts to elaborate global space-time scenarios may be well appreciated if one acknowledges that the developments of Modern Cosmology rely, basically, upon two types of empirical foundation:

(i) the occurence of redshifts, seemingly of cosmological origin, in the spectra emmitted by distant objects (Hubble expansion);

(ii) the detection of the homogeneous Microwave Background Radiation (MBR) of $2.7^{o}K$.

These two facts are to be accounted for the widespread acceptance enjoyed along the last decades by the standard Hot Big-Bang model (HBB) which, besides incorporating these features, has also succeeded in providing a quantitative explanation for the relative abundances of the elements in the Universe [1].

According to the HBB model, after its emergence out of an initial singular state - separate a finite time interval H_o^{-1} from us, where H_o is the Hubble parameter - the Universe goes into an expanding phase with geometrical features, described through Einstein's equations of gravitation, corresponding to the spatially homogeneous and isotropic Friedman-Robertson-Walker (FRW) model, given by the line element $ds^2 = dt^2 - A^2(t)d\sigma^2$.

Therefore, the cosmological redshift mentioned in (i) would result from the coupling between the eletromagnetic and gravitational fields: photons (described by the propagation 4-vector K_μ) move along paths identified to the null geodesics of the FRW geometry, and consequently the frequency $\omega = k_\mu V^\mu$ of the radiation, seen by an observer V^μ co-moving with the hipersurface of homogeneity, varies in inverse proportion to the Universe radius A(t). Thus, the origin of the observed redshift is attributed to the expansion of the Universe.

On the other hand the existence of the MBR quoted in (ii), presently characterized by the spectrum of a black-body in thermal equilibrium at temperature $T = 2.7^oK$, is understood as the remnant of a hot, dense primordial phase of the cosmic evolution. Indeed, according to the standard model, in the initial stages of the cosmic expansion electromagnetic radiation becomes the main responsible for the curvature of space-time; furthermore, its black-body character would be preserved along the entire history of the Universe. This would happen since the photon energy density is proportional to the fourth power of the temperature, $\rho_\gamma \sim T^4$, and at early times should prevail over the contribution of the matter content, for $\rho_M \sim T^3$. Moreover, since the law of energy conservation gives that $\rho_\gamma \sim A^{-4}$, it follows that the equilibrium temperature T of the radiation varies in inverse proportion to the Universe radius A(t), in the same way as the frequency ω. This in turn implies that the thermal spectrum of the radiation, described by the black-body distribution $dN_\omega = [\exp \frac{\omega}{T} -1]^{-1}$, would be preserved throughout the expanding era (at least as long as these photons interact through gravitation only), once $\frac{\omega}{T}$ = constant.

However, it is hard to understand how the concept of a thermal equilibrium might be valid in the near vicinity of an eventual initial singularity [2]. In the usual scheme the idea of an equilibrium configuration (and so the notion of an equilibrium temperature) is simply assumed to be appropriate, with no regard to the possibility of an ulterior confirmation.

This assumption becomes still more difficult to accept if one reminds that in the standard model such radiation gas provides the most important contribution to the curvature of space-time at primeval epochs, as we mentioned above, since a self-gravitating gas would hardly mantain a thermodynamical equilibrium configuration. The introduction of an inflationary era, sandwiched between radiation-dominated phases, represents no further improvement on this situation, of course [3].

Another far-reaching consequence of the standard approach concerns total entropy conservation along the evolution of the Universe, which stems from the assumed thermalized photon configuration and also from the fact that the total number of photons in the Universe must be a constant, if particle-antiparticle annihilation is neglected - which seems to be quite reasonable under ordinary conditions. In effect, from the conclusion that the energy density of the photons evolves according to $\rho_\gamma \sim A^{-4}$ it follows that the photon number density $n = \frac{N}{V}$ varies in time exclusively in virtue of the expansion of the Universe. Setting $\theta = \frac{\dot{V}}{V}$ one gets that $\dot{n} + n\theta = 0$, and hence that $\dot{N} = \frac{dN}{dt} = 0$. If this is indeed true, then the ratio n_γ/n_B between the number densities of photons and baryons existing in the Universe becomes an universal constant, which is estimated today to be of the order of 10^9. Thus in the standard scenario this number stands for an intrinsic characteristic imprinted on the actual Universe, requiring either an explanation on the basis of a more fundamental principle or else to be assigned to an inaccessible set of initial conditions. This question may be alternatively formulated in terms of the total entropy S of these photons, which in the same token should represent a "primordial" quantity. Many attempts of elucidating the origin of such imprint have been suggested in recent times, notably the proposal of the so-called inflationary models quoted above.

In the present paper we will examine the consequences of a new cosmological scenario in which the above difficulties may be solved in a simple and direct way. Indeed, according to this new scenario the problem of thermalizing a self-gravitating radiation field disappears - because the radiation-dominated phase of the cosmic evolution might simply have never occurred. How could one conciliate the absence of this phase with the great ammount of photons we observe today? Alternatively, where did these photons come from, and what is the origin of today's great value of their total entropy? In the standard theory these questions have simple and definitive answers: photons were created at the Big-Bang (that is, they are of primordial origin) in such a huge quantity that the present high value of the entropy per baryon is automatically explained away.

An alternative conjecture - certainly a less dramatic one, but still highly speculative - has been proposed by Sunyaev and Zel'dovich some years ago [4]. They argue that "... lower mass black hole evaporation, through the Hawking mechanism, before nucleosynthesis, could produce the MBR". This suggestion could give support to the concept of a cold beginning of the Universe; in the present context, however, we shall be concerned with a different line of investigation and proceed to the elaboration of an interaction picture in which the behaviour of the gravitational field affects the number of photons existing in the Universe at a given cosmic epoch (in section 2 we will show how such cosmological mechanism of photon creation may be implemented in the case of a conformally-flat, expanding Universe). As a consequence of the resulting non-conservation of the number of cosmic photons, the value of the ratio n_γ/n_B becomes a function of cosmic time, and so its current evaluation $n_\gamma/n_B \sim 10^9$ represents neither an actual physical constant nor a primordial characteristic of Nature. Therefore, according to the present reasoning the justification of this figure does not constitute a fundamental difficulty, since it manifests but an inconspicuous, transient feature elicited by an occasional mensuration.

In order to establish our scenario on a proper foundation and accomplish the demonstration of the preceding statements we shall proceed through the following conceptual steps: in section 2 we introduce the subject of both minimal and direct couplings between electromagnetic and gravitational fields, and compare some of their most prominent cosmological consequences; then we elaborate the proof that a conformally flat expanding Universe may in fact produce photons. In section 3 we make use of a statistical treatment (comprising the introduction of a non-null chemical potential for the photons, due to their interaction with gravity) to give a detailed description of the resulting supply of cosmic radiation (in terms of a gas of photons non-minimally coupled to space-time curvature), and we also provide an outline of the main thermodynamical properties of such gas, in comparison with the conventional black-body behaviour. In section 4, finally, we give a general overview of the most relevant aspects afforded by the present scheme and conclude with some further remarks on the consequent modifications of the standard cosmological picture.

2 - PHOTON CREATION BY GRAVITATIONAL FIELDS

It has been claimed by many scientists that the gravitational field associated to the FRW geometry cannot change the number of photons comprised in the Universe. Which are the reasonings that seem to support this assertion, and which is the degree of certainty it is to be creditted with when one attempts to describe the actual world? In order to answer these questions, let us first review some important results that will supply the basis for our analysis.

Undoubtedly, the proper domain of the photon concept is quantum theory; however, an useful classical analogue of this concept is immediately available if it is admitted that, under certain circunstances, electromagnetic disturbances may be effectivelly characterized by their high frequency limit (geometrical optics approximation). This is accomplished by means of the standard expansion of the electromagnetic potential A_μ in terms of a small dimensionless quantity ε (such that $\varepsilon^2 << \varepsilon$):

$$A_\mu = \mathrm{Re}[(a_\mu + \varepsilon b_\mu + \varepsilon^2 c_\mu + \ldots)e^{\frac{i}{\varepsilon}\phi}] , \tag{1}$$

which is a suitable representation of processes in which the phase of the electromagnetic disturbance changes much faster than the amplitude variation along a typical length of the region under consideration or, in the case of curved space-times (i.e., when gravitational interaction is taken into account) along a length provided by $L \sim [\text{curvature}]^{-2}$.

In order to follow the evolution of the above expansion of the A_μ field one needs to know its dynamics, that is, to learn how the propagation of such wave is affected by the gravitational field. According to the general program of Field Theory in Curved Spaces there are two principles that may be chosen as possible guides for this task:

(1) the Hypothesis of Minimal Coupling (HMC)

(2) the Hypothesis of Direct Coupling to the Curvature (HDCC).

The HMC approach - which has been formulated in a very precise manner by Einstein in the beginnings of the Theory of General Relativity, more than 60 years ago - had its origin in the ambition of enlarging the scope of the Equivalence Principle so that this Principle could be extrapolated to the rank of a genuine producer of physical laws. Due to the good results it has since provided, and to the persistent (and perhaps a little exaggerated) use of Occam's razor, later physicists inadvertently began to consider HMC as <u>the</u> true coupling between eletromagnetic and gravitational fields, and not as just one possible candidate for one's explanation of the observed phenomena. In fact, other schemes (such as HDCC) are equally capable of explaining the current set of observational evidence, as we shall see later on.

The standard procedure for implementing HMC may be found in any modern textbook on gravitation theory [5]. It suffices to withdraw the requirement of a Minkowskian space-time structure and to rewrite Maxwell's equations in a covariant fashion; then, from the Lagrangian given by

$$L = -\frac{1}{2}\sqrt{-g}\; F_{\mu\nu}F^{\mu\nu} = -\frac{1}{2}\sqrt{-g}\, g^{\alpha\rho}g^{\beta\sigma}F_{\alpha\beta}F_{\rho\sigma} \tag{2}$$

where $g \equiv \det[g_{\mu\nu}]$ and $F_{\mu\nu} \equiv A_{\mu,\nu} - A_{\nu,\mu}$, as usual, one obtains

$$F^{\mu\nu}{}_{;\nu} = 0, \tag{3a}$$

$$F_{\mu\nu;\alpha} + F_{\nu\alpha;\mu} + F_{\alpha\mu;\nu} = 0. \tag{3b}$$

HMC therefore supplies a simple and straightforward prescription for the generalization of the well-known results of Special Relativity in flat Minkowski space to curved space-times. Remark that, according to this recipe, functionals of the curvature do not appear explicitly in the dynamical picture of the evolution of the field $F_{\mu\nu}$.

Now in the HDCC case the dynamical equations of motion are derived from a functional expression comprising both electromagnetic and curvature contributions. When such functional is taken to be linear in the curvature and of the second order in $F_{\mu\nu}$ there are seven possible combinations to be considered [6]; however, the actual spectrum of possibilities may be strongly restricted if additional conditions (such as the conservation of certain symmetries) must be satisfied. For example, if it is demanded that gauge invariance should be preserved, then candidates $L_1 = \sqrt{-g}\; RA_\mu A^\mu$ and $L_2 = \sqrt{-g}\; R_{\mu\nu}A^\mu A^\nu$ are eliminated; if parity conservation is required as well, then $L_3 = \sqrt{-g}\; R\; F_{\mu\nu}\overset{*}{F}{}^{\mu\nu}$ and $L_4 = \sqrt{-g}\; R_{\mu\nu\rho\sigma}F^{\mu\nu}\overset{*}{F}{}^{\rho\sigma}$ (where the asterisk stands for the dual operation) are also rejected. In this case, there remains only $L_5 = \sqrt{-g}\ R\ F_{\mu\nu}F^{\mu\nu}$, $L_6 = \sqrt{-g}\ R_{\mu\nu}F^{\mu}{}_{\sigma}F^{\sigma\nu}$ and $L_7 = \sqrt{-g}\ R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ as viable choices. Moreover, since here the electromagnetic field will be treated as a test-field - that is, neglecting its influence back on space-time curvature - in the background of a conformally - flat FRW-type metric, these three competitors may be further reduced to just one. In fact, since the Weyl conformal tensor vanishes for a FRW geometry, triplet L_5, L_6, L_7 reduces to just a pair; on the other hand, once the conformal factor is time-dependent only, one is free to choose either L_5 or L_6. In view of the preceding reasonings, hereafter we will consider that electromagnetic processes in the presence of an external gravitational field are adequatelly described by the non-minimal Lagrangian

$$L_{EM} = \sqrt{-g}\ [-\tfrac{1}{2}\ F_{\mu\nu}F^{\mu\nu} + \xi\ RF_{\mu\nu}F^{\mu\nu}]\ , \tag{4}$$

where ξ is a constant with dimensions[length]^2. The fundamental origin of this parameter may be atributed to the presence of the curved background, and therefore its precise value should depend on the fine structure constant $\alpha = \frac{e^2}{\hbar c}$ and/or on the gravitational constant k. Another possible justification concerns the occurrence of tidal effects on the photon, due to quantum one-loop vacuum polarization in a curved background space-time, which should be taken into account by the inclusion of curvature-dependent terms in the equations of motion of Electrodynamics [7].

The first interesting consequence of the adoption of Lagrangian eq. (4) concerns the energy-momentum tensor of the electromagnetic field which, besides the usual Einstein-Maxwell tensor $E_{\mu\nu} \equiv F_{\mu\alpha}F^{\alpha}{}_{\nu} + \frac{1}{4}\ g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$, acquires a new contribution given by

$$\frac{1}{\xi} T_{\mu\nu} = F^2 G_{\mu\nu} - 2R F_{\mu\sigma}F^{\sigma}{}_{\nu} - g_{\mu\nu} \Box F^2 + F^2{}_{,\mu;\nu} \qquad (5)$$

where $F^2 \equiv F_{\mu\nu}F^{\mu\nu}$, $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is Einstein's tensor, $\Box$ is the generalized D'Alembertian operator and the semi-colon stands for covariant differentiation.

Different authors have called attention upon a remarkable property of the above tensor: it does not vanish in the flat geometry limit. In effect, even when the metric tensor $g_{\mu\nu}$ reduces to the Minkowskian form $\eta_{\mu\nu}$ there still survives a track of the variational procedure in the expressions of $T_{\mu\nu}$:

$$\frac{1}{\xi} \overset{(o)}{T}_{\mu\nu} \equiv - \eta_{\mu\nu} \overset{(o)}{\Box} F^2 + F^2{}_{,\mu,\nu} \quad , \qquad (6)$$

where $\overset{(o)}{\Box}$ is the usual D'Alembertian of Special Relativity. However, the same does not occur with the equation of motion of the electromagnetic field which, in the flat space limit, reverts to the standard Maxwellian form associated to HMC. In fact, from the action corresponding to eq. (4) one obtains

$$F^{\mu\nu}{}_{;\nu} = 4\xi(RF^{\mu\nu})_{;\nu} \quad , \qquad (7)$$

which reproduces eq. (3a) when R=0. Incidentally, it seems worth at this point to make a comment on a misleading, though widespread, belief about HDCC: in the literature, it is frequently argued that the introduction of curvature-dependent terms in the equations of motion of the field $F_{\mu\nu}$ would result in a breakdown of charge conservation [8]. Although such reasoning could in effect be used as a guiding principle in order to select certain types of direct coupling, it surely cannot lead to the exclusion of all kinds of couplings derived from HDCC - as one can easily see through the inclusion of an external current J^{μ} into Lagrangian eq. (4). Indeed, from eq. (7) it follows that such current would be conserved, just as in the customary approach.

Remark also that taking the trace of eq. (5) one obtains

$$T^{\alpha}_{\alpha} = \xi\,(RF^2 - 3\,\Box F^2). \qquad (8)$$

Thus, contrary to the minimal coupling case, in the present non-minimal scheme gravitational interaction induces a non-vanishing trace for the energy-momentum tensor of the electromagnetic field. Nevertheless, a null value for the trace can still be achieved if the field invariant F^2 vanishes. This requirement is fulfilled, for instance, by radiation fields which satisfy, in the average, the condition $\langle E^2 \rangle = \langle H^2 \rangle$; hence, the usual hydrodynamical treatment of the electromagnetic field still leads to the equation of state $P = \frac{1}{3}\rho$. In section 3, we shall use this property to describe a photon gas interacting, via HDCC, with gravity.

In possession of both sets of dynamical field equations, eq. (3a,b) and eq. (7), arising from HMC and HDCC, respectively, we can proceed to investigate the evolution of the quantities a_μ, b_μ, etc. appearing in the decomposition eq. (1). Denoting the variation of the phase ϕ by the vector $K_\mu = D_\mu\phi$, from

HMC one obtains for the main terms of the evolution equation the following relations [9]:

$$2a^{\mu}{}_{;\lambda}K^{\lambda} + a^{\mu}K^{\lambda}{}_{;\lambda} + iK^2 b^{\mu} = 0, \tag{9a}$$

$$(a^2 K^{\mu})_{;\mu} = 0, \tag{9b}$$

$$K^2 = 0, \tag{9c}$$

where we have set $a^2 = a_{\mu}a^{\mu}$, $K^2 = K_{\mu}K^{\mu}$; while in the HDCC case the corresponding equations are

$$(1-4\xi R)(2a^{\mu}{}_{;\lambda}K^{\lambda}+a^{\mu}K^{\lambda}{}_{;\lambda}+iK^2 b^{\mu})-4\xi R_{,\lambda}(K^{\lambda}a^{\mu}-K^{\mu}a^{\lambda}) = 0, \tag{10a}$$

$$[(1 - 4\xi R)a^2 K^{\mu}]_{;\mu} = 0, \tag{10b}$$

$$K^2 = 0. \tag{10c}$$

The first conclusion - and perhaps the most important one - provided by the above relations is that in both kinds of coupling photons move along (null) geodesics. This is truly a relevant result since it explicitly attests that the majority of the observations involving electromagnetic radiation and gravitational fields may be satisfactorily explained by either HMC or HDCC (e.g., the redshift observations quoted in section 1). Inasmuch as eqs. (9c) and (10c) are indistinguishable, both the minimal procedure and the present approach concur in that the trajectories followed by photons when immersed in a gravitational field shall be identified to the null geodesics of the given background geometry.

Eqs. (9b) and (10b), however, are evidently quite different. This remarkable aspect is, in fact, the fulcrum of the present work and brings into evidence the conceptual richness afforded by non-minimal scenarios. It is a direct consequence of eq. (9b) (hence, of HMC) that the number N of photons comprised in a volume V, given by the formula $N = \int a^2 \omega dV$, is a constant of motion. Here, the radiation frequency ω is defined in the usual way, $\omega = K_\mu V^\mu$, with respect to an observer endowed with four-velocity V^μ. Thus, in an expanding FRW Universe the density $n = \frac{N}{V}$ of the photons obeys an equation of evolution such that

$$\dot{n} + n\theta = 0, \tag{11}$$

in which the expansion factor $\theta \equiv \frac{\dot{V}}{V}$ gives a measure of the variation of volume V with cosmic time. Eq. (11) therefore guarantees the conservation of the number of photons, $\dot{N} = \frac{dN}{dt} = 0$.

In the HDCC case, however, a different result is achieved. Indeed, for a FRW geometry eq. (10b) yields

$$\dot{n}(1 - 4\xi R) + n\theta = 4\xi n(\dot{R} + R\theta). \tag{12}$$

This expression shows that the gravitational field may affect the total number of photons in the Universe. Although such result has been obtained in a simple, classical fashion, in terms of the geometrical optics approximation eq. (1), its quantum version can still be valid. In effect, many authors have argued that, in view of the conformal invariance of the scalar product associated to the Hilbert space of quantum operators, massless particles described by conformally invariant equations of motion cannot be produced by conformally flat gravitational fields; thus, since Maxwell's equations keep their conformally invariant character when generalized, via HMC, to curved space-times, they conclude that an expanding FRW-like Universe is not capable of creating photons [10]. All of these claims, however, rely on the use of HMC; therefore, the above conclusion is valid under HMC conditions only. Now it is easy to see that in the present HDCC approach such reasonings cannot be sustained, once conformal symmetry is broken. For instance, if one considers that throughout the history of the Universe the background geometry is determined solely by a matter configuration ρ_M (such that $T_{\mu\nu} = \rho_M V_\mu V_\nu$), if follows that the number of photons at the singular origin $t = 0$ is null, increasing continually afterwards until a saturation value N_0 is reached and remaining constant thereon, since $N \sim \frac{t^2}{\delta^2 + t^2}$, $\delta^2 = \frac{16}{3} |\xi|$ (for $\xi < 0$).

3 - THE CHEMICAL POTENTIAL OF THE PHOTON AND ITS COSMIC CONSEQUENCES

After demonstrating the actual viability of photon production by the gravitational field associated to an expanding homogeneous Universe, the natural step ensuing is the attempt to quantify such process. However, instead of pursuing the usual approach and put under scrutiny the quantum evolution equation for the photon vacuum, we shall follow another line of inquiry here: starting from the present, observed values of the MBR, we will proceed to investigate, through an statistical analysis, the possible influence of a variation in the number of photons upon the characteristics of such radiation field.

In condensed matter physics, changes in the number of elementary constituents of a given reaction are costumarily taken into account through the introduction of a "chemical potential" term [11]. Nevertheless, the behaviour of photons interacting with matter is characterized by a null value of the chemical potential, once photons can be emitted or absorbed at any rate in an arbitrary reaction. Let us now pose the question: would such charactheristic still be valid for the photon-gravity interaction?

Of course, the standard approach provides an inequivocal answer: if the concept of a vanishing chemical potential for the photon holds locally, then a straightforward use of the Equivalence Principle would extend its validity to any circumstances whichever. Due to the current trend of esteeming the Equivalence Principle as a de facto generator of physical laws, such statement became uncontroversial and thus acquired a generalized acceptance. However, as indicated in the previous section, a strict adhesion to empirical criteria leads to a wider conclusion, since the ensemble of observational data presently available on electromagnetic processes in gravitational fields does not suffice to establish HMC as the true and only type of admissible coupling between these fields. Therefore, it seems worth to examine the HDCC alternative hereunto developed. We are thus led to consider the the statistical distribution function of a boson gas endowed with a chemical potential μ:

$$dN_{\omega} = \frac{1}{\exp[\frac{\omega-\mu}{T}]-1} \quad . \tag{13}$$

According to the present HDCC scenario, the total number of photons in the Universe is affected by gravitational interaction; hence, in eq. (13) the total chemical potential μ may be conveniently split into two independent parts:

$$\mu = \mu_0(P,T) + \Delta\mu \quad , \tag{14}$$

where $\Delta\mu$ is the gravitationally - induced contribution and $\mu_0(P,T)$ is the flat-space component which, in view of the arguments quoted above, should vanish. To proceed, one needs to consider the question: which is the presumable form of functional dependence of $\Delta\mu$ on the curvature? Instead of attempting to answer this question for arbitrary configurations, we will restrict ourselves to the special cases in which space-time structure can be adequately represented by FRW geometries. Furthermore, as we have previously stated, in the present study photons will be treated as test-particles in a background gravitational field, thus contributing negligibly to the matter-energy content responsible for space-time curvature. Such premise requires that throughout the history of the Universe - even at primordial epochs of great compression - the energy density ρ_γ of the photons should remain very small. We shall see latter on that such regularization of the photon energy density, in the case of a FRW model, will follow as a natural consequence of photon number non-conservation.

Due to the spatial homogeneity of the FRW model, $\Delta\mu$ shall depend on cosmic time only. It seems reasonable to suppose that $\Delta\mu$ may be written as a combination of powers of the unique curvature parameter available, the expansion factor θ. This feature allows us to preliminarly consider the form

$$\Delta\mu = -b^2\,\theta, \tag{15}$$

in which $b^2 = \text{const.}$ and the minus sign arises from the bosonic nature of photons and from the fact that we presently live in an expanding era (i.e., $\theta > 0$). In this way, the arrow of time provided by the cosmic expansion coincides with the thermodynamical arrow, as we shall see soon. Remark that, since the inclusion of higher powers of θ does not affect qualitatively our forthcoming results, eq. (15) can be accepted at least as a good first approximation.

From Lagrangiam eq. (4) it follows that photons travel along null geodesics, as we have shown in section 2. Also, as in the standard model, frequency ω varies like $\omega \sim A^{-1}$; we will further assume that temperature T behaves in the same manner.
The thermodynamical potential $\Omega = -PV$ is provided by the expression

$$\Omega = \frac{VT}{\pi^2}\int_0^\infty \omega^2\,\ell n\,[1 - e^{-\frac{(\omega-\mu)}{T}}]\,d\omega\,, \tag{16}$$

in which μ is given by eq. (15). Calling $\beta \equiv -\frac{\mu}{T} = \frac{b^2\theta}{T}$, we thus obtain

$$\Omega = \frac{-2}{\pi^2} VT^4 \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^4} . \tag{17}$$

This kind of infinite series will appear frequently in the remaining of this paper; they will be denoted by

$$L\ (\beta,s) \equiv \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^s} . \tag{18}$$

In the Appendix some useful properties of these series are listed. Particularly interesting for our future developments is a regularization procedure which consists in the conversion of $L(\beta,s)$ into a power series in β, with coefficientes proportional to the Riemann Zeta function

$$\zeta(s) \equiv \sum_{m=1}^{\infty} \frac{1}{m^s} , \tag{19}$$

evaluated away from its poles. In order to apply this result, let us first recall the usual expression for the radius of the Universe in the standard FRW model, given by $A(t) = A_o t^q$, where the parameter q varies in the range $0 < q < 1$. This expression is in good agreement with all observational data, including the evidence about primordial cosmic abundances of chemical elements. Accordingly, the functional dependence of β with respect to cosmic time goes like $\beta \sim t^{q-1}$. Thus, for very long time intervals $(t \to \infty)$ factor β becomes extremely small, and we obtain

$$\lim_{\beta \to o} \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^4} = \zeta(4), \tag{20}$$

where $\zeta(4)$ is the corresponding form assumed by Riemann's Zeta function. Therefore, potential Ω in this limiting condition approaches the value

$$\Omega_\infty = \frac{-\pi^2}{45} VT^4 \ . \tag{21}$$

Hence, we see that HDCC distribution eq. (17) converges assymptotically to a black-body spectrum. Since $\beta << 1$ implies that $\frac{\hbar\theta}{k_B T} << \frac{1}{b^2}$, where k_B is Boltzmann's constant, and present MBR observations show that cosmic photons behave very nearly like black-body radiation, we achieve the condition $b^2 << 10^{-5}$.

From eq. (17) for the potential Ω a straightforward calculation provides the expressions of other relevant thermodynamical quantities. In Table I below we list their values for both HMC and HDCC cases; remark that in the limit $\beta \to 0$ (that is, for large values of t) HDCC relations coincide with those derived from HMC. Particularly noteworthy is the preservation of the equation of state $P = \frac{1}{3}\rho$, which in the HMC case arises from the condition $T^\alpha_\alpha = 0$, and in the HDCC case from the vanishing, on the average, of the trace $T^\alpha_\alpha = \xi(RF^2 - 3\Box F^2)$ of the electromagnetic energy-momentum tensor.

TABLE I

Thermodynamical Quantities	HMC case (Photon gas minimally coupled to gravity)	HDCC case (Photon gas non-minimally coupled to gravity)
Energy density	$\rho = \frac{\pi^2}{15} T^4$	$\rho = \frac{6}{\pi^2} T^4 \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^4}$
Pressure	$P = \frac{1}{3}\rho$	$P = \frac{1}{3}\rho$
Total number of photons	$N = \frac{2}{\pi^2} \zeta(3) VT^3$	$N = \frac{2}{\pi^2} VT^3 \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^3}$
Free energy	$F = \frac{-\pi^2}{45} VT^4$	$F = \frac{-2}{\pi^2} VT^4 \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^4} + N\mu$
Total entropy	$S = \frac{-4F}{T}$	$S = \frac{-4F}{T} + 3 \frac{N\mu}{T}$
Specific entropy $s = \frac{S}{N}$	$s = \frac{4}{3} \frac{\rho v}{T}$	$s = \frac{1}{T} (\frac{4}{3} \rho v - \mu)$
Thermodynamical potential $\Omega = - PV$	$\Omega = F$	$\Omega = F - N\mu$
Evolution of the number density of photons $n = \frac{N}{V}$	$\dot{n} + n\theta = 0$	$\dot{n} + n\theta = \frac{-2b^2}{\pi^2} T^2 R_{oo} \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^2}$

Table I: Thermodynamical quantities – comparison between HMC case and HDCC case (which includes the creation of photons by the expansion of the Universe). Note that for large values of cosmic time (or $\frac{\theta}{T} \ll \frac{1}{b^2}$) HDCC formuli go into HMC ones. Note also the definition $v = \frac{1}{n}$.

Using Actor's result (see the Appendix) the equation of evolution for the density $n = \frac{N}{V}$ may be written as

$$\dot{n} + n\theta = \frac{-2b^2}{\pi^2} T^2 R_{oo} L(\beta,2) =$$

$$= \frac{-2b^2}{\pi^2} T^2 R_{oo} [\frac{\pi^2}{6} - \beta - \frac{\beta^2}{4} + \beta \ln \beta - \sum_{\ell=1}^{\infty} \frac{\beta^{2\ell+1}}{(2\ell+1)!} \zeta(1-2\ell)] . \tag{22}$$

Remark that for $R_{oo} < 0$ there occurs a mechanism of particle creation, that is, $\dot{N} = (\dot{n} + n\theta)V > 0$. This is the case if Einstein's equations describe the metric evolution and the strong energy condition $T_{00} + 3T^{\mu}{}_{\mu} > 0$ is satisfied – or equivalently, if the energy-momentum tensor of the cosmic fluid corresponds to a matter-dominated configuration, $T_{\mu\nu} = \rho_M V_\mu V_\nu - P_M h_{\mu\nu}$, with $\rho_M + 3P_M > 0$. Given that today (i.e., for large times) the condition $\beta << 1$ holds, it follows that in the present matter-dominated configuration eq. (22) may be approximated by

$$\dot{n} + n\theta \cong \frac{b^2}{6} (k_B T)^2 k \rho_M , \tag{23}$$

where k is Einstein's constant; therefore, in the present epoch the production of photons is almost insignificant, $\frac{\dot{N}}{N} \sim 10^{-65} b^2$.

Let us make some further comments on the results displayed at Table I. At the beginning of the Universe (i.e., at $t = 0$), the total number of photons N vanishes, as well as the total entropy S. Thus we arrive at the qualitatively new result we have announced at the Introduction: according to the HDCC approach developed here neither the total number of photons nor their total entropy are in fact conserved quantities, and so the ratio $\frac{n_\gamma}{n_B}$ between the number of photons and baryons comprised in the Universe depends on the cosmic epoch. As the Universe expands, gravity's capacity to create photons progressively decreases, and at later times (i.e., for $\frac{\theta}{T} \ll \frac{1}{b^2}$) the total number of photons approaches a constant value (indeed, if we make use of the lower possible limit $b^2 \sim 10^{-5}$ we obtain that the present rate of photon creation is $\frac{\dot{N}}{N} \sim 10^{-70}$). The effects of the presence of a non-null chemical potential gradually vanish and the photon gas correspondingly acquires the black-body character manifested by current MBR observations. Thus in the HDCC scenario presented here the observed value of the entropy S does not constitute a crucial cosmological problem (as it has been claimed to be from the point of view of the standard approach), representing but a fortuitous value associated to the present epoch.

4 - CONCLUSION AND FINAL REMARKS

The generalized belief that the gravitational field assigned to an homogeneous expanding Universe cannot be taken as the (main) responsible for the generation of the observed ammount of cosmic photons has left few alternative routes open for the explanation of the origin of such radiation content. The conventional approach has given a simple and seemingly definitive answer to this question: these photons were created at the Big-Bang, together with space-time continuum itself, and therefore they constitute veritable primordial entities. In spite of the undeniable empirical efficiency afforded by the standard model, however, this kind of inscrutable statement cannot in fact be considered as quite satisfactory from the theoretical standpoint.

In the present article we pursue an alternative proposal, based upon the hypothesis of direct coupling to the curvature (HDCC), according to which the problematic provenance of cosmic photons may be envisaged from a new angle. In effect, the study of non-minimal (direct) couplings between electromagnetic and gravitational fields leads in a natural way to the elaboration of an interaction scheme in which gravitational fields may produce photons. Thus, in the case of an homogeneous Friedman-like configuration, we have obtained a quasi-standard scenario which, besides reproducing appropriately the main observational features of the conventional approach, further incorporates a cosmological mechanism of photon creation such that the total number of photon comprised in the Universe changes according to the cosmic expansion. This is, in principle, a genuinely new aspect requiring further examination, since it points to a fresh realm of conceptual possibilities yet to be explored in full. For instance, a quantum description of electromagnetic fields satisfying HDCC is certainly a future priority; nevertheless, in this work we have chosen a purely classical approach and took into consideration, through a statistical analysis, the thermodynamical properties implied by the present non-minimal scheme.

Let us briefly review the most important results provided by such analysis. Both kinds of observational evidence that currently give empirical support to nearly all of the attempts towards the establishment of a global space-time model (namely, the occurence of cosmological redshifts and the detection of the MBR of 2.7^{o}K) fit perfectly well within our approach. In effect, HDCC photon trajectories coincide with the null geodesics of the background geometry, in the same fashion as HMC ones, and thus redshift observations are explained just as in the conventional case.

On the other hand, the introduction of a gravity-induced, non-vanishing chemical potential term in the distribution function of a photon gas directly coupled to the curvature leads to the apparition of an expansion-driven mechanism of photon creation which affects the evolution of this photon gas in such a way that, for large times, its behaviour progressively approaches that of a black-body - a feature that can be associated to the occurrence of an all-pervading cosmic black-body radiation field, just as we observe today.

Thus, the arrow of entropy variation of this photon gas is determined by the arrow of the expansion of the Universe;

furthermore, the ratio between the numbers of photons and baryons is evidently not a constant, but rather a function of cosmic time. This in turn implies that current attempts to elucidate the precise origin of a "fundamental" characteristic of the Universe, consubstantiated in the observed value $\frac{n_\gamma}{n_B} \sim 10^9$ of this ratio, no longer constitute rightful speculations, since this figure stems from a fortuitous mensuration and not from a presumptive primordial ordering imprinted on the actual Universe.

The present, initial stage of development of the new cosmological scenario proposed here is certainly still incomplete, thus requiring many further improvements; nevertheless, we believe that the preliminar results sketched above clearly indicate that HDCC - based models deserve to be studied at length. Elsewhere, we have already investigated other interesting features provided by non-minimal schemes; for instance, in this paper we have not considered HDCC configurations in which, due to non-minimal coupling, photons can induce substantial changes on the behaviour of a gravitational field at the neighborhood of the point of maximum contraction, which may eventually lead to the preclusion of the initial singular state traditionally displayed by the standard HBB model [12]. A phase of cosmic contraction, prior to the present expanding era, would surely imply drastic modifications on this scenario [13].

In a forthcoming paper [14] we will attempt to show that the adoption of non-minimal couplings between the two long-range fields yet known to physicists, gravity and electromagnetism, might presumably lead to the solution, at one stroke, of many outstanding problems of standard Cosmology, such as the presence of the initial singularity, the occurence of causal horizons and, as we have seen here, the need for an explanation, on a fundamental basis, of the observed value of the cosmic entropy afforded by present MBR observations.

- APPENDIX

In general, the expression $L(\beta,s) \equiv \sum_{n=1}^{\infty} \frac{e^{-m\beta}}{m^s}$ requires a very difficult treatment. To obtain the important thermodynamical quantities which are necessary to describe the system constituted by our photon gas in an expanding Universe, one needs to evaluate those infinite series for very small values of β (which means, for very high values of the cosmical time). A very efficient method to deal with these cases has been presented recently by A.Actor [15]. He obtains regularized formuli for infinite series of the form $\sum_{n=1}^{\infty} \frac{f(x)}{m^s}$, for some special functions $f(x)$. We quote only some of Actor's results which are particularly useful for our calculations.

$$\sum_{m=1}^{\infty} \frac{\sinh m\beta}{m^{2N+2}} = \sum_{a=o}^{\infty *} \frac{\beta^{2a+1}}{(2a+1)!} \zeta(2N-a+1)+$$

$$+ \frac{\beta^{2N+1}}{(2N+1)!} [\gamma + \psi(2N+2) - \ln\beta] \qquad (A.1)$$

for $N = 0,1,2,\ldots$.

The asterisk* above the summation symbol means that the value a=N is excluded. The reason is that in this case the Riemann Zeta function reduces to the divergent expression $\zeta(1)$. The process of regularization used by Actor gives precisely the extra term envolving the Euler number γ, by the use of an integral expression for the series A.1. The additional term is the result of the contribution to the integral, in the complex plane, of the residue of the pole $\zeta(1)$.

We can also evaluate the series:

$$\sum_{m=1}^{\infty} \frac{\cosh m}{m^{2N+1}} = \sum_{a=o}^{\infty *} \frac{\beta^{2a}}{(2a)!} \zeta(2N-2a+1) +$$

$$+ \frac{\beta^{2N}}{(2N)!} [\gamma + \psi(2N+1) - \ell n\beta] \qquad \text{(A.2)}$$

for $N = 0,1,2,\ldots$.

In these expressions $\zeta(s)$ is the Riemann Zeta function

$$\zeta(s) = \sum_{m=1}^{\infty} \frac{1}{m^s} = \frac{1}{(1-2^{1-s})\Gamma(s)} \int_0^{\infty} \frac{x^{s-1}}{e^x+1} dx \quad . \qquad \text{(A.3)}$$

Some special values which are useful for our considerations are:

$$\zeta(0) = -\frac{1}{2}$$

$$\zeta(1) = \infty$$

$$\zeta(2) = \frac{\pi^2}{6}$$

$$\zeta(3) = 1,202$$

$$\zeta(4) = \frac{\pi^4}{90}$$

The function $\psi(Z)$ which appeared in (A.1,2) is the so-called Psi (Digamma) Function.

$$\psi(Z) = \frac{d}{dz} [\ell n\ \Gamma(Z)] = \frac{\Gamma'(Z)}{\Gamma(Z)} \qquad \text{(A.4)}$$

which for integer values of the argument gives

$$\psi(1) = -\gamma \quad (\gamma \text{ is the Euler Number}).$$

$$\psi(n) = -\gamma + \sum_{m=1}^{n-1} \frac{1}{m} \quad . \tag{A.5}$$

The recurrence formula needed to obtain some special values of interest is given by

$$\psi(Z+1) = \psi(Z) + \frac{1}{Z} \, . \tag{A.6}$$

The above expressions yield the special formuli which appeared frequently in our calculations in the present paper:

$$\sum_{m=1}^{\infty} \frac{\sinh m\beta}{m^4} = \frac{1}{6}\beta^3\left(\frac{11}{6} - \ell n\beta\right) + \sum_{a=0}^{\infty *} \frac{\beta^{2a+1}}{(2a+1)!}\zeta(3-2a) \tag{A.7}$$

$$\sum_{m=1}^{\infty} \frac{\cosh m\beta}{m^4} = \frac{\pi^4}{90} + \frac{\zeta(3)}{2}\beta^2 - \frac{1}{48}\beta^4 \tag{A.8}$$

$$\sum_{m=1}^{\infty} \frac{\sinh m\beta}{m^3} = \frac{\pi^2}{6}\beta - \frac{1}{12}\beta^3 \tag{A.9}$$

$$\sum_{m=1}^{\infty} \frac{\cosh m\beta}{m^3} = \frac{3}{4}\beta^2 - \frac{1}{2}\beta^2 \ell n\beta + \sum_{a=0}^{\infty *} \frac{\beta^{2a}}{(2a)!}\zeta(3-2a) \tag{A.10}$$

$$\sum_{m=1}^{\infty} \frac{\sinh m\beta}{m^2} = \beta(1 - \ell n\beta) + \sum_{n=0}^{\infty *} \frac{\beta^{2a-1}}{(2a-1)!}\zeta(3-2a)$$

$$\sum_{m=1}^{\infty} \frac{\sinh m\beta}{m^2} = \beta(1 - \ell n\beta) + \sum_{a=0}^{\infty *} \frac{\beta^{2a-1}}{(2a-1)!}\zeta(3-2a) \tag{A.11}$$

$$\sum_{m=1}^{\infty} \frac{\cosh\, m\beta}{m^2} = \frac{\pi^2}{6} - \frac{1}{4}\,\beta^2 \qquad \text{(A.12)}$$

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