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THREE REGGEON (STRING) VERTEX FOR N.S. SECTOR WITH BOSONIZED GHOSTS

by

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ABSTRACT: We construct the complete BRST invariant Sciuto-Della Selva-Saito three reggeon vertex for Neveu-Schwarz sector with bosonized reparametrization ghosts and fermionized SUSY ghosts.

Key-words: Superstring theory; Dual models.

I - INTRODUCTION

We believe nowdays that certain superstring theories are finite quantum theories unifying gravity with other forces. Since quantization of gravity may be relevant in nature we may expect that these superstring theories can also have some relevance.

One of the outstanding problems in superstring theories is to prove this finiteness to any order in perturbation theory. In the old days of the string theories the starting point for computing multiloops was the N-reggeon vertex[1] and an arbitrary multiloop amplitude[2] could be achieved by sewing together arbitrary legs by the insertion of a propagator. This program had serious difficulties with the unphysical states. Now, we understand that we can overcome these difficulties[3] by incorporating the covariant BRST approach[4].

It was shown in reference [5] that a basic ingredient for computing BRST-invariant N-string amplitude is the BRST--invariant three-reggeon vertex[6].

The Witten's vertex for Neveu-Schwarz-Ramond (N.S.R.) sector was given in reference [7] and in this letter we compute the complete BRST-invariant three-reggeon vertex for N.S. sector with "bosonized" ghosts[8]. This vertex has two parts, for the first one, not proportional to the fermionic coordinate, we construct an ansatz and prove the BRST invariance. For the second part, that one proportional to the fermionic coordinate, we construct a BRST invariant operator which connects the first and the second part of the vertex.

II - THE VERTEX

The supersymmetric extension of BRST invariant Sciuto--Della Selva-Saito's vertex given in reference [6] is

$$V = {}_{(2)}\langle q=0;\ O_a;\ O_\psi;\ N_R=3;\ N_{ss}=-2| :\exp I: \tag{1}$$

with

$$I = \oint dZ \Big\{ -DX^{(2)}(z,\theta)X^{(1)}(1-z-\phi\theta, i\theta-i\phi) + B^{(2)}(z,\theta)C^{(1)}(1-z-\phi\theta, i\theta-i\phi) + - iC^{(2)}(z,\theta)B^{(1)}(1-z-\phi\theta, i\theta-i\phi) \Big\} . \tag{2}$$

The superfields which appears in (2) are given by:

$$\begin{aligned} X^\mu(z,\theta) &= x^\mu(z) + \theta\ \psi^\mu(z) \\ B(z,\theta) &= \beta(z) + \theta\ b(z) \\ C(z,\theta) &= c(z) + \theta\gamma(z) \end{aligned} \tag{3}$$

with contractions

$$\begin{aligned} \langle X^\mu(z,\theta)X^\nu(w,\phi)\rangle &= -\ g^{\mu\nu}\log(z-w-\theta\phi) \\ \langle B(z,\theta)C(w,\phi)\rangle &= \frac{\theta-\phi}{z-w-\theta\phi} \end{aligned} \tag{4}$$

where $x^\mu(z)$ and $\psi^\mu(z)$ the coordinate field of the string and its supersymmetric (N.S.) partner respectively. The fermionic fields $b(z)$ and $c(z)$ are the reparametrization ghosts[8], $\beta(z)$ and $\gamma(z)$ the commuting spinor ghosts of local supersymmetry[8a,c,d]. The integration in (2) $\oint dZ$ is a shortening for $(2\pi i)^{-1}\oint dz \int d\theta$

with θ and ϕ fermionic coordinates. The Susy covariant derivative D is:

(5) $$D = \partial_\theta + \theta\partial_z \quad .$$

The state appearing on the left in (1) is the vacuum of the oscillators in the supercoordinate $X^\mu(Z)$ given by (3) with vanishing center-of-mass position of the string and has ghost number $N_R = 3$, $N_{SS} = -2$ for the fermionic and bosonic ghost respectively, following the approach of reference [8a,c,d].

It is convenient to rewritte (2) in terms of superfield's components:

(6) $$I = A + \phi B$$

with

$$A = \frac{1}{2\pi i}\oint_0 dz \Big\{ -x'^{(2)}(z).x^{(1)}(1-z) + b^{(2)}(z)c^{(1)}(1-z) - c^{(2)}(z)b^{(1)}(1-z) + i\psi^{(2)}(z).\psi^{(1)}(1-z) + i\beta^{(2)}(z)\gamma^{(1)}(1-z) - i\gamma^{(2)}(z)\beta^{(1)}(1-z) \Big\}$$

(7)

$$B = \frac{1}{2\pi i}\oint_0 dz \Big\{ ix'^{(2)}(z).\psi^{(1)}(1-z) - \psi^{(2)}(z).x'^{(1)}(1-z) + \beta^{(2)}(z)c'^{(1)}(1-z) - \gamma^{(2)}(z)b^{(1)}(1-z) - ib^{(2)}(z)\gamma^{(1)}(1-z) + ic^{(2)}(z)\beta'^{(1)}(1-z) \Big\}$$

In such a way that (1) can be written as:

(8) $$V = V_0 + V_1\phi \quad .$$

Now, we pass to compute the bosonized-fermionized form of the vertex. In reference [8a,c,d] it is shown that it is convenient to fermionize the Susy ghosts in order to treat the covariant vertices of superstring and in ref. [9] it was shown that both the computation and the result of N-bosonic string amplitude simplifies greatly if the reparametrization ghosts are bosonized. In the representation where the ghosts are "bosonized" we have[8]:

(9) $$\begin{aligned} b(z) &= c_2{:}e^{-\sigma(z)}{:} \qquad c(z) = c_2{:}e^{\sigma(z)}{:} \quad . \\ \gamma(z) &= c_1{:}e^{\Phi(z)}\eta(z){:} \qquad \beta(z) = c_1{:}e^{-\Phi(z)}\xi'(z){:} \end{aligned}$$

with c_i cocycles, $\sigma(z)$ and $\Phi(z)$ conformal fields with conformal dimension zero giving the dimensions -2 and 1 for the conformal fields $b(z)$ and $c(z)$ respectively. The fields $\eta(z)$ and $\xi(z)$ have dimensions 1 and zero respectively which together to $\Phi(z)$ gives dimensions -1/2 for $\gamma(z)$ and 3/2 for $\beta(z)$ [8,a,c,d].

These new conformal fields, we have introduced, have the contractions[8,a,c,d]:

(10) $$\begin{aligned} \langle\sigma(z)\sigma(w)\rangle &= -\langle\Phi(z)\Phi(w)\rangle = \ln(z-w) \\ \langle\xi(z)\eta(w)\rangle &= \frac{1}{z-w} \quad . \end{aligned}$$

The BRST charge, Q, in the fermionic string, given by[8,a,d]

(11) $$Q = \oint dZ{:}\left[-\frac{1}{2}\,CDX.X' + \frac{1}{4}\,C(DC)(DB) - \frac{3}{4}\,CC'B\right]{:}$$

can be expressed in terms of the "bosonized" ghosts (9) by using the method explained in ref. [9], its value is:

$$Q_{BOS} = \oint dZ\ j(z)$$

(12)
$$j = :\Big\{ -\tfrac{1}{2} c_2\, e^{k^{(2)}.W}\, \psi'.\psi - \tfrac{1}{4} c_2\, e^{(2k^{(1)}-k^{(2)}).W}\, \eta'\eta + c_2\, e^{k^{(2)}.W}\, \xi'\eta + \tfrac{1}{2} c_1\, e^{k^{(1)}.W}\, \eta\psi.W' - \tfrac{1}{2} c_2 e^{k^{(2)}.W} W'.W' + + \tfrac{1}{2} c_2\, e^{k^{(2)}.W} (3k^{(2)}-2k^{(1)}).W'' \Big\} : \quad .$$

Where we have introduced the twelve component Veneziano field $W^{\mu}(z)$

(13) $$W^{\mu}(z) = (x^{\mu}(z), \Phi(z), \sigma(z))$$

which can be written in terms of oscillators as:

(14) $$W^{\mu}(z) = q^{\mu} - i\alpha_0^{\mu}\ \ln z + i \sum_{n\neq 0} \tfrac{1}{2}\, \alpha_n^{\mu}\, z^{-n} \qquad \mu = 1,\ldots,12$$

Having the canonical comutation relations

(15)
$$[\alpha_m^{\mu}, \alpha_n^{\nu}] = m\ g^{\mu\nu}\ \delta_{m,-n}$$
$$[q^{\mu}, \alpha_0^{\nu}] = i\ g^{\mu\nu}$$

with contraction:

(16) $$\langle W^{\mu}(z) W^{\mu}(w) \rangle = -\ g^{\mu\nu}\ \ln(z-w)$$

$g^{\mu\nu}$ being $g^{\mu\nu} = \mathrm{diag}(1,\ldots,1,1,-1)$, with twelve entries. The vectors $k^{(1)}$ and $k^{(2)}$ having as the only non-zero components the eleventh and twelveth respectively, i.e.,

$$k^{(1)\mu} = (0,\ldots,0,1,0)$$
(17)
$$k^{(2)\mu} = (0,\ldots,0,0,1)$$

It is simple to see that in order to keep the hermicity properties:

$$c^{\dagger}(z) = z^{2}c(1/z) \quad ; \quad b^{\dagger}(z) = z^{-4}\, b(1/z) \quad ;$$
(18)
$$\gamma^{\dagger}(z) = z\gamma(1/z) \quad ; \quad \beta^{\dagger}(z) = -\, z^{-3}\, \beta(1/z) \quad ,$$

in the new "bosonized" language (9), we must require that:

(19) $$\alpha_0^{\dagger} = \alpha_0 + i\ (-2k^{(1)}+3k^{(2)})$$

which means a non-hermitean center-of-mass momentum for the eleventh and twelveth components.

If we construct the bosonized version of the ghost number which, following reference [9], turns out to be:

(20) $$N_R = ik^{(2)}\cdot\alpha_0 \quad ; \quad N_{SS} = -ik^{(1)}\cdot\alpha_0 \quad ,$$

with N_R and N_{SS} the reparametrization and Susy ghost number, we see that the non-hermiticity of the momentum in (19) is associated to the charge asymmetry of the system[8,d]:

(21) $$N_R^{\dagger} = -N_G + 3 \quad ; \quad N_{SS}^{\dagger} = -N_{SS} - 2$$

which has as a consequence the non-trivial scalar product[8a,c,d]

(22) $${}_{gh}\langle 0; N_R = 3\ ,\ N_{SS} = -2|0\rangle_{gh} = 1 \quad .$$

Now let us construct the "bosonized" version of the vertex (1). We do this by parts. The bosonized version of the first term of V, V_0, in (8) is:

$$V_0^{bos} = {}_{(2)}\langle\gamma| :\exp J: \tag{23}$$

with:

$$J = -\oint \frac{dz}{2\pi i}\Big[W^{(1)}(1-z).W'^{(2)}(z)+i\psi^{(1)}(1-z).\psi^{(2)}(z) + \eta^{(1)}(1-z)\xi^{(2)}(z)-\xi^{(1)}(1-z)\eta^{(2)}(z)\Big] \tag{24}$$

where the vacuum in the left hand side of (23) is

$${}_{(2)}\langle\gamma| \equiv {}_{(2)}\langle q^\mu=0;Gh|(-1)^{N_R^{(2)}\sum_i N_i^{(1)}}(-1)^{N_{SS}^{(2)}\sum_i N_i^{(1)}} \tag{25}$$

with

$${}_{(2)}\langle q^\mu=0;Gh| = \sum_{n,m\in Z} {}_{(2)}\langle q^\mu=0;\ N_R^{(2)}=n, N_{SS}^{(2)}=m; O_\psi, O_\xi, O_\eta| \quad ; \tag{26}$$

the vacuum on the right hand side of (26) is the vacuum of the oscillators of W^μ, ψ^μ, ξ, η with vanishing eigenvalue of q^μ and this lead us to have the sum over all ghost number. When n or m is odd on right hand side of (26) the vacuum term has a fermionic character (See ref. [9]) then must anticommute with all fermions in fock space 1, the cocycles introduced in the right hand side of (25) play this role with $\sum_i N_i^{(1)}$ being the sum over all fermion number in fock space 1 present in the theory.

Proceeding as in the case of the bosonic string with bosonized ghosts[9] it is possible to show the BRST invariance of the vertex V_0^{bos} in (23):

(27) $$\left[Q^{(1)}_{bos},V^{bos}_{0}\right] = (-1)^{\sum_i N_i^{(1)}} V^{bos}_0 Q^{(2)}_{bos} \quad .$$

The proof is tedious, although straightforward. The details will not be presented here.

In order to construct the bosonized version of the second term in (8), ϕV_1, let us define an operador P as:

(28) $$P = -\oint_0 dz\ [x'(z).\psi(z)+\beta(z)c'(z)-\gamma(z)b(z)] \quad .$$

The operator can be easily proved to be BRST invariant, i.e.

(29) $$[Q,P] = 0 \quad ,$$

and if we analise the conformal and ghost properties of P we see that it has exactly what is needed to construct V_1 from V_0. In fact if we analise the simplest case, the vertex for the emission of a tachyon from a N.S. string

$$V = e^{ik.x} + i\theta k.\psi e^{ik.x} \quad ,$$

we see that $i\theta k.\psi e^{ik.x} = [\theta P,e^{ik.x}]$.

The bosonized version of P is:

(30) $$P^{(i)}_{bos} = -\oint_0 dz{:}\ W'^{(i)}(z).\psi^{(i)}(z){:}-c_2c_1\oint_0 dz{:}$$
$$\left[e^{(k^{(2)}-k^{(1)}).W^{(i)}(z)}\ k^{(2)}.W'^{(i)}(z)\xi'^{(i)}(z)\ + \right.$$
$$\left. -\ e^{(k^{(1)}-k^{(2)}).W^{(i)}(z)}\ \eta(z)\right]{:} \quad .$$

Now, it is simple to have the bosonized form of ϕV_1, what we have to do is to compute the following commutator:

$$(31) \qquad V_{(1)}^{(bos)}\phi = [\phi P_{bos}^{(1)}, V_0^{bos}]$$

and the result is:

$$V_{(1)}^{bos} = {}_{(2)}\langle\gamma| :S:$$

$$S = e^J \oint_{z=1} dz \Big\{ W'^{(2)}(1-z).\psi^{(1)}(z) + iW'^{(1)}(z).\psi^{(2)}(1-z)$$

$$+ c_2 c_1 \; e^{(k^{(2)}-k^{(1)}).[W^{(2)}(1-z)+W^{(1)}(z)]} \;\cdot$$

$$(32) \qquad \cdot \; [-k^{(2)}.W'^{(1)}(z)\xi'^{(1)}(z) + k^{(2)}.W'^{(2)}(1-z)\xi'^{(1)}(z)$$

$$+ k^{(2)}.W'^{(1)}(z)\xi'^{(2)} - k^{(2)}.W'^{(2)}(1-z)\xi'^{(2)}(1-z)] \; +$$

$$+ c_2 c_1 \; e^{(k^{(1)}-k^{(2)}).[W^{(2)}(1-z)+W^{(1)}(z)]} [\eta^{(1)}(z) - \eta^{(2)}(1-z)] \Big\}$$

and

$$(33) \qquad V_{bos} = V_0^{bos} + V_1^{bos}\,\phi \qquad ,$$

where V_0^{bos} is given by (23). The term V_1^{bos} is automatically BRST invariant because $P_{bos}^{(1)}$ is BRST invariant (29) and V_0^{bos} is BRST invariant (27).

III - CONCLUSIONS

We have computed the BRST invariant three string, Sciuto's vertex for the Neveu-Schwarz sector when the reparametrization ghosts are bosonized and Susy ghosts fermionized. The vertex is made up of the conformal fields of the theory, the first part V_0^{bos} (23) it was computed by the analysis of the conformal weight and by requiring BRST invariance. For the second part it was constructed an auxiliary BRST invariant operator P_{bos} which generates the second part from the first one.

It looks simple, now, to introduce the spin fields[8a,c,d] and so to construct the BRST invariant Neveu-Schwarz-Ramond vertex, we hope to report on it in a future publication[10].

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NOTE ADDED

After this work was completed we received a preprint (KA-THEP-5-1987) by U. Carow-Watamura and S. Watamura which presents the part not proportional to the fermionic coordinate, V_0, of the Susy extension of Caneschi-Schwimmer-Veneziano vertex with bosonized ghosts.