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SPIN-GLASS IN LOW DIMENSION AND THE MIGDAL KADANOFF APPROXIMATIONS

by

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Abstract

We study the spin glass problem within the Migdal Kadanoff approximation of the hyper cubic lattices. Using various technics, both analytical and numerical, we perform the real space renormalisation of the problem. We find that a Spin Glass transition occurs in 3 dimensions while it does not occur in two dimensions. The specific heat critical exponent for the transition is found to be large and negative in agreement with the experimental results.

Key-words: Spin-glass; Renormalisation; Monte Carlo.

I. Introduction

The controversy about the existence or nonexistence of a spin-glass (SG) phase in three dimensions has presented, recently, new evidences in favour of the existence of a transition [1,2,3,4,5,6]. These evidences have been obtained by several methods e.g. numerical simulations [4,5], properties of a T= o fixed point [2,3,6] and others [1,21].

However, powerfull methods as renormalisation group or Monte-Carlo simulations lead, in the last few years, to contradictory results [7,8,9,10,11,12,13]. In the Monte-Carlo simulation several problems occur, related to the long relaxation time of the metastable states when the temperature is lowered, which lead almost always to inconclusive results [12,13]. In the particular case of real space renormalisation group (RSRG), the last results give, systematically, a lower critical dimension greater than three for the existence of a spin-glass phase [9,10,11]. In general, the RSRG uses the Edwards-Anderson model [14],

$$H = -\sum_{<ij>} J_{ij} S_i S_j \quad S_i = \pm 1 , \quad (1.1)$$

where the exchange energy J_{ij} is a quenched random variable whose probability law is in the form of two [7] or three [8,9,10,11], Dirac's deltas . To simulate the cubic (or hypercubic) lattice, the Migdal - Kadanoff approximation [15,16] has been largely used. Within this approximation we obtain renormalisations that are equivalent to the formulation of the same model on an adequate hierarchical lattice [17,18]. The question is that, generally, the probability law for J_{ij} changes after renormalisation .

Here, two approches are possible :

a) to choose a simple probability distribution (usualy a sum of Dirac's deltas) and to do the approximation that the final distribution is of the same shape as the initial one [7,9,10,11] ; then to check how the first moments of the distribution change in the renormalisation process . The advantage of this method is that we can obtain analytical expressions for the renormalised parameters ; the disavantage is that we cannot control the errors caused by this kind of approximation.

Practically all RSRG work on spin-glass at low dimensionalitiy use this scheme .

b) to choose an initial probability distribution and to follow it in the renormalization process .This approach has been used in conductance problems [19,20]. The advantage of the method is that we do not limit the distributions space (as in the first case) however, analytical results are not generally available and the distribution must be followed by numerical methods.

This second scheme has been used to study the SG transition only in high[22] or infinite[23] dimensionality.

In this paper we shall use and **unify** both approaches : First we shall formulate an analytical RSRG (for the nearest neighbour Edwards-Anderson model in the Migdal-Kadanoff approximation for hypercubic lattices) with a fixed probability distribution. Then, we shall make with the same model, and same lattice - a kind of Monte-Carlo renormalisation group using several initial probability distributions in order to obtain their numerical evolutions.

The phase diagram of both approaches exhibits the spin-glass transition in three dimensions-but not in two- in agreement with the last results obtained by others methods [1,2,3,4,5,6].

Finally, we want to stress that the nature of the spin-glass phase in these models (characterized by short range interactions - nearest neighbours only) is rather different from the mean field one (long range interactions)[25,26,27,28,29] . E.Gardner[23] has shown, in a model with short range interactions in infinite dimension (Migdal-Kadanoff approximation for an infinite dimensional hypercubic lattice), that the behaviour expected for the mean field spin-glass (replica symmetry breaking, Almeida-Thouless line, ultrametric distance amoung the minima of the energy, etc) are not reproduced. As our model presents short range interaction and as we work with a Migdal-Kadanoff approximation of finite hypercubic lattices (square and cubic), the same conclusions are reached and we do not have mean field-like behaviour in the spin-glass phase.

This paper is divided in the following sections :

In section II, we formulate the model, show some interesting relations and derive some exact bounds. We use a even gaussian distribution to obtain numerical values for upper and lower bounds on the critical temperature of the spin-glass phase.

In section III we define our RSRG in assuming that a determined distribution is form-invariant during the renormalisation (only their moments are changed). For each chosen distribution we can determine the SG critical temperature (Tc) and the specific heat critical exponent (α). This

RSRG can be defined for all dimensions d (for d=2 the critical temperature is zero).

We also define a continuous version of RSRG in dimension d.

In section IV our version of the MCRG (Monte Carlo RG) is constructed and the *trajetory* of some distributions are shown. We shall show that all distributions evolve towards some defined trajetory previously defined in section III.

The phase diagram exhibits a phase transition in three dimensions.

II. Model and some properties

We consider an Edwards-Anderson type[14] Hamiltonian (see eg. 1.1), on a hierarchical lattice (HL) with N branches, each of them being compound of b bonds in series (figure 1).

On each site of the HL there is an Ising spin and each bond represents the nearest neighbour interaction.

The Migdal-Kadanoff approximation to hypercubic lattices consists to put $N=b^{d-1}$ (d being the dimension of hypercubic lattice).

We numerate each bond by two indices : the latin one indicate the branches and the greek one numerate the bonds in serie inside each branch.

The exchange energy, $K_{i\alpha}$ ($i = 1,\ldots,N$, $\alpha = 1,\ldots,b$; $K_{i\alpha} = J_{i\alpha}/k_BT$), is a quenched random variable whose probability distribution $P(K_{i\alpha})$ is even. The renormalized probability distribution is given by :

$$P_r(K) = \int\left(\prod_{i=1}^{N}\prod_{\alpha=1}^{b} dK_{i\alpha}P(K_{i\alpha})\right)\delta\left(K-\sum_{j=1}^{N}\tanh^{-1}\Big(\prod_{\beta=1}^{b}\tanh K_{j\beta}\Big)\right) \quad (2.1)$$

$$K_{i\alpha} \in (-\infty,+\infty), \quad \forall\, i,\alpha$$

where δ is a Dirac's delta. This renormalisation can be decomposed into two steps :

a)The first one corresponding to the serie array of bonds inside a branch i is given by :

$$P_s(K_i) = \int\left(\prod_{\alpha=1}^{b} dK_{i\alpha}P(K_{i\alpha})\right)\delta\left(K_i-\tanh^{-1}\prod_{\beta=1}^{b}\tanh K_{i\beta}\right) \quad (2.2a)$$

b)The second one corresponding to the collection of N parallel branches is given by :

$$P_r(K) = \int \left(\prod_{i=1}^{N} dK_i P(K_i) \right) \delta \left(K - \sum_{l=1}^{N} K_l \right) \quad (2.2b)$$

The second step specified by eq. (2.2b) presents a gaussian as non-trivial form-invariant distribution (since it is a convolution equation). By this we mean that if the initial distribution in equation (2.2b) is an even gaussian (with variance σ) then the final distribution , P_r will be also an even gaussian (with variance $\sqrt{N}\sigma$).

The involution given by eg. (2.2a) presents a non-trivial form-invariant distribution :

$$P(K) = \frac{[\delta(K-K_0)+\delta(K+K_0)]}{2},$$

K_0 being a constant .

By trivial distributions we mean the Dirac's delta centred at K= 0 (infinite temperature) or at K= $\pm\infty$ (zero temperature). It is clear that if we start with a (even) gaussian distribution, the serie transformation leads to a different distribution. After the parallel transformation we approach again a gaussian distribution (in the sense that their moments obey relations among themselves that are almost the same as in the gaussian case). The higher the dimension, the more the form-invariant distribution approaches the gaussian one[22]. In the limiting case when the dimension is going to infinity, the form-invariant distribution of the problem is gaussian[23].

Remark : We can consider the central limit theorem expressing the fact that the gaussian is a form-invariant distribution for a special type of hierarchical lattice shown in figure 2 [24]. All other types of hierarchical lattices shown here have also their form-invariant distribution (that are not gaussians).

This suggests that we have a kind of central limit theorem for each HL, each one giving a different distribution, the gaussian case being one among others (but whithout doubt the more interesting).

The transformation given by equation (2a) has some interesting properties, which can be written as :

$$<(thK)^{2n}>_{P_s} = (<(thK)^{2n}>_P)^b \quad , \forall n \in \mathbf{Z} \qquad (2.3)$$

$$<\ln|thK|>_{P_s} = b<\ln|thK|>_P, \qquad (2.4),$$

valid for any distributions P . P_s is obtained from P by eq. (2.2a), and $<...>_P$ ($<...>_{P_s}$) is the average obtained with the distribution P (P_s).

III. Bounds

a) Inequalities

We shall first derive some inequalities for b=2. The extension for all b is straightforward.

We have, for each bond, a value of the exchange energy $K_{i\alpha}$. If two bonds are in serie we call K_m (K_M) the value of the exchange energy with lowest (greatest) absolute value. The resultant exchange energy of the two bonds in serie (K_s) satisfies the relation :

$$\tanh K_s = \tanh K_m . \tanh K_M \qquad (3.1)$$

Our basic inequality follows from the remark that $0 <|\tanh x|< 1$ which leads to :

$$\tanh^2|K_m| \le \tanh|K_s| \le \tanh|K_m| \qquad (3.2)$$

The equation (3-2) then gives :

$$\frac{\ln[\cosh(2|K_m|)]}{2} \le |K_s| \le |K_m| \qquad (3.3)$$

Now, the probability distribution for the minimum value of two random variables of distribution P is given by :

$$P_m(Km) = \int\int P(K1)P(K2)\delta(K\text{-}Min(K1,K2))dK1dK2$$

and can be calculated in each particular case, either analytically, or numerically .

Remark : Collet and Eckmann[22] have an inequality as (3.3) but they use $|K_m|-\ln(2)/2$ instead of $\ln[ch(2|K_m|)]/2$. In fact they use the first two terms of the low temperature ($|K_m| \to \infty$) expansion for $\ln[ch(2|K_m|)]/2$.

Using the property that the square of $\ln[ch(2|K_m|)]/2$ is a convex function of K_m^2, we obtain, taking the mean square of each member of equation (3.3), our main inequality :

$$\frac{\ln^2[ch(2\sigma_m)]}{4} \leq \sigma_s^2 \leq \sigma_m^2 \qquad (3.4)$$

where $\sigma_s^2 = < K_s^2 >_{P_s}$ and $\sigma_m^2 = < K_m^2 >_P$.

Let us remark that when we use dimensionless definition of the temperature (using $k_B=1$) and $<J^2>=1$ to fix the energy scale, one has : $\sigma=1/T$. This will be used for the rest of the paper .

b) Gaussian bounds

It is clear that the zero-centered gaussian distribution (GD) is certainly a good approximative distribution to P_r and this is so because the paralell transformation (2.2b),when repeted, brings almost all distributions to the gaussian one (central limit theorem).

To verify how the series transformation (2.2a) leads P_s away from a GD if the initial distribution (P) is a GD we take advantage of the fact that when σ_s is not too small (large T) P_s and P_m are very close together.(up to exponentialy small terms). Now we calculate the fourth over second moment ratio . We obtain (numericaly) :

$$\frac{<K_m^4>_{P_m}}{3<K_m^2>_{P_m}^2} \cong 1.13 \qquad (3.5a)$$

This moment's ratio is 1 for a GD

Moroever,we have numericaly estimated this quantity for the exact distribution P_r, constructed with the technics of section V . We have found :

$$\frac{<K_r^4>_{P_r}}{3<K_r^2>_{P_r}^2} \cong 1.05 \pm .1 \qquad (3.5b)$$

This shows that the series transformation (for b=2) does not bring P_s far away from a GD when the initial one is itself a GD. Clearly the final distribution P_r is even more close to a GD that P_s , justifying widely the utilisation of a GD as a (good) approximation of the true form-invariant distribution.

We will see another justification of this approximation with the Monte-Carlo results, in section V.

Within this approximation (i.e. that P, and P_r are GD but without any approximation for P_s.), we can obtain bounds for the variance of P_r (σ_r). If we multily each member of equation (3.4) by N (in this section b=2 thus $N=2^{d-1}$) we get :

$$N\frac{\ln^2[ch(2\sigma_m)]}{4} \leq N\sigma_s^2 \leq N\sigma_m^2 \qquad (3.6)$$

This, because :

$$N\,\sigma_s^2 = \sigma_r^2 \qquad \text{(central limit)} \quad (3.7)$$

where σ (σ_s, σ_r) is the variance of the gaussian distribution P (P_s, P_r).

Now, using :

$$\sigma_m^2 = 0.37\sigma^2 \qquad \text{(numerical)}, \quad (3.8)$$

the equation (3.6) can be written as :

$$N\frac{\ln^2[ch(2\sigma_m)]}{4\sigma^2} \leq \left(\frac{\sigma_r}{\sigma}\right)^2 \leq 0.37N \qquad (3.9)$$

The result is shown in figure 3 in terms of $T=1/\sigma$ and $Tr=1/\sigma_r$.

For N=4 (d=3) we can see that if $T<T^*$ then $(\sigma_r/\sigma)>1$ exhibiting the divergence of the variance (Spin glass phase). The critical temperature must satisfy $Tc\geq T^*$.

For N=2 $(\sigma_r/\sigma) < 1$ for all temperatures implying that $\sigma_r \to 0$ indicating the paramagnetic phase.

We have, then, a spin-glass phase in three dimensions but not in two, the lower critical dimension being $d_c \sim 2.3$ ($N \sim 2.5$).

Of course this numerical value depends of our GD approximation.

IV. Renormalisation-group

a) Discrete renormalisation group

In this section we shall fix the shape of the probability distribution.

However,insteed of a sum of Dirac's deltas we shall assume, in view of discussion in section III.b, that P, P_s and P_r are centred gaussian distributions, with recpective variances σ, σ_s and σ_r . Within this approximation equation (2.3) turns out to be (with n=1) :

$$\langle \tanh^2 K \rangle_{g(\sigma s)} = \langle \tanh^2 K \rangle^b_{g(\sigma)}\,, \qquad (4.1)$$

where $g(\sigma)$ is a gaussian distribution of width $\sigma = 1/T$.

Now,if we define the funtion f(T) as :

$$f(T) = \langle \tanh^2 K \rangle_{g(\sigma)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-x^2/2} \tanh^2\left(\frac{x}{T}\right) dx \qquad (4.2)$$

The equation (4.1) can be written as :

$$f(Ts) = f^b\,(T) \qquad (4.3)$$

As, from equation (3.7), $\sigma s = \frac{\sigma r}{\sqrt{N}}$,we have :

$$Ts = \sqrt{N}\, Tr \qquad (4.4)$$

The equations (4.3) and (4.4) give us a renormalisation equation for the temperature :

$$f(\sqrt{N}\, Tr) = f^b\,(T) \qquad (4.5)$$

Remark : We have a very simple approximation for the function f(T) as

$$f_{ap}(T) = \frac{1}{1+T^2+\sqrt{\frac{2}{\pi}}T} \quad , \qquad (4.6)$$

which gives the same high and low temperature asymptotic behaviour than f(T), namely :

$$f(T) \cong \begin{cases} 1-\sqrt{\frac{2}{\pi}}T & \text{if } T\text{---->} 0 \qquad (4.7a) \\ \frac{1}{T^2} & \text{if } T\text{---->}\infty \qquad (4.7b) \end{cases}$$

In figure 4 we compare the numerical forms of f(T) and f_{ap} (T).

The equations (4.5) and (4.7) give us the asymptotic behaviour of T_r as a function of T:

$$Tr \sim \begin{cases} \frac{b}{\sqrt{N}}T & \text{if } T\text{---->} 0 \qquad (4.8a) \\ \frac{1}{\sqrt{N}}T^b & \text{if } T\text{---->}\infty \qquad (4,8b) \end{cases}$$

In figure 5 we show, schematically, the renormalised temperature T_R as a function of T for d =3 , b=2 (N=4) and d=2 , b=2 (N=2).

For d=3 we obtain a critical temperature $T_c \sim 0.95$. Above this value : T_r -->∞(σ_r -->0), and below it,T_r -->0 (σ_r--> ∞).

This shows the existence of a spin-glass phase transition . The specific heat critical exponent α ($C \sim |T-Tc|^{-\alpha}$) is calculated and we find $\alpha \sim -10$, showing that there is no divergence in the specific heat at $T = T_c$, in accordance with the experimental results. For d=2, we do not obtain a SG phase, T_r--->∞ (σ_r--->0) for all T≠0 (paramagnetic phase).

The lower critical dimension is $d_{LCD} \sim 2.6$ (for the spin glass phase).

Remark : We also observe that in the limit d--> ∞ (for b fixed) then N-->∞ and $Tc \sim \sqrt{N}$ which is precisely the mean field spin glass [23] result.

b) Continuous renormalisation group.

We can analyse equation (4.5) when b-->1 (it is intuitive that if the number of series elements tends to one the gaussian approximation is better). If we write $b = 1+\varepsilon$, (ε -->o) then :

$$N \sim 1+\varepsilon(d-1) \qquad (4.9a)$$

$$f^b(T) \sim f(T)(\, 1+\varepsilon \ln f\,(T)) \qquad (4.9b)$$

and equation (4.5) can be writen as :

$$\lim_{\varepsilon \to 0} \frac{1}{f'(T)} \frac{f(Tr)-f(T)}{\varepsilon} \equiv \frac{\partial T}{\partial b} = -\frac{d-1}{2}\,T + \frac{f(T)\ln f(T)}{f'(T)}, \qquad (4.10)$$

where f' (T) means $\frac{\partial f}{\partial T}$

If we make $\frac{\partial T}{\partial b} = 0$ (scale invariance) we obtain the critical temperature $T_c(d)$.

The derivative of the last term of equation (4.10) with respect to the temperature, evaluated at $T = T_c$ (d), gives us the specific heat critical exponent . For d = 3 , we have :

$$Tc \sim 1 \qquad (4.11a)$$

and

$$\alpha \sim -6. \qquad (4.11b)$$

For d=2 we find Tc = 0.

V Numerical results

In order to test numericaly the preceeding results we have performed a randomized renormalisation group :

We consider a starting sample of 8^4 values of K : $K_i^{(4)}$, whose values are calculated according to a given law of probability P(K), which depends upon the temperature and on a possible parameter (see 3d-model) ; then, regrouping 8 by 8 the initial values of this sample, we calculate the 8^3 renormalised values of the coupling, $K_i^{(3)}$, and so on, and so on until we reach the last value $K_i^{(0)}$,.

This operation is repeated 100 times in order to obtain a statistically significant result for the last step of the renormalisation. Moroever,the stability of the results have been tested with a high statistical sample (1000).

At each step of the renormalisation we compute the effective value of the temperature, $T = \sqrt{1/<K^2>}$, and the mean value of the square hyperbolic tangent of the coupling : $t2= <\tanh^2 K>$. The result is ploted on a [t2-T] graph (figs. 6) where the f function has been drawn together with the transition line which seems to common to all models.

We have tried 5 different models, three of them had a continous probability distribution, and two were of a discrete type :

a-model : The starting probability is continous and constant over a certain range of K :

$$P(K) = N\,\theta(|K_0|-|K|)\ ,\ \text{with}\ N = 1/2K_0\ ,\ K_0 = \sqrt{3}/T_0, \qquad (5.2)$$

θ is the standart step function.

b-model : The starting probability distribution is continous and exponential :

$$P(K) = N\exp(|K|/K_0)\ \text{with}\ N= K_0/2\ ,\ K_0 = 1/\sqrt{2}T_0, \quad (5.1)$$

T_0 is the initial temperature of the model .

g-model : The starting probability is gaussian :

$$P(K) = \frac{T_0}{\sqrt{2\pi}} \exp(-K^2T_0^2/2) \qquad (5.3)$$

2δ-model : The starting probability is now discrete and is the proper vector of the serie operation (see section II) :

$$P(K) = \frac{\delta(K-K_0)+\delta(K+K_0)}{2}, \text{ with } K_0 = 1/T_0. \qquad (5.4)$$

3δ-model : We add to the two deltas model a non zero probability for K to be 0 : p (dilution probability)

$$P(K) = p\delta(K) + (1-p)\frac{\delta(K-K_0)+\delta(K+K_0)}{2}, \text{ with } K_0 = \frac{1}{T_0\sqrt{1-p}} \qquad (5.5)$$

Notice that for this last model the probability distribution depends on two parameters, which allows us to explorate a great part of the [t2-T] plane with the starting point of the renormalisation (fig. 6-3d).

We have first checked that our results are not sensitive to small non zero values of the mean value of the random variable <K> ; in fact, in this case, <K> goes rapidly to zero with the renormalisation . An extended study of the phase space [T,<K>] will be published elsewere .

Now the results are shown in figures 6-a to 6-3d, and exhibit the following trends :

-The limiting distribution (after 5 renormalisations) can not in general be distinguished from the theoretical curve of section IV, which exhibit the fact that the proper vector of the renormalisation group is very close to a standard gaussian .

-Each starting model has its own transition temperature : ~.85 for the g-model, ~.7 for the b-one, ~1 for the a-model and ~1.2 for the 2δ-model : The transition temperature for the 3δ-model depends on the α parameter and is 0 when one reach the percolation value : p = .68 (t2=.32) .

-The transition temperature for the two delta model is within the error bars of the heavy numerical results of Ogielsky et al [5] which have used the 2d-model on a real three dimensional lattice . This suggests that the Migdal Kadanoff approximation of the standart (3d) lattice is quite good in the spin-glass case .

VI. Conclusion

We formulate a short range interaction Ising spin-glass on hierarchical lattices (with N branches and b steps in each branch) and analyse the existence of a spin-glass phase. The hierarchical lattices are chosen in order to give the Migdal-Kadanoff approximation for hypercubic lattices when $N=b^{d-1}$.

We obtain some rigorous bounds for the variances of the final (renormalized) probability distributions in terms of the initial ones. These bounds allow us to show that the centered gaussian distribution for the quenched random exchange energy is a good approximation (at least for b=2) of the true form invariant probability distribution of the renormalisation transformation.

With the gaussian distributions we can estimate the bounds for the critical temperature Tc. In three dimensions we have $0.35 < Tc$, which prove without ambiguity, the existence of a spin-glass phase transition.

In two dimensions these bounds give Tc=0.

Using a property of the renormalisation transformation and the gausssian approximation, we define a discret renormalisation group (RG) on the temperature. This RG gives us the spin-glass phase at d = 3 (Tc ~0.85) but not in two dimensions. The specific heat critical exponent is calculated (α is negative) and the lower critical dimension is $d_c \sim 2.5$.

A continuous version of this RG is performed which gives the same qualitative results.

Finally we make a Monte-Carlo RG on hierarchical lattices. We chose some distributions and follow their *trajectory* in a adequate space under the MCRG transformation. We observe that almost all distributions tend to the same invariant line in the [t2-T] space (the f function defined in IV).

In three dimensions there is a saadle-node fixed point on this line (it gives the critical temperature for the form-invariant distribution). Its stable manifold gives us the critical surface separating the spin-glass phase from the paramagnetic one. The unstable eigenvalue gives us the critical exponent α. The *trajectory* of the gaussian distribution is practically on the invariant line supporting the gaussian approximation.

Also, these results remain unchanged if we take a distribution with a nonzero (small) mean value showing the stability of the fixed points.

In two dimensions there is no sadle node fixed point. There is no spin-glass phase in this case.

Therefore, by differents methods (bounds, discret and continuous RG and MCRG) we have consistently obtained the existence of a spin-glass phase in three dimensions but not in two.

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FIGURES CAPTION

Figure 1: Schematic representation of an hypercubic latice with N parallel links made of b bonds .

Figure 2 : The renormalisation process associated with the central limit theorem .

Figure 3 : Upper and lower bounds of the σ_r/σ ratio as a function of the temperature ; upper curves : 3 dimentional case, lower curves : 2 dimensions .

Figure 4 : True and approximate function f(T)

Figure 5 : The renormalised temperature as a function of T ; upper curve ,d=2, lower curve d=3 .

Figures 6 :The numerical renormalisation process in the [t2-T] plane for various models . The continuous line is the f function, the dasched line is the transition line for all models .

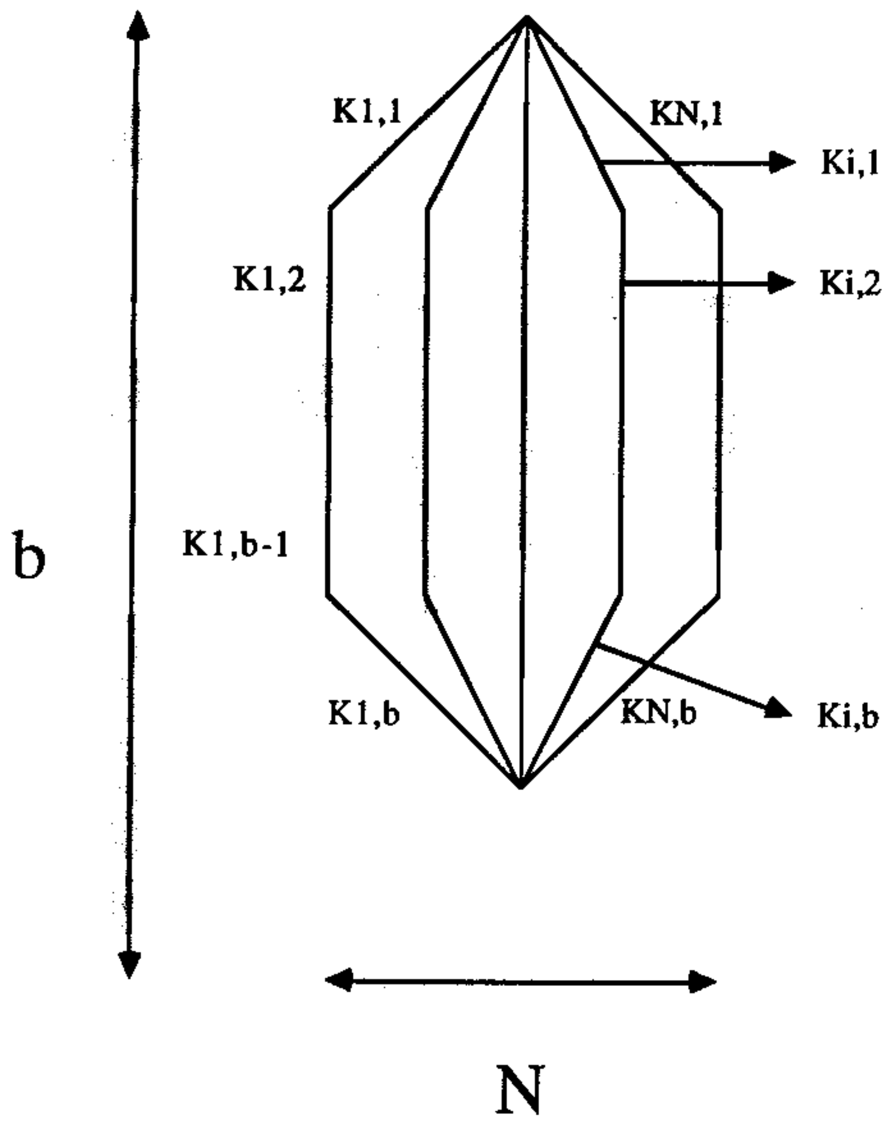


figure 1

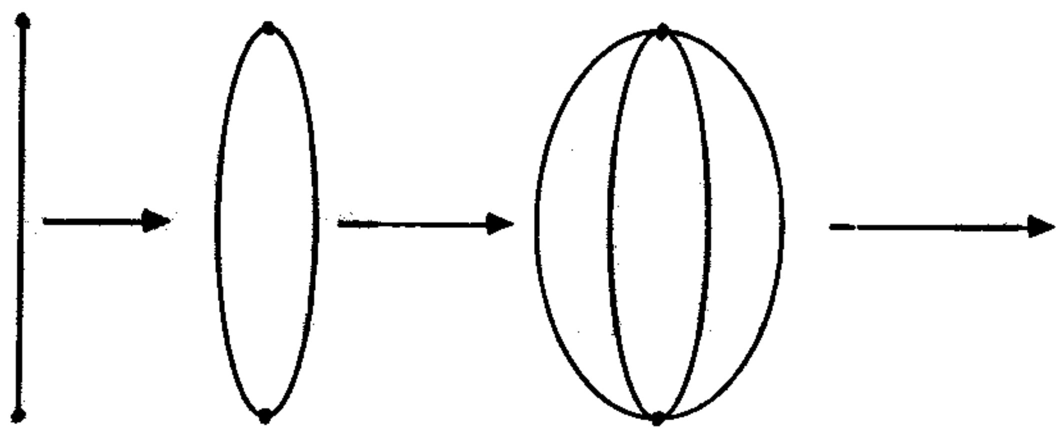
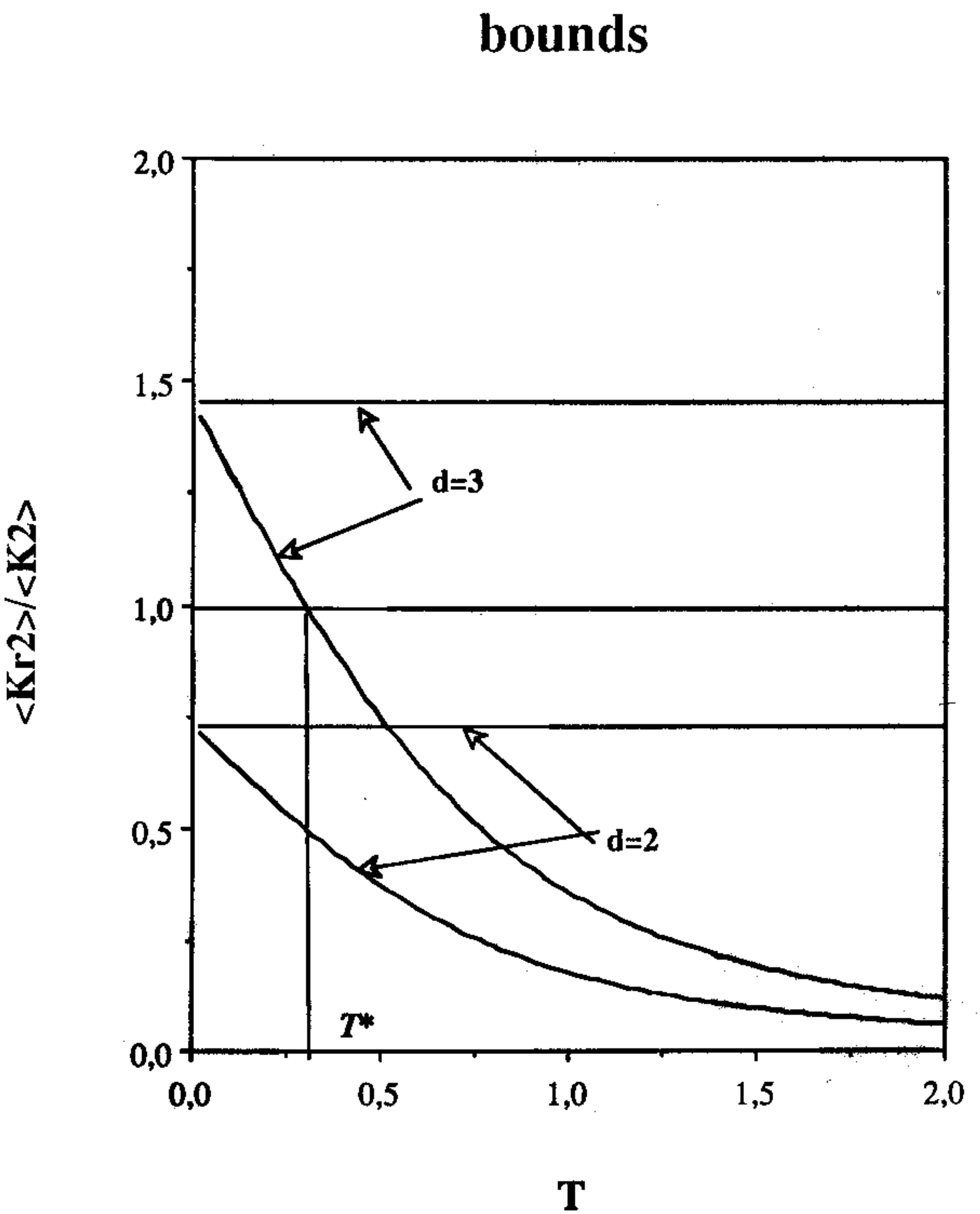


figure 2



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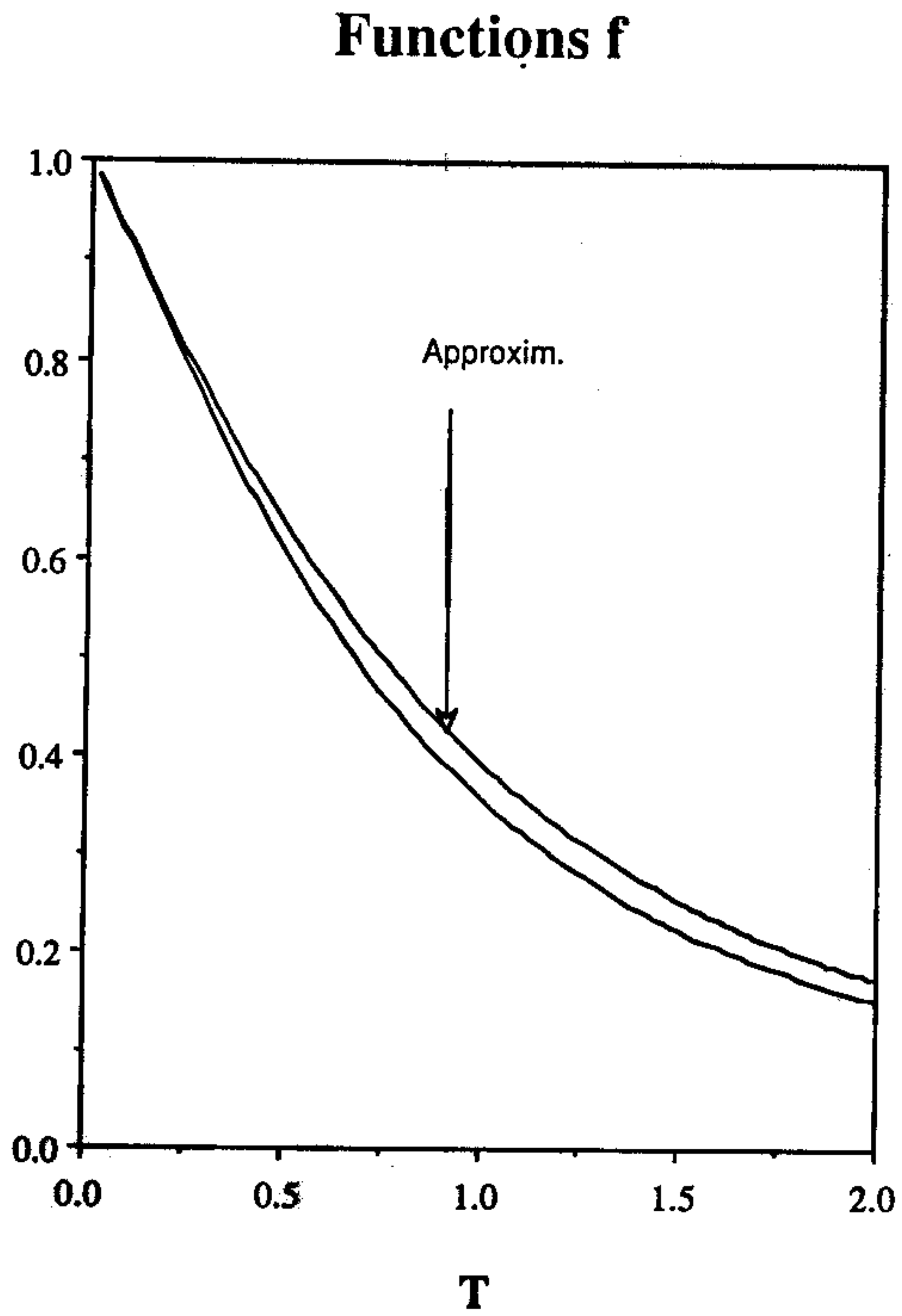

figure 4

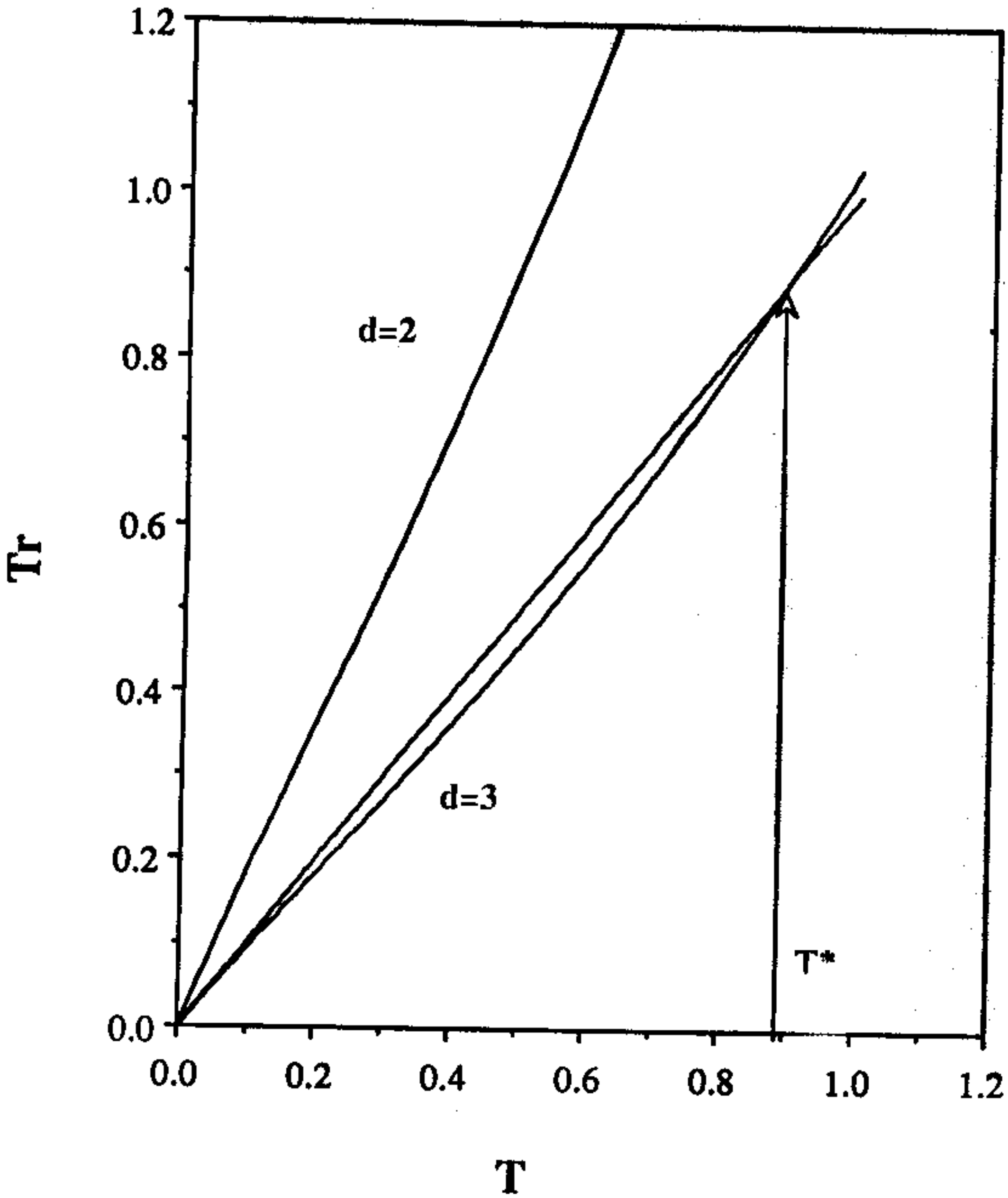

figure 5

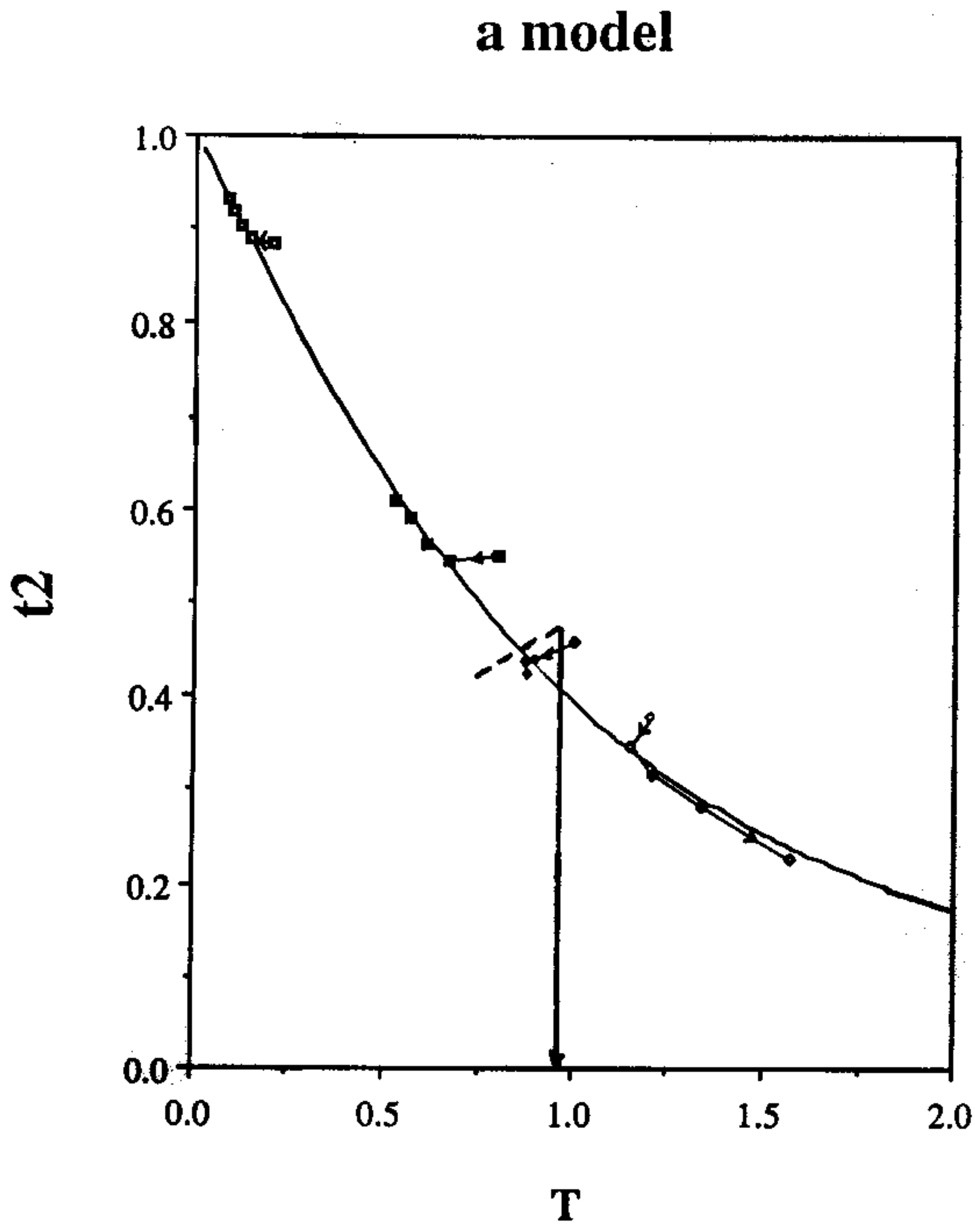

figure 6-a

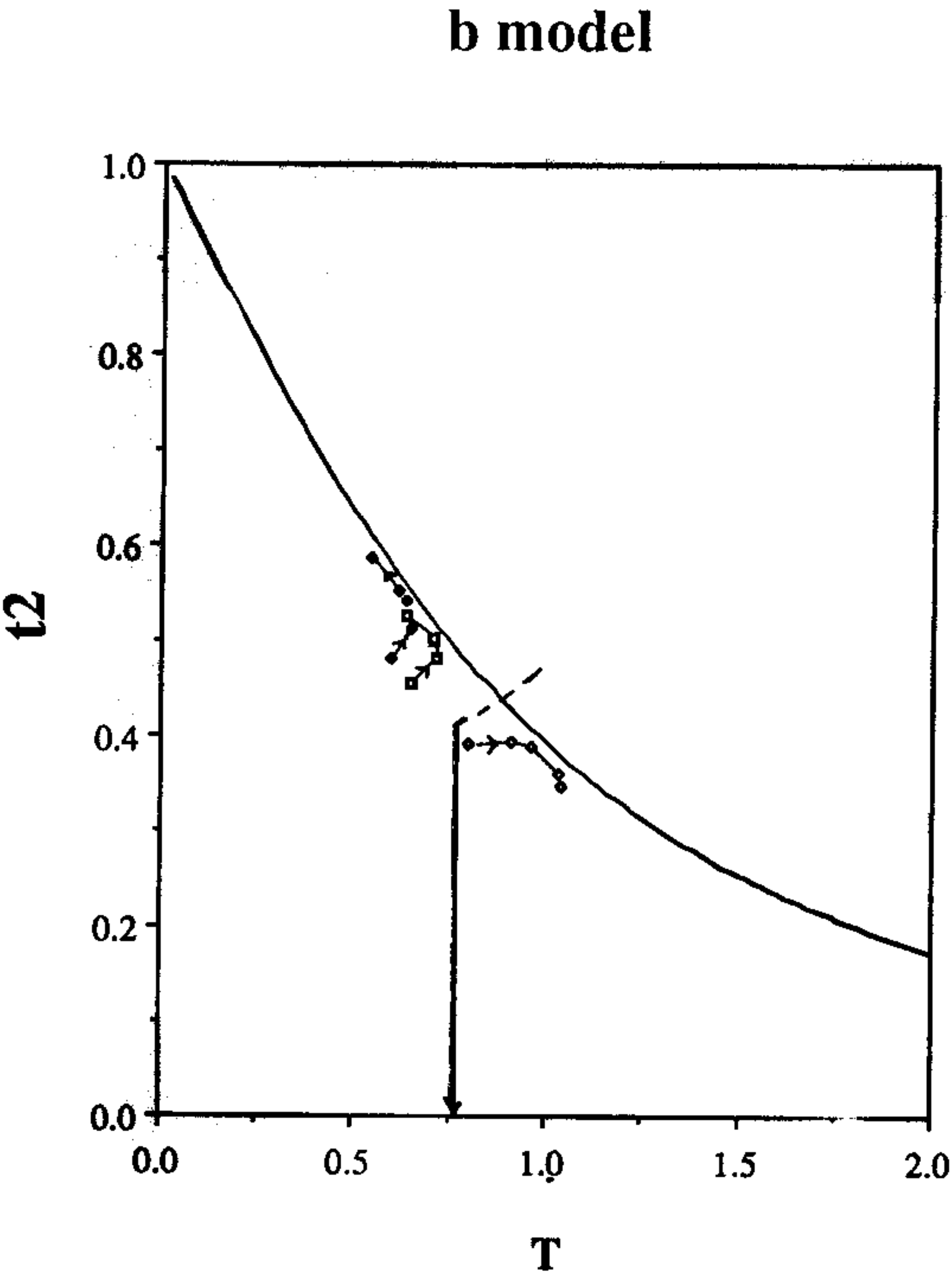

figure 6-b

g model

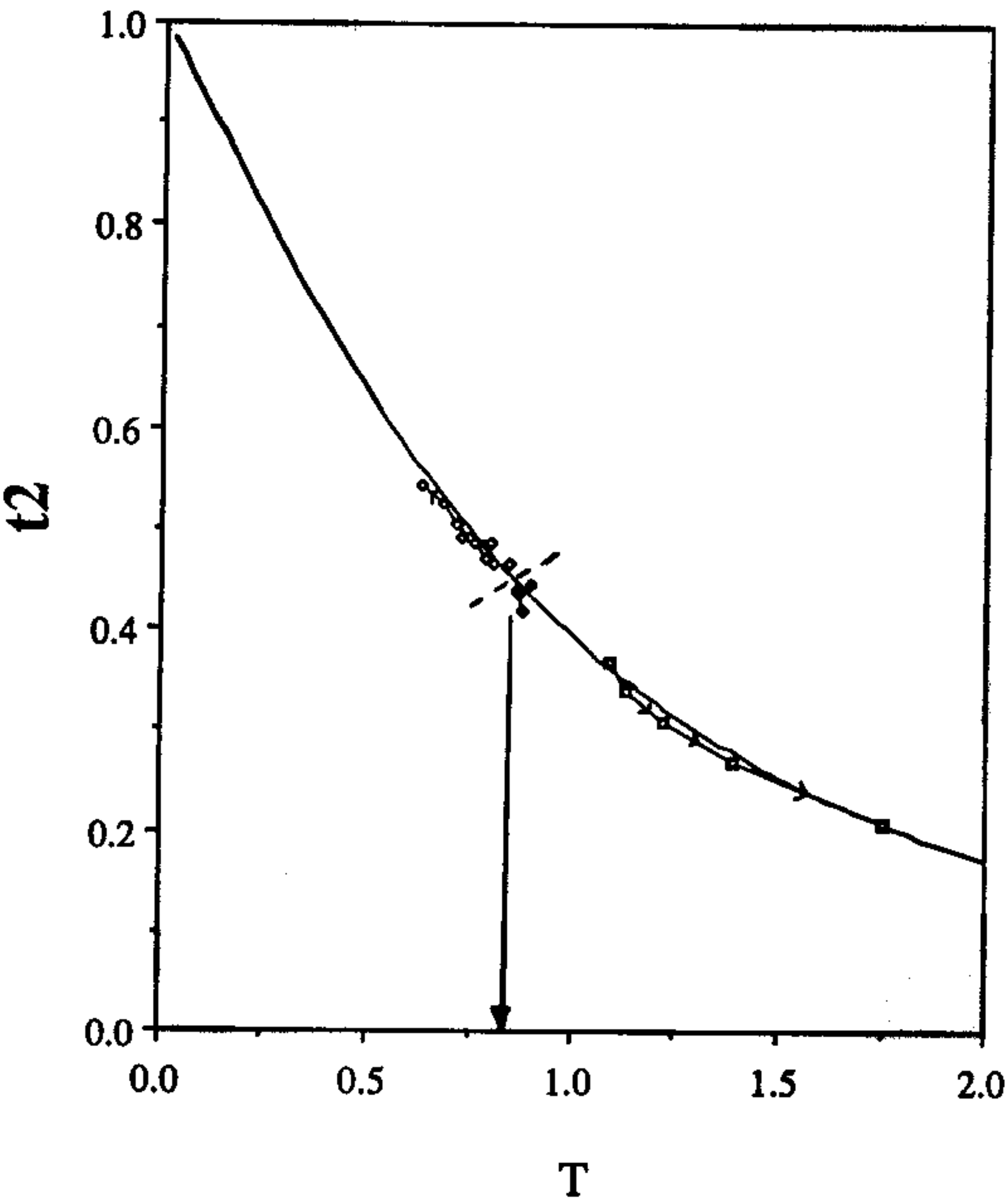

figure 6-g

2δ

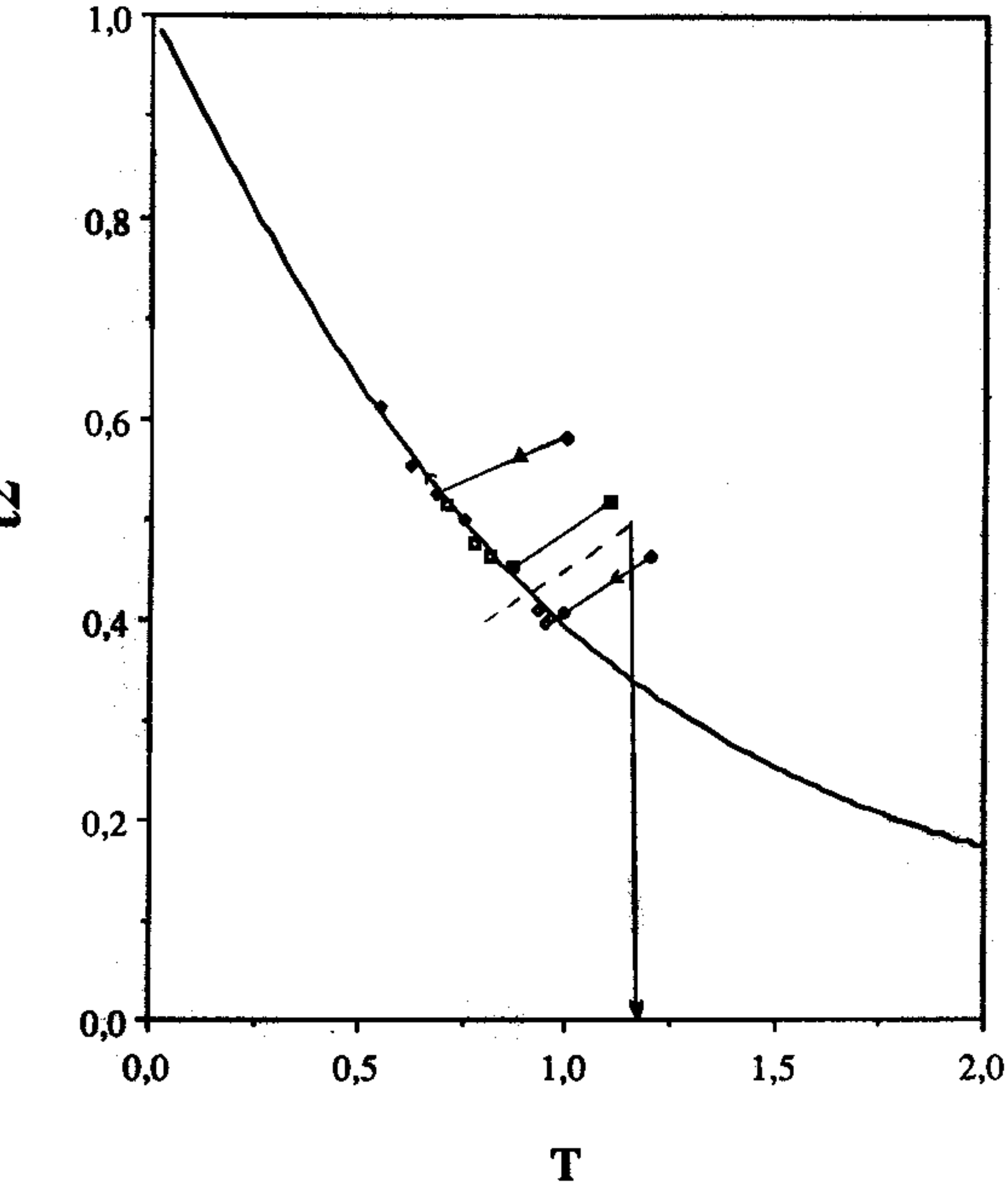

figure 6-2d

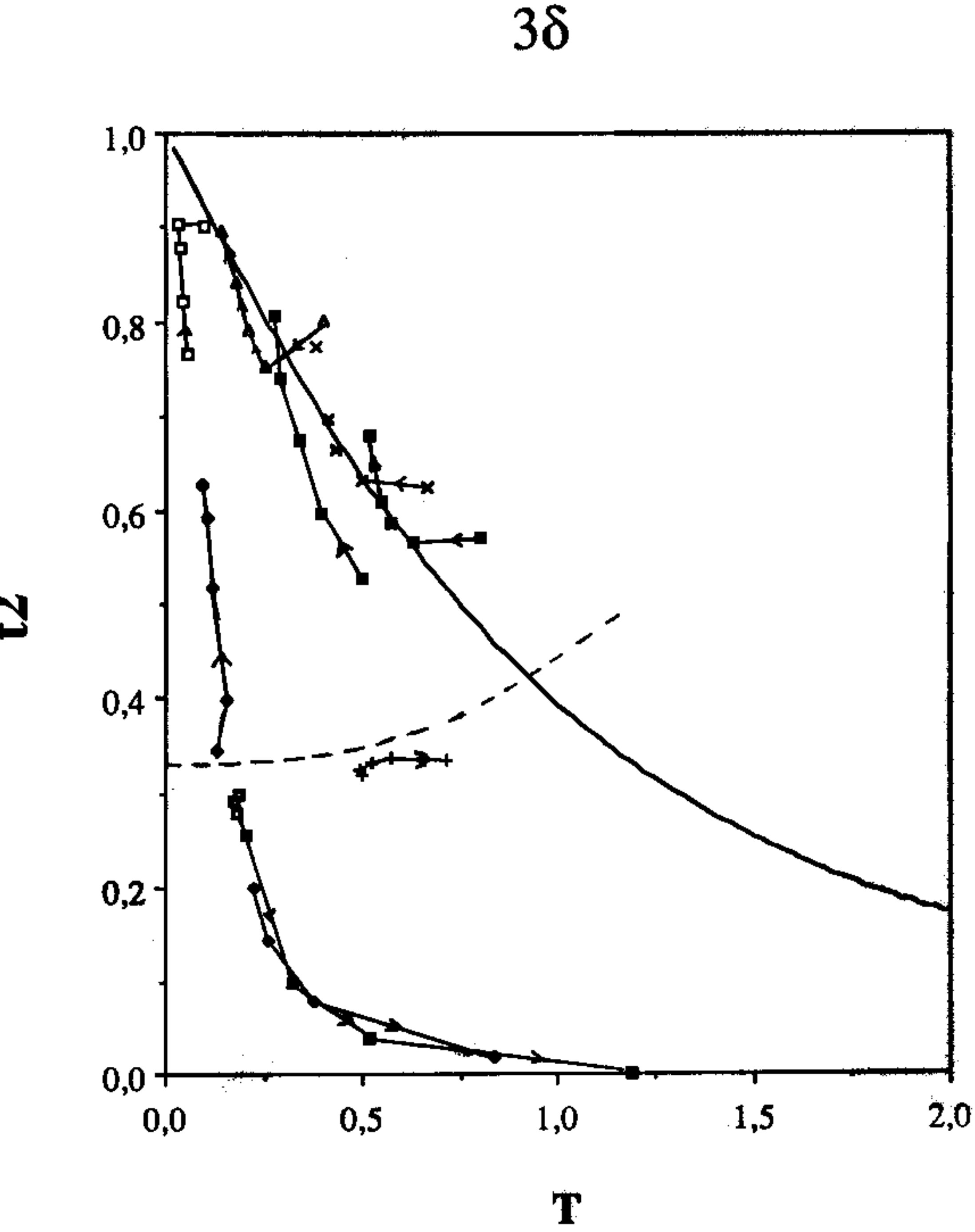

figure 6-3d

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