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CONTRIBUTION OF EXCHANGE CURRENTS TO THE ELASTIC πD SCATTERING IN THE Δ_{33} RESONANCE REGION

by

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Abstract

We calculate, using Feynman diagrams, the contribution of pion exchange currents to the $\pi D \rightarrow \pi D$ scattering in the region where the Δ_{33} resonance is dominant. The results show that the contribution of this mechanism to the differential cross-section, although small, is comparable with that of the double scattering.

Key-words: Deuteron; Pion; Exchange currents; Elastic scattering; Differential cross-section, Intermediate energy.

1. Introduction

Most of theoretical models used to describe the πD elastic scattering are based essentially either on the Faddeev Integral Equations [1] or on the Multiple Scattering Series [2]. In the last years and in parallel to the increasing sophistication of the theoretical models [3], a great number of experiences have been performed generating measures of high precision [4]. However, in the region of intermediate energies, neither models of Faddeev type nor models of Multiple Scattering Series type succeed to describe satisfactorily the experimental data for energies higher than 180 MeV and large scattering angles [5], seeming to indicate that other mechanisms like the baryonic resonances [6], the absortion of pions [7] or the exchange currents [8], can become important. The first two mechanisms, when included to Faddeev type models, don't make the agreement between theoretical and experimental curves better [9]. The role of the exchange currents has been analised over all in the region near threshold, aiming to determine the scattering lenght [10], while there exist very few articles in concerning the energy region between the threshold and the resonances [11,12]. The goal of this paper is to estimate the relevance of the exchange currents in the region where the Δ_{33} resonance is dominant.

Besides the diagrams of single (SS) and double scattering (DS) (see fig. 1), usually considered, we will calculate the current exchange diagrams (EC) containing at least one Δ_{33} resonance [11] (see fig. 2). The difficulty in calculating these EC diagrams forced us to make certain approximations, mainly concerning the internal loop integrals. In order to estimate coherently the relative weight of each diagram we will mantain basically the same set of approximations also for the single and double scattering diagrams. We will use as starting point the impulse approximation [13] and a semi relativistic formalism in which the scattering amplitudes are calculated from covariant expressions, while the proton-neutron-deuteron (pnd) vertex is decribed via non-relativistic phenomenological wave function.

In Section 2 we present the expressions for the amplitudes and explicit the approximations used to calculate the differential cross-section. Our results are discussed in section 3, while in the Appendix we present the details of the calculus sketched in section 2.

2. Scattering amplitude

Usually, in reactions involving one deuteron, the calculations are performed in the rest frame of the deuteron, where the non-relativistic limit for the pnd vertex is taken. Here, as we have deuteron in the initial (D) and in the final state (D') we will use the Breit (or brick wall) frame, that treats the deuterons symmetrically:

$$\vec{D} = - \vec{D}' = \vec{\Delta}$$

where $\vec{\Delta}$ is the momentum of the incident deuteron.

We outline here the calculations for the EC diagrams. The amplitude for the SS and DS are obtained in a very similar way and will be given at the end of this section.

The quadrimomenta of the particles are defined in figs. 1 and 2. We take a pseudoscalar (γ_5) coupling for the pion-nucleon-nucleon (πNN) and pion-delta-delta ($\pi\Delta\Delta$) vertices. Form factors are introduced according to Wolf prescription [14].

The covariant amplitude corresponding to the $N\Delta$ diagram (fig. 2a) is

$$T^{N\Delta}_{\lambda'\lambda} = -\iint \frac{d^4f_1}{(2\pi)^4}\cdot\frac{d^4f_2}{(2\pi)^4}\,\mathrm{Tr}\Big[\frac{i(\not{N}'+m)}{(N'^2-m^2)}\, g^2_{\pi N\Delta}\,\frac{1}{(M_\Delta^2-Q'^2)}\,K'^{\nu}S_{\nu\mu}(Q')K_i^{\mu}$$

$$\frac{i(\not{P}+m)}{(P^2-m^2)}\, g_{\pi NN}\,\gamma_5\,\frac{i(\not{N}+m)}{(N^2-m^2)}\, g_{pnd}\,\frac{\Gamma.\varepsilon_\lambda}{\sqrt{2}}\,\frac{i(-\not{M}+m)}{(M^2-m^2)}$$

$$g_{\pi NN}\gamma_5\,\frac{i(-\not{M}'+m)}{(M'^2-m^2)}\, g_{pnd}\,\frac{\Gamma.\varepsilon_\lambda{}'}{\sqrt{2}}\,\frac{i}{(K_i^2-\mu^2)}\Big]$$

where $m(\mu)$ is the mass of the nucleon (pion) and $M_\Delta = m_\Delta - i\Gamma_\Delta/2$, $m_\Delta = 1232$ MeV being the resonance mass and Γ_Δ, its width [14].

The quantities $g_{\pi NN}$, $g_{\pi N\Delta}$ and g_{pnd} are, respectively, the pion-nucleon-nucleon, pion-nucleon-delta and proton-neutron-deuteron coupling constants; ε_λ and Γ are the polarization and the relativistic vertex of the deuteron:

$$\Gamma_\mu = F_1(N^2;M^2)\gamma_\mu + F_2(N^2;M^2)f_\mu$$

where the form factors F_1 and F_2, are normalized such as $F(m^2,m^2) = 1$ [15].

The operator $S_{\mu\nu}$ is the spin part of the Rarita-Schwinger propagator [16].

$$S_{\mu\nu}(Q) = \frac{1}{3}(\not{Q} + m_\Delta)\left[\frac{2}{m_\Delta^2} Q_\mu Q_\nu - 3g_{\mu\nu} + \gamma_\mu\gamma_\nu + \frac{1}{m_\Delta}(\gamma_\mu Q_\nu - \gamma_\nu Q_\mu)\right]$$

We have chosen as loop variables the momenta relative to Fermi motion:

$$f_1 = \frac{1}{2}(N - M)$$

$$f_2 = \frac{1}{2}(N' - M')$$

As the deuteron is composed of two weakly bound nucleons, we will consider them as free in the spin factor. Moreover, in order to perform the integral over the "energy" $f_{o,i}$ we will use the Gross approximation [17] and put one of them on-mass-shell. This corresponds to make the following substitutions:

$$\not{N} + m = 2m \sum_n \bar{u}_n(N) u_n(N)$$

$$\frac{1}{M^2 - m^2} \rightarrow - i\pi \frac{1}{M_o} \delta(M_o - \sqrt{m^2 + \vec{M}^2})$$

Taking the non-relativistic limit for the pnd vertex [18] we have

$$g_{pnd}\bar{u}_n(N)\frac{1}{(N^2 - m^2)} \frac{\gamma \cdot \varepsilon_\lambda}{\sqrt{2}} v_m(M) = \frac{1}{2m}(32\ m\pi^3)^{1/2} \psi_\lambda^{n,m}(\vec{f})$$

where $\psi_\lambda^{n,m}(\vec{f})$ is the phenomenological wave-function of the deuteron, for which we use the McGee parametrization [19].

With the approximations above, the amplitude become

$$T^{N\Delta}_{\lambda'\lambda} = -\frac{m^3}{2\pi^3} \sum_{spins} \iint d^3f_1 d^3f_2\, \psi^{*n'm'}_{\lambda'}(\vec{f}_2) F_{m'm} G_{n'n} \psi^{nm}_{\lambda}(\vec{f}_1)\,\cdot$$

$$\cdot \frac{1}{(m^2+\vec{M}^2)^{1/2}} \cdot \frac{1}{(m^2+\vec{M}'^2)^{1/2}} \cdot \frac{1}{(K_i^2-\mu^2)} \cdot \frac{1}{(P^2-m^2)} \cdot \frac{1}{(M_\Delta^2-Q'^2)} \qquad (2.1)$$

where

$$F_{m'm} = g_{\pi NN} \bar{v}_m(-M)\gamma_5 v_{m'}(-M') \qquad (2.2)$$

$$G_{n'n} = g_{\pi NN}\, g^2_{\pi N\Delta}\, \bar{u}_{n'}(N')K'^{\mu} S_{\mu\nu}(Q')K_i^{\nu}(\not{P}+m)\gamma_5 u_n(N) \qquad (2.3)$$

As the wave-function has a rapid exponential fall-off we will take out the integral the spin factors, evaluating them for some value of $\vec{f}_i\,(i = 1,2)$ appropriately chosen [20]. Here, we take $\vec{f}_1 = \vec{f}_2 = 0$ [12]. So, the amplitude reads:

$$T^{N\Delta}_{\lambda'\lambda} = -\frac{m^3}{2\pi^3} \frac{1}{(m^2+\vec{\Delta}^2/4)} \sum_{spins} F_{m'm} G_{n'n} I^{m'm;n'n}_{\lambda'\lambda} \qquad (2.4)$$

where

$$I^{m'm;n'n}_{\lambda'\lambda} = \iint d^3f_1 d^3f_2\, \psi^{*m'n'}_{\lambda'}(\vec{f}_2)\psi^{mn}_{\lambda}(\vec{f}_1) \frac{1}{(P^2-m^2)} \frac{1}{(M_\Delta^2-Q'^2)} \frac{1}{(K_i^2-\mu^2)} \qquad (2.5)$$

The detailed calculations of $F_{m'm}$, $G_{n'n}$ and $I_{\lambda'\lambda}$ are given in the Appendix.

The amplitudes corresponding to the other EC diagrams are also given by (eq. 2.4) provided we make the following substitu tions:

Diagram ΔN (fig. 2b)

$$G_{n'n} = g_{\pi NN} g^2_{\pi N\Delta} \bar{u}_{n'}(N')\gamma_5(\not{P}' + m) K^{\mu}_{i} S_{\mu\nu}(Q) K^{\nu} u_n(N)$$

Diagram ΔΔ (fig. 2c)

$$G_{n'n} = g^2_{\pi N\Delta} g_{\pi\Delta\Delta} \bar{u}_{n'}(N') K'^{\mu} S_{\mu\alpha}(Q')\gamma_5 S^{\alpha\nu}(Q) K_{\nu} u_n(N)$$

and $1/(M^2_{\Delta} - Q^2)$ instead of $1/(P^2 - m^2)$ in (eq. 2.5)

For the pion-delta-delta ($g_{\pi\Delta\Delta}$) coupling constant we take [21]

$$\frac{g_{\pi\Delta\Delta}}{4\pi} = 20$$

The amplitude for the SS calculated with the same set of approximations is

$$T^{SS}_{\lambda'\lambda} = -4m \sum_{spins} \delta_{m'm} G_{n'n} I^{m'm;n'n}_{\lambda'\lambda}$$

where $\delta_{m'm} = 1(0)$ if $m' = m(m' \neq m)$ and

$$G_{n'n} = \bar{u}_{n'}(N') \left[A + B\left(\frac{\not{K} + \not{K}'}{2}\right) \right] u_n(N) \qquad (2.6)$$

$$I^{m'm;n'n}_{\lambda'\lambda} = \int d^3 f \psi^{*m'n'}_{\lambda'}(\vec{f} - \vec{\Delta}/2) \psi^{mn}_{\lambda}(\vec{f} + \vec{\Delta}/2)$$

with $G_{n'n}$ being the pion-nucleon(πN) amplitude [22] (see Appendix) and $I_{\lambda'\lambda}$ the deuteron form factor [23]. We remove $G_{n'n}$ from the integral $I_{\lambda'\lambda}$ for $\vec{f} = \vec{\Delta}/2$, in order that the quantization axis for the πN interaction coincides with the one of πD reaction [12] $(\vec{N} = -\vec{N}' = \vec{\Delta})$.

The covariant amplitude corresponding to DS, with our approximations, reads

$$T^{DS}_{\lambda'\lambda} = i\,\frac{m^3}{2\pi^3}\,\frac{1}{(m^2 + \vec{\Delta}^2/4)}\,\sum_{spins} G_{m'm}\,G_{n'n}\,I^{m'm;n'n}_{\lambda'\lambda}$$

where $G_{m'm}$ and $G_{n'n}$ are the πN amplitudes and

$$I^{m'm;n'n}_{\lambda'\lambda} = \iint d^3f_1\,d^3f_2\,\psi^{*m'n'}_{\lambda'}(\vec{f}_2)\,\psi^{mn}_{\lambda}(\vec{f}_1)\,\frac{1}{(K_1^2 - \mu^2)} \qquad (2.7)$$

This integral is calculated in the Appendix. For the DS we remove the πN amplitudes from the integral for $\vec{f}_1 = \vec{f}_2 = 0$.

Finally, the differential cross-section in the center of mass system is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 S}\,\sum_{\lambda\lambda'}\left|T^{SS}_{\lambda'\lambda} + T^{DS}_{\lambda'\lambda} + T^{EC}_{\lambda'\lambda}\right|^2$$

where $T^{EC}_{\lambda'\lambda}$ is the sum of the EC amplitudes and $S = (K + D)^2$.

In our formulae the isospin factors are implicit.

3. Conclusions

We show in fig. 3 and 4 the differential cross sections corresponding to the contributions of the SS (continuous curve), DS (dashed curve) and EC (dash-dotted) at the energies of T_{π}^{lab} = 254 MeV and 292 MeV respectivelly. As expected, the single scattering is dominant for almost all the angular range from $\theta = 0^0$ to 180^0. However, as one can see, at these energies and large angles ($\theta > 50^0$) the exchange currents give contributions of the same order or even greater than the double scattering.

The angular behaviour of the EC differential cross-section (figs. 3 and 4, dotted curve) can be understood as follows: the EC amplitude has a spin factor $F_{m'm}$ (eq. 2.2) proportional to the momentum transfer that vanishes in the forward direction (see Appendix). Besides that, the spin factor $G_{n'n}$ (eq. 2.3) has a spin-flip part G_{+-} cancelled for the diagrams $N\Delta$ and ΔN and small for the diagram $\Delta\Delta$, while the spin-non-flip parts G_{++} are all proportional to the vector $\vec{S}$ which is normal to $\vec{\Delta}$ and zero at $\theta = 180°$ (see Appendix). This explain why the EC contribution is significative mainly in the region $\theta \sim 90°$.

We show in figs. 5 and 6 the experimental data of reference [4] at $T_\pi = 254$ MeV and 292 MeV and the differential cross-sections obtained adding coherently to the SS (continuous curve) the DS (dashed curve) and the DS + EC (dash-dotted). As we see the cross-section is dominated by the single scattering but the inclusion of the DS and of the EC introduces up to 20% of change at large angles. The main effect of the EC is to produce a steeper cross-section in the region where the minimun occurs ($\theta \sim 90^0$). The discrepancy between our results and the experimental data, even at small angles, is due to the approximation used for the loop integral in the single scattering amplitude. In fact it is well know [24] that at these energies, to fit the data it is very important to keep the πN amplitude in the integral (2.6). However as our goal was to estimate the weight of the EC relative to the SS and DS we decided to choose the same set of approximations for all diagrams. A more refined model for the SS, which is overestimated here, will certainly increase the role of the EC.

Finally we point out that the inclusion of the deuteron D-wave in the EC amplitude (eq. 2.5) affects very little the results [25] (see Appendix).

Appendix

A - Spin amplitudes

Pion-nucleon (πN)

The spin-non-flip (G_{++}) and spin-flip (G_{+-}) parts of the πN amplitude (eq. 2.6) in the Breit frame are given by [25]

$$G_{++} = \frac{N_o}{m}(A + m\,\frac{K_o}{N_o}\,B)$$

$$G_{+-} = -\,i\,\frac{|\vec{S}|\vec{\Delta}|}{m}\,B$$

where A and B are the Dirac invariant amplitudes [22], K(N) is the pion (nucleon) quadrimomentum and $\vec{S} = \vec{K} + \vec{N}$.

We choose the z axis as the direction of $\vec{N}$ and the y axis as the direction of $\vec{S}$.

Exchange current (EC)

For the spin amplitudes $F_{m'm}$ and $G_{n'n}$ defined by eq. 2.2 and eq. 2.3 we have (without the coupling constants)

$$F_{n'n} = \bar{u}_{n'}(M')\gamma_5 u_n(M) = \frac{|\vec{\Delta}|}{2m}\,\chi^+_{n'}\sigma_z\chi_n$$

where $|\vec{\Delta}|$ is proportional to the momentum transfer t:

$$|\vec{\Delta}| = -\frac{t}{4m}$$

and

$$G_{n'n} = C_1 \bar{u}_{n'}(N')\gamma_5 u_n(N) + C_2 \bar{u}_{n'}(N')K'\gamma_5 u(N)$$

or

$$G_{n'n} = C_1 \frac{|\vec{\Delta}|}{2m} \chi^+_{n'}\sigma_z\chi_n - C_2(|\vec{S}| \frac{N_o}{2m} \chi^+_{n'}\sigma_y\chi_n + |\vec{\Delta}|\chi^+_{n'}\sigma_z\chi_n)$$

where $\chi(\sigma)$ is the Pauli spinor (matrix); C_1 and C_2 have different expressions according to the EC diagram considered. For the NΔ diagram (see fig. 2a):

$$C_1 = \frac{1}{3}[4mC_3 + (\mu^2 + 4K.N)C_4 + (2K'.N + 4N.N)C_5 + (m\mu^2 + 4mK.N)C_6]$$

$$C_2 = \frac{1}{3}[C_3 + mC_5 + (2K'.N + 4N'.N - 4m^2)C_6]$$

where

$$C_3 = \frac{\mu^2}{m_\Delta}(Q.K_i - mm_\Delta) + \frac{Q.K_i}{m_\Delta}[m(m+m_\Delta) + 2K.N] + (m+m_\Delta)C_7$$

$$C_4 = C_7 + 2K.N - \frac{m}{m_\Delta} Q.K' + \frac{(m+m_\Delta)}{m_\Delta}(mm_\Delta + Q.K_i)$$

$$C_5 = \mu^2 - \frac{(m+m_\Delta)}{m_\Delta} Q.K'$$

$$C_6 = (m+m_\Delta) - \frac{1}{m_\Delta} Q.K'$$

$$C_7 = \frac{2}{m_\Delta^2}(Q.K')(Q.K_i) - K'.K_i - 2K'.N$$

For the ΔN diagram (see Fig. 2b) we have the following relations:

$$G_{++}^{\Delta N} = G_{++}^{N\Delta}$$

$$G_{+-}^{\Delta N} = - G_{+-}^{N\Delta}$$

For the $\Delta\Delta$ diagram (see Fig. 2 c):

$$C_1 = \frac{1}{9}(C_3 - mC_4 + mC_5 - m^2 C_6)$$

$$C_2 = \frac{1}{9}(C_7 - mC_8 + mC_9 - m^2 C_{10})$$

where, now,

$$C_3 = C_{11} + 2(K.N)C_{12} + (m + m_\Delta)C_{13}$$

$$C_4 = -\frac{4(K.N)}{m_\Delta^2}(3K.Q + 2m_\Delta^2) + (m + m_\Delta)C_{12} + C_{14}$$

$$C_5 = (m + m_\Delta)C_{12} - C_{15}$$

$$C_6 = \frac{4}{m_\Delta}[K.Q - (m + m_\Delta)m_\Delta]$$

$$C_7 = (m + m_\Delta)C_{11} + \mu^2 C_{13} + 2(K.N)C_{15}$$

$$C_8 = -\frac{4(K.N)}{m_\Delta^2}(K.Q)(3m + 5m_\Delta) + \mu^2 C_{12} + (m + m_\Delta)C_{14}$$

$$C_9 = -\mu^2 C_{12} + (m + m_\Delta)C_{15}$$

$$C_{10} = \frac{2(K.Q)}{m_\Delta^2}(3\mu^2 - 3m^2 - 5m_\Delta^2 - 8mm_\Delta) + 4\mu^2$$

with

$$C_{11} = -(2K'.N + 4N'.N - 4m^2)C_{16} - (\mu^2 + 4K.N)C_{17} + (m + m_\Delta)C_{18} - 2C_{19}$$

$$C_{12} = \frac{2}{m_\Delta}(2Q'.Q - K.Q) + \frac{(m + m_\Delta)}{m_\Delta^2}(2Q'.Q - 7K.Q + 4m_\Delta^2) + C_{16} + 2C_{17}$$

$$C_{13} = -\frac{1}{m_\Delta^2}(2K'.N + 4N'.N - 4m^2)(2Q'.Q - 7K.Q + 4m_\Delta^2) + (m + m_\Delta)C_{17} - C_{18} - 2C_{20}$$

$$C_{14} = \frac{1}{m_\Delta^2}\Big[4K.Q(3m^2 - 2Q'.Q) + (\mu^2 + 4K.N)(2Q'.Q - 7K.Q + 4m_\Delta^2)$$

$$- 2m_\Delta^2(6K'.Q - 4K.N - \mu^2)\Big] + (m + m_\Delta)C_{16} + 2C_{18}$$

$$C_{15} = \frac{1}{m_\Delta^2}\Big[6K.Q(2K'.N + 4N'.N - 2m^2) - (\mu^2 + 4K.N)(2Q'.Q - 7K.Q$$

$$+ 4m_\Delta^2) - 4K.Q(2Q'.Q - K.Q) + 12m_\Delta^2 K'.Q\Big] - (m + m_\Delta)C_{16} - 2C_{18}$$

$$C_{16} = -\frac{1}{m_\Delta^3}\Big[2K.Q(K.Q + 2Q'.Q) + m_\Delta^2(2K.Q - 2K'.Q + 4K'.N - 8K.N - 5\mu^2\Big]$$

$$C_{17} = \frac{4}{m_\Delta}(2N.N + K'.N - K.N - 2m^2)$$

$$C_{18} = \frac{2(k.Q)}{m_\Delta^4}\Big[2(K.Q)(Q'.Q) - m_\Delta^2(2K.Q' + 2K.Q - K.N - 5K'.N - 2N'.N$$

$$+ \mu^2 - m^2\Big] + \frac{1}{m_\Delta^2}\Big[(Q'.Q + m_\Delta^2)(\mu^2 - 4K.N) + K'.K(5m_\Delta^2 - 2Q'.Q)\Big]$$

$$C_{19} = \frac{1}{m_\Delta}\Big[2K.Q(2K.Q - 3K.N + K'N + 2N'N - \mu^2 - 3m^2) + Q'.Q(\mu^2 + 4K.N)\Big]$$

$$C_{20} = \frac{1}{m_\Delta^2}\left[2K.Q(Q'.Q - K.Q) - m_\Delta^2(K'.Q - K.Q + 4N'.N + \mu^2 - 6m^2)\right]$$

B - Integrals

Double Scattering (DS)

In the DS amplitude we have to calculate the integral (eq.2.7)

$$I_{\lambda'\lambda} = \int d^3f_1 d^3f_2 \psi^*_{\lambda'}(\vec{f}_2)\psi_\lambda(\vec{f}_1)\frac{1}{K_i^2 - \mu^2}$$

Taking

$$\vec{f}_1 = \frac{\vec{q}_1}{2} - \vec{q}_2$$

$$\vec{f}_2 = -\frac{\vec{q}_1}{2} - \vec{q}_2$$

we have, in the coordinate space,

$$I_{\lambda'\lambda} = 2\pi^2\int |\vec{r}|e^{i|\vec{K}||\vec{r}|}J(|\vec{r}|)d|\vec{r}|$$

where

$$J(|\vec{r}|) = \int e^{i\vec{S}.\vec{r}}\psi^*(\vec{r})\psi(\vec{r})d\hat{r}$$

and we can write

$$e^{i\vec{S}.\vec{r}} = 4\pi\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} i^\ell j_\ell(|\vec{S}||\vec{r}|)Y_{\ell,m}(\hat{S})Y^*_{\ell,m}(\hat{r}).$$

where $j_\ell(x)$ is the spherical Bessel functions and $Y_{\ell,m}(\Omega)$ the spherical harmonics.

Considering the S and D waves and calling S,K,r the modulus of the vectors $\vec{S},\vec{K}$ and $\vec{r}$ respectively, we have

$$I_{\lambda'\lambda} = 2^{-2}\{aa'\int h(K r)u^2(r)j_0(S r)d r - \sqrt{4\pi}\left[\sum_{\beta=-2}^{2} a'by_{2,\beta}(\hat{S}) + \right.$$

$$\left. + \sum_{\beta=-2}^{2}(-1)^{\beta}ab'y_{2,\beta}(\hat{S})\right]\int h(K r)u(r)\omega(r)j_2(S r)d r + (-1)^{\alpha}\left[bb'\right.$$

$$\sqrt{5}\ <2,2;-\alpha,\alpha|0,0>\int h(k r)\omega^2(r)j_0(S r)d r + \sqrt{4\pi}\sqrt{\frac{10}{7}}\sum_{\beta=-2}^{2}(-1)^{\beta}bb'$$

$$\left.<2,2;\beta-\alpha,\alpha|4,\beta>y_{2,b}(\hat{S})\int h(K r)\omega^2(r)j_4(S r)d r\right]$$

where a(a') and b(b') are the Clebsh-Gordan coefficients for the initial (final) deuteron:

$$a = <\frac{1}{2}\ \frac{1}{2}\ mn|1\lambda>$$

$$b = <2,1;\lambda - m - n, m + n|1\lambda><\frac{1}{2}\ \frac{1}{2}\ mn|1,m+n>$$

and u(ω) is the S(D) McGee wave function [19]. In addition,

$$\alpha \equiv \lambda - m - n$$

$$\alpha' \equiv \lambda' - m' - n'$$

$$h(K r) = \frac{e^{iKr}}{r}$$

Exchange current (EC)

The integral that appears in the calculation of the EC amplitude (eq. 2.5) is

$$I_{\lambda',\lambda} = \iint d^3 f_1 d^3 f_2 \psi^*_{\lambda'}(\vec{f}_2)\psi_\lambda(\vec{f}_1)\frac{1}{(P^2-m^2)}\frac{}{(M^2_\Delta - Q^2)}\frac{}{(K^2_i - \mu^2)}$$

where $P = P(f_1)$, $Q = Q(f_2)$ and $K_i = K_i(f_1, f_2)$.

We don't know how to solve analiticaly this integral specially when the D-wave is considered. So, in a first step we will only consider the S-wave. Furthermore, as the pion pole is outside the integration region and varies slowly (M and M' are now on-mass-shell) we can disregard the f_1-dependence of $K_i(f_1, f_2)$, and consequently decouple the integrals. Then we have

$$I_{\lambda',\lambda} = \int d^3 f_2 \psi^*_{\lambda'}(\vec{f}_2)\frac{1}{(M^2_\Delta - Q^2)}\frac{1}{(k^2_i - \mu^2)}\int d^3 f_1 \psi_\lambda(\vec{f}_1)\frac{1}{(P^2 - m^2)}$$

This integral is calculated in detail in Ref. [26].

We might have considered a second alternative to solve this integral, including the deuteron D-wave. The starting point is to remove completely the pion pole of the integral for $\vec{f}_1 = \vec{f}_2 = 0$. The new integral is now similar to that of the DS. The angular integration can be calculated analytically and the radial part is then performed numerically. The result obtained is not very far from the one presented here [25].

Figure Captions

Fig. 1 - Single (SS) and Double Scattering (DS) diagrams in $\pi D \rightarrow \pi D$.

Fig. 2 - Exchange currents (EC) diagrams for $\pi D \rightarrow \pi D$ scattering in the Δ_{33} region: a) nucleon-delta ($N\Delta$); b) delta-nucleon (ΔN); c) delta-delta ($\Delta\Delta$).

Fig. 3 - Differential cross-section for $\pi D \rightarrow \pi D$ scattering at $T_\pi^{LAB.} = 254$ MeV. The continuous, dashed and dash-dotted curves correspond respectively to the SS, DS and EC isolated contributions.

Fig. 4 - idem for $T_\pi^{LAB.} = 292$ MeV.

Fig. 5 - Differential cross-section for $\pi D \rightarrow \pi D$ scattering at $T_\pi^{LAB.} = 254$ MeV. The experimental data are those of ref. [4]. Continuous curve: SS; dashed curve: SS + DS; dash-dotted curves: SS + DS + EC contribution.

Fig. 6 - Idem for $T_\pi^{LAB.} = 292$ MeV.

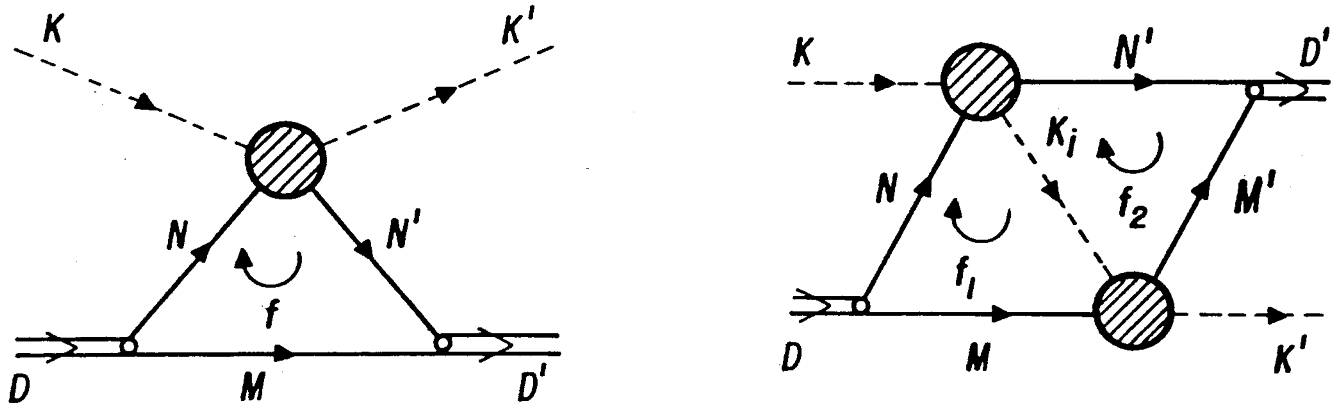


FIG. 1

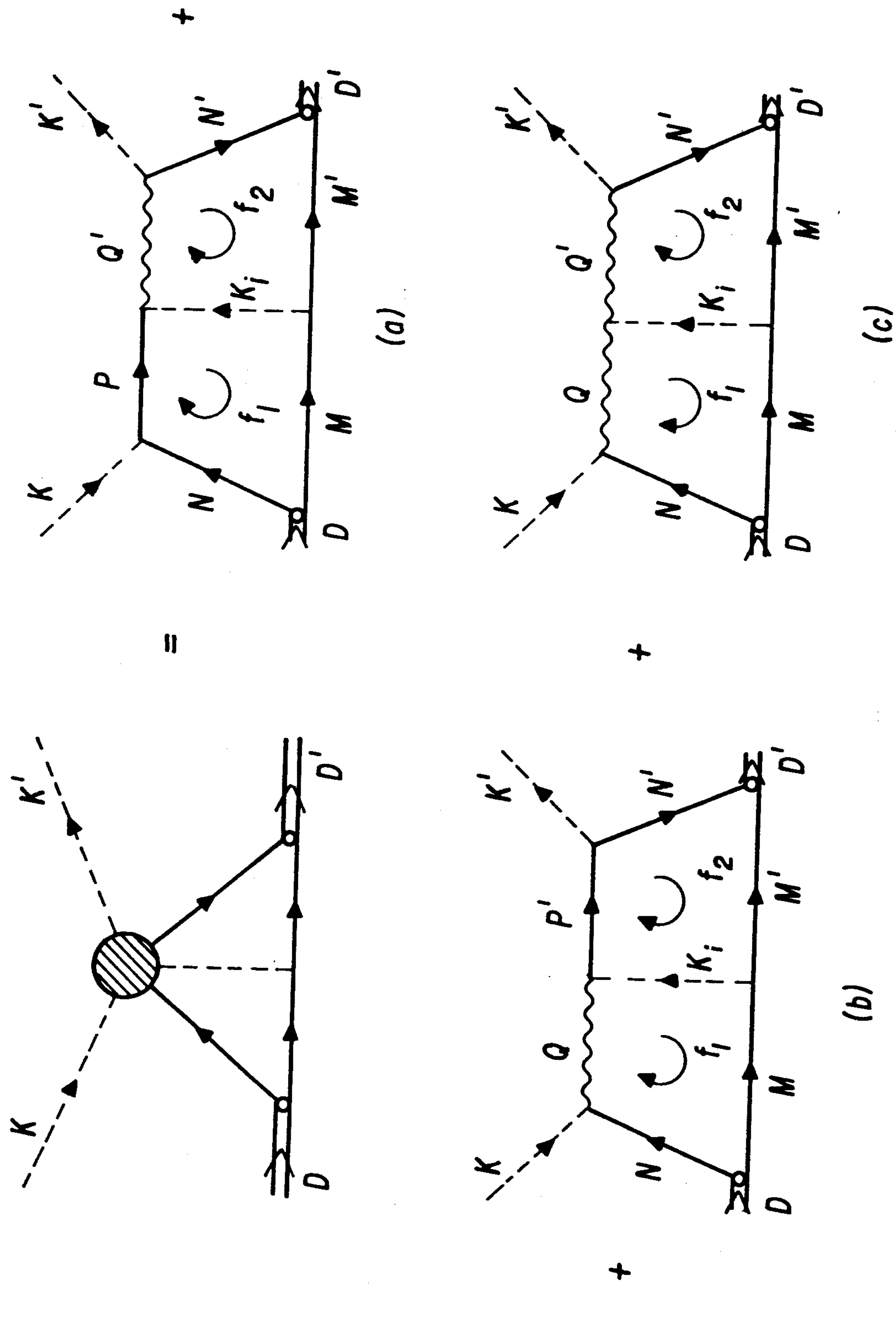


FIG. 2

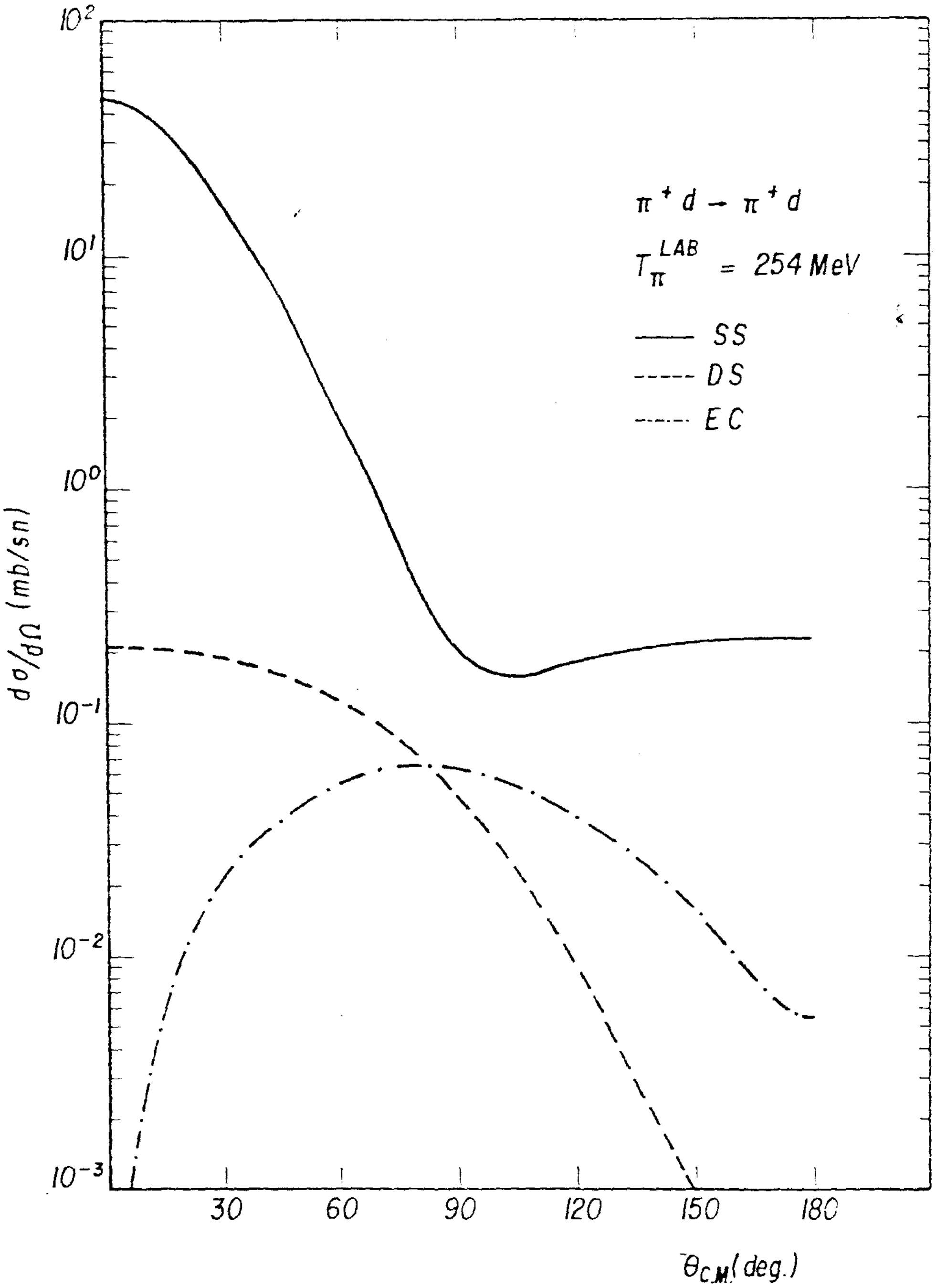

10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$d\sigma/d\Omega$ (mb/sn)
$\pi^+ d \rightarrow \pi^+ d$
$T_\pi^{LAB} = 254$ MeV
SS
DS
EC
30
60
90
120
150
180
$\theta_{C.M.}$(deg.)

FIG. 3

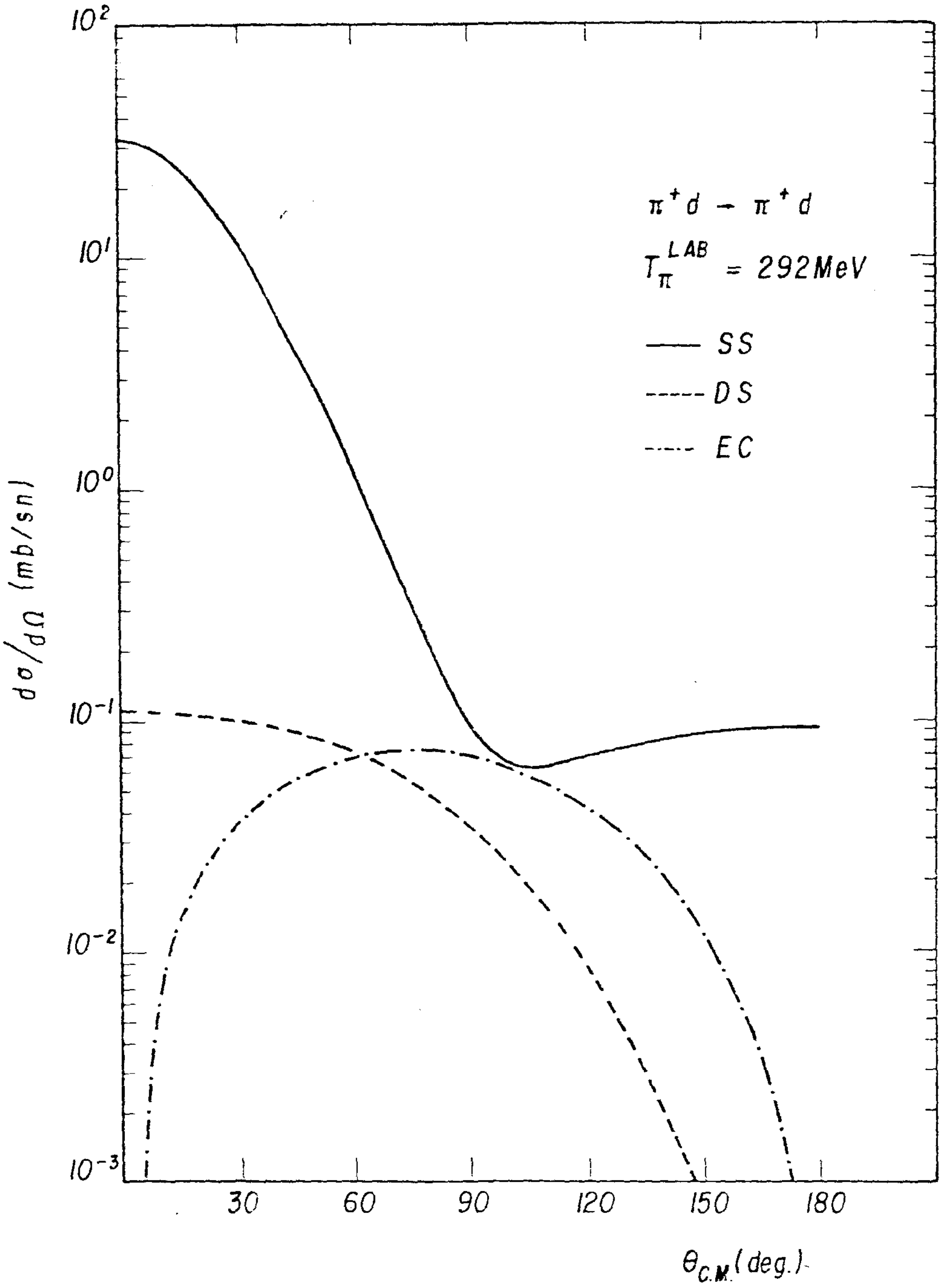

10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$d\sigma/d\Omega$ (mb/sr)
$\pi^+ d \to \pi^+ d$
$T_\pi^{LAB} = 292 MeV$
SS
DS
EC
30
60
90
120
150
180
$\theta_{C.M.}$ (deg.)

FIG. 4

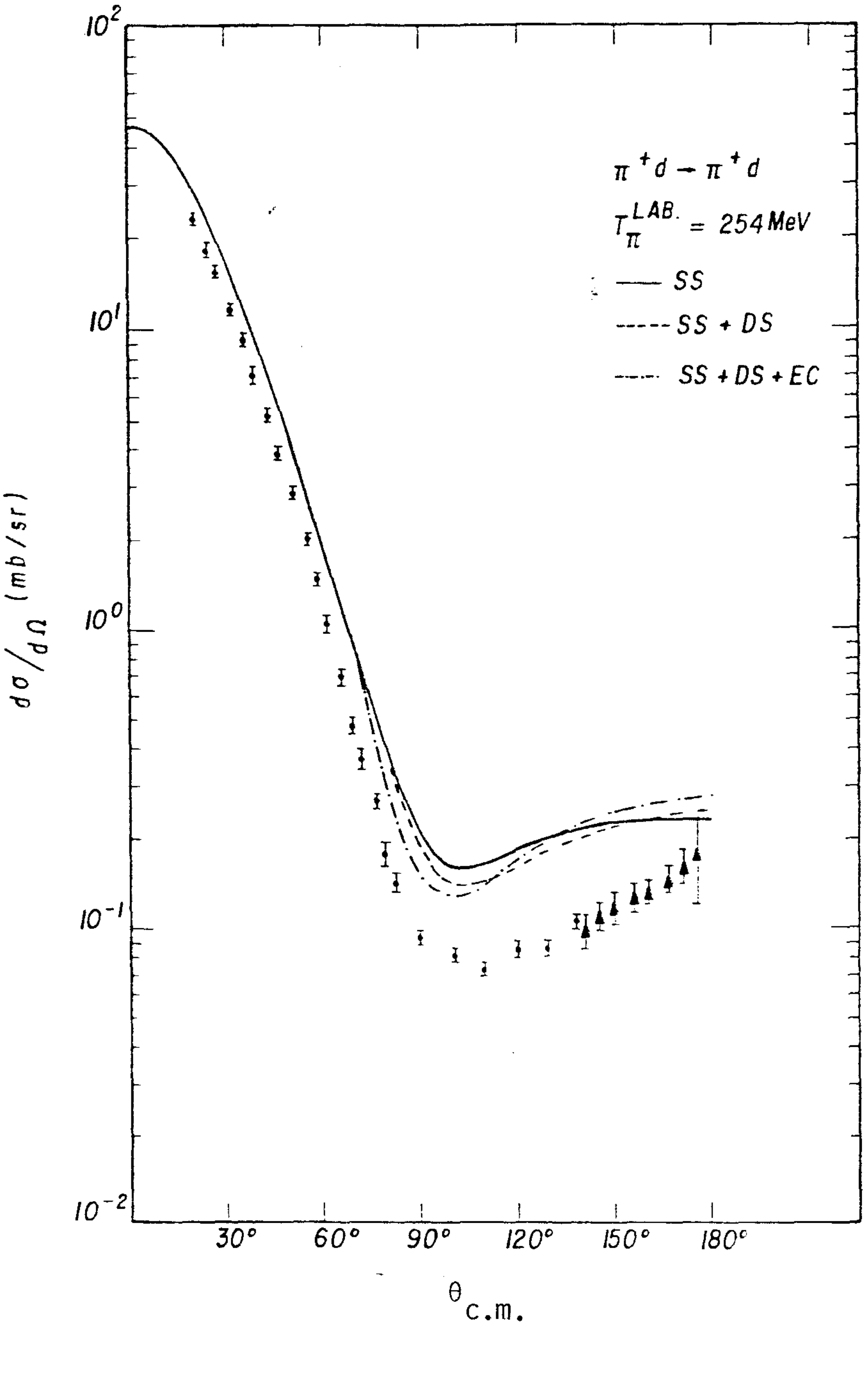
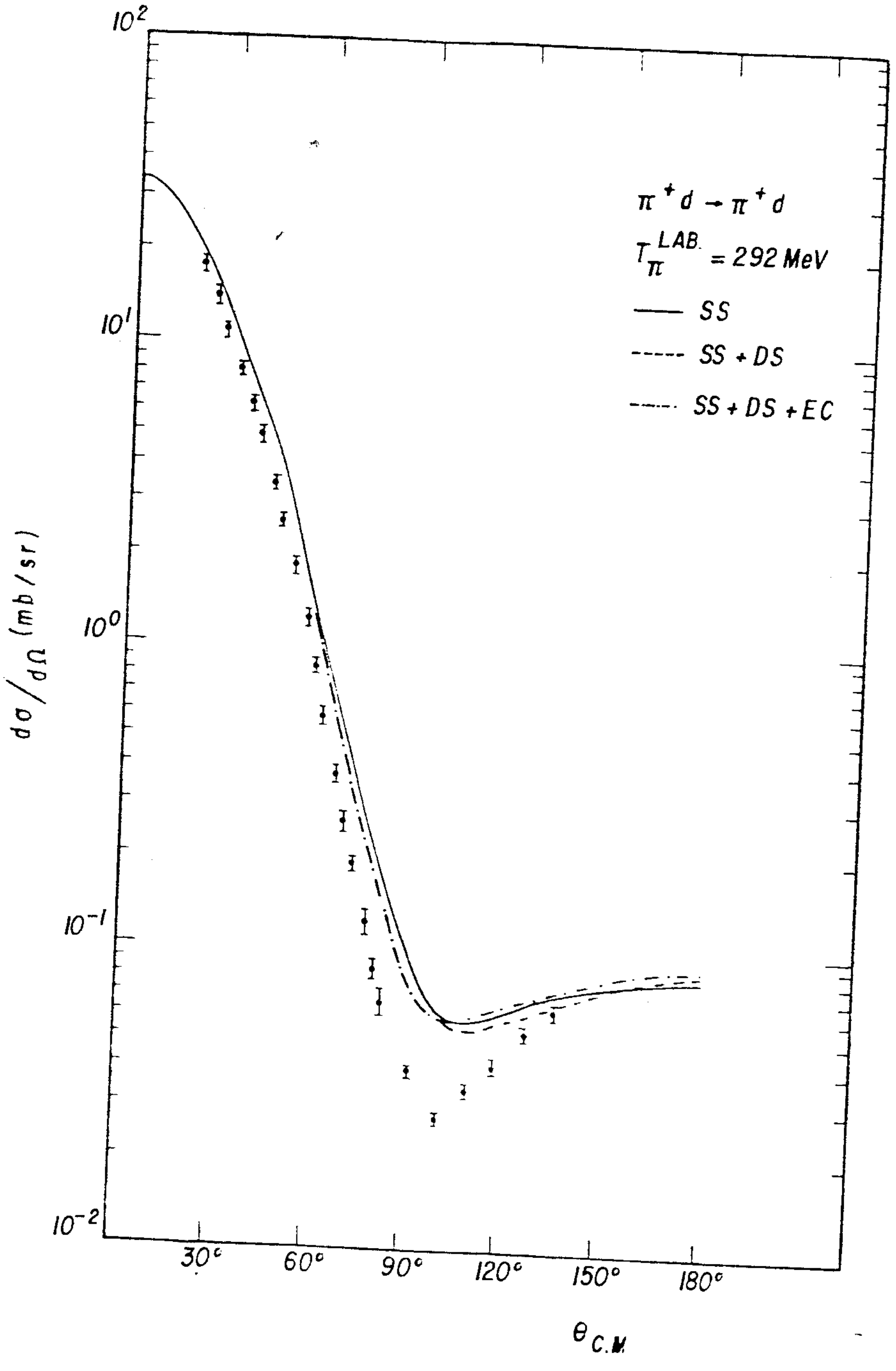

$\pi^+ d \to \pi^+ d$
$T_\pi^{LAB.} = 254\,MeV$
SS
SS + DS
SS + DS + EC
10^2
10^1
10^0
10^{-1}
10^{-2}
$d\sigma/d\Omega$ (mb/sr)
30°
60°
90°
120°
150°
180°
$\theta_{c.m.}$

FIG. 5



10^2
10^1
10^0
10^{-1}
10^{-2}
$d\sigma/d\Omega$ (mb/sr)
$\pi^+ d \to \pi^+ d$
$T_\pi^{LAB.} = 292$ MeV
SS
SS + DS
SS + DS + EC
30°
60°
90°
120°
150°
180°
$\theta_{C.M.}$

FIG. 6

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