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A MODIFIED MASS-YIELD FORMULA FOR INTERMEDIATE-ENERGY PHOTOSPALLATION OF MEDIUM-WEIGHT NUCLEI

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Very recently yields of a number of photon-induced spallation residuals have been experimentally determined in our Laboratory (1) for ^{51}V, ^{55}Mn, ^{nat}Fe (91.68% ^{56}Fe) and ^{59}Co targets, by using the activation method and γ-spectroscopy. In ref. (1) the yields, expressed as mean cross sections per photon, $\bar{\sigma}_K$, in the energy range 0.3 GeV-1 GeV, were successfully analysed mainly in terms of charge-dispersion curves. For the sake of conciseness the reader is referred to (1) as far as experimental conditions of exposures, target thicknesses, detection techniques, monitoring devices and the errors involved in the measurements are concerned.

Mass-yield distributions of more than 100 spallation products were obtained, which showed the well known pattern of spallation, i.e. an exponential decrease of the yields with increasing number ΔA of emitted nucleons ($\Delta A = A_t - A_p$, A_t and A_p being the mass numbers of the target nucleus and product nucleus, respectively), and identical yield distribution curves which fitted fairly well the experimental points for different A_p at fixed charge numbers Z.

Also we found quite similar values of the slopes K of the yield-surface ridges, regardless of the target masses. The measured values were, in fact, K = 1.31 for ^{51}V, 1.33 for ^{55}Mn, 1.32 for ^{nat}Fe, and 1.33 for ^{59}Co and, by taking into account the error that affects such measurements, the conclusion was reached of a substantial constancy of K within the mass range investigated, which is rather limited.

The slope K of the yield-surface ridge represents the decrease of yield per unit increase of ΔZ and can be expressed as

$$K = (\sigma_{p1}/\sigma_{p2})^{1/\Delta Z_p} \quad , \tag{1}$$

where σ_{p1} and σ_{p2} are the yields of two different products equally displaced from, and on the same side of, the β-stability valley, and ΔZ_p represents the difference $(Z_t - Z_{p2}) - (Z_t - Z_{p1}) = Z_{p1} - Z_{p2}$, Z_t, Z_{p1}, and Z_{p2} being the charge numbers of target, first spallation product chosen, and the second one, respectively ($Z_{p1} > Z_{p2}$, since K is always higher than unity).

Clearly K is a very useful parameter in evaluating the rate with which the yields of spallation residuals decrease with increasing ΔZ (and, consequently, ΔA) for a given target nucleus.

Several multiparameter analytical functions have been proposed up now (2-6) to calculate the cross section of any spallation product as a function of A_t , A_p , Z_p , but no use of K has been made, at least directly.

The aim of this work was to search for a rather simplified formula which included K among the other parameters.

In view of this, the mean cross sections of ref.(1) have been multiplied by the factor $(K)^{\Delta Z}$, with $K = (1.32 \pm 0.02)$ and $\Delta Z = Z_t - Z_p$ .This procedure brought all the parabolas of the different mass-yield distributions at fixed Z_p up to the same horizontal level. Then we have plotted the quantities $\sigma^* = \bar{\sigma}_K (K)^{\Delta Z}$ versus $(A_p - A_s)$ in a semilog graph. The difference $A_p - A_s$

represents the distance of the mass number of the spallation residual from the mass number A_s at the centre of the stable valley for $Z = Z_p$ .As a first approximation, we used for A_s the experimental peak values of the mass-yield distribution curves of ref.(1). In doing this we considered only product nuclides with $\Delta A \geq 2$ and $\Delta Z \geq 1$ and analysed, thus, 96 mean cross sections per photon in the energy range 0.3 GeV - 1 GeV.

All the ^{51}V, ^{55}Mn, ^{nat}Fe, and ^{59}Co σ^* experimental data were found, in this way, to lie very nearly on a parabola and therefore we assumed, for this peculiar yield distribution, the following expression to be valid

(2) $$\sigma^* = B \exp\{-C(A_p - A_s)^2\} .$$

A least-squares analysis has been carried out in order to determine the parameters B and C, by putting eq.(2) in the form

(3) $$\ln\sigma^* = \ln B - C(A_p - A_s)^2,$$

which simplified a good deal the calculation, since it establishes a linear dependence of $\ln\sigma^*$ on the quantity $(A_p - A_s)^2$.

By successive iterations we got

(4) $$B = 1100 \pm 25 ,$$

(5) $$C = 0.250 \pm 0.005$$

and a coefficient of correlation $r = -0.98$, which confirmed the original assumption of a linear correlation between $\ln\sigma^*$ and $(A_p - A_s)^2$.

In the course of the analysis we were also able to find the following relation between the parameter B and the target mass number A_t

(6) $$B = aA_t\bar{\sigma}_N A_t^{-1} = a\bar{\sigma}_N ,$$

with

(7) $$a = 4.26 \pm 0.09$$

and

(8) $$\bar{\sigma}_N = 258\ \mu b ,$$

$\bar{\sigma}_N$ being the mean total cross section of the interaction of photons with a free nucleon, whose value has been taken from the paper of Damashek and Gilman (7). The product $A_t\bar{\sigma}_N$ in eq.(6) represents the mean total inelastic yield of the γ-nucleus interaction.

Finally, we deduced a Z-dependence of A_s of the type

(9) $$A_s = (2.27 \pm 0.07)Z - (2.20 \pm 0.09) .$$

The latter expression gives values of A_s which are compared, in table I, with similar ones obtained by means of different assumptions (4,8) and with others experimentally determined (1,9). As one can see, very good agreement is found among calculated and experimental A_s values, with the exception of those reported in ref.(4).

From the above reported considerations, eq.(2) may now be rewritten as

(10) $$\sigma^{*}_{calc} = a\bar{\sigma}_N \exp\{ -C(A_p - A_s)^2\} .$$

In fig. 1 the trend of σ^{*}_{calc} is shown as a function of $(A_p - A_s)$ .For the sake of comparison, the experimental σ^{*} values are also reported. The FWHM of the parabola is about 3.3 $(A_p - A_s)$ units with a maximum of 1100 μb. The estimated errors in σ^{*}_{calc} range between 10% and 50%.

From eq.(10) one obtains

$$\bar{\sigma}_{\kappa calc} = (a\bar{\sigma}_N \,/\, (K)^{\Delta Z})\exp\{-C(A_p - A_s)^2\} \,, \tag{11}$$

which, by substituting to the parameters a, $\bar{\sigma}_N$, C, and A_s the numerical values of eq.(7), (8), (5), and (9), respectively, and by assuming K = (1.32 ± 0.02), gives the absolute mean cross section per photon (in μb) for the production of the nuclide A_p from a given A_t , in the energy range 0.3 GeV - 1 GeV.

In eq.(11), C defines the width of each mass-yield curve at fixed Z_p and can be compared with the same parameter found by Kumbartzki *et al.* (^{4}) (there indicated as R), whose value is 0.29 ± 0.07 for ^{51}V and 0.37 ± 0.08 for ^{nat}Fe. Reasonable agreement is found for ^{51}V only. On the other hand, we found (^{1}), for the mass-yield distribution curves from ^{51}V, ^{55}Mn, ^{nat}Fe, and ^{59}Co, the same widths, as has already been said. Besides, the values given by Kumbartzki *et al.* (^{4}) refer to the energy range 1 GeV -2 GeV.

A good test of the validity of eq.(11) may consist in calculating the ratios R between experimental and calculated mean cross sections per photon and their displacements from unity. We considered the set of 96 experimental values of ref.(^{1}) and another set of 19 values measured by Bülow et al. (9) for ^{51}V in approximately the same energy range. The result of the calculation is shown in fig. 2. It has been found that 53% of the experimental cross sections are reproduced by eq.(11) within a factor 1.25, 80% within a factor 1.5, and 95% within a factor 2. Moreover, we compared the measured yields of ^{24}Na photoproduction from different targets ($19 \leq Z_t \leq 29$) (9,10,11) with those calculated by means of eq.(11). Fig. 3 allows to compare the experimental values with the calculated trend for $A_p = 24$ and $Z_p = 11$. Other measured yields, such as those of ^{18}F and ^{22}Na from ^{39}K and ^{40}Ca targets (10), seem to be rather well reproduced by eq.(11) within a factor about 1.5.

All these considerations allow us to consider the five-parameter formula (11) as an useful tool in predicting photospallation cross sections in the energy range 0.3 GeV - 1 GeV with a fairly good accuracy for target masses between 50 and 60. Reasonable good accuracy is also obtained for the range of masses 39 -65.

The lack of a rather large amount of experimental data on photospallation of heavier nuclei neither permits to check the validity of eq.(11) for higher target masses, nor it allows any attempt to find a similar formula which be valid for that mass range.

In concluding this note we wish to point out that eq.(11) is somewhat simpler than other formulae, which need for a different set of parameters for each target nucleus and do not furnish, however, results better than those here discussed.

REFERENCES

(1) V.di Napoli, F.Salvetti, M.L.Terranova, J.B.Martins, O.A.P.Tavares and H.G.de Carvalho : Journ.Inorg.Nucl.Chem. (in press). See also : Centro Brasileiro de Pesquisas Físicas Report N A0014/7, May 1977.

(2) G.Rudstam : Z.Naturf., 21a,1027(1966).

(3) R.G.Korteling and A.A.Caretto,Jr. : Journ.Inorg.Nucl.Chem.,29, 2863(1967).

(4) G.J.Kumbartzky, U.Kim and C.K.Kwan : Nucl.Phys.,A 160,237(1971); G.J.Kumbartzky and U.Kim : Nucl.Phys., A 176,23(1971).

(5) A.C.Pappas, J.Alstad and E.Hagebø : Radiochemistry (Ed. A.G. Maddock), Butterworths (London, 1972), p.291.

(6) G.G.Jonsson and K.Lindgren : Phys.Scripta, 7,49(1973).

(7) M.Damashek and F.J.Gilman : Phys.Rev.D ,1,1319(1970).

(8) R.D.Evans : The Atomic Nucleus , Mc Graw-Hill (New York,1969), p.292.

(9) B.Bülow, B.Johnsson, M.Nilsson and B.Forkman : Z.Physik, A 278, 89(1976).

(10) V.di Napoli, G.Rosa, F.Salvetti, M.L.Terranova, H.G.de Carvalho, J.B.Martins and O.A.P.Tavares : Journ.Inorg.Nucl.Chem., 37,1101 (1975).

(11) A.Järund, B.Friberg and B.Forkman : Z.Physik, 262, 15(1973).

Table I. - Calculated and experimental values of the mass number A_s at the centre of the stability valley as a function of Z.

Element	Z	Calculated (a)	Calculated (b)	Calculated present work	Experimental ref.(9)	Experimental ref.(1)
Ne	10	21.0	20.6	20.5		20.5
Na	11	23.1	22.6	22.8	23.0	22.7
Mg	12	25.3	24.6	25.0	25.3	25.0
Al	13	27.5	26.6	27.3	27.3	27.3
Cl	17	36.4	34.6	36.4	36.2	36.2
Ar	18	38.6	36.6	38.7	38.6	38.4
K	19	40.9	38.5	40.9	40.8	40.8
Ca	20	43.1	40.5	43.2	43.2	43.2
Sc	21	45.4	42.4	45.5	45.5	45.4
V	23	50.1	46.3	50.0		50.0
Cr	24	52.4	48.3	52.3		52.4
Mn	25	54.7	50.2	54.5		54.5
Fe	26	57.1	52.1	56.8		57.0

(a) Values calculated from $Z = \frac{1}{2}(A_s - 0.0060A_s^{5/3})$, quoted in ref.(8).

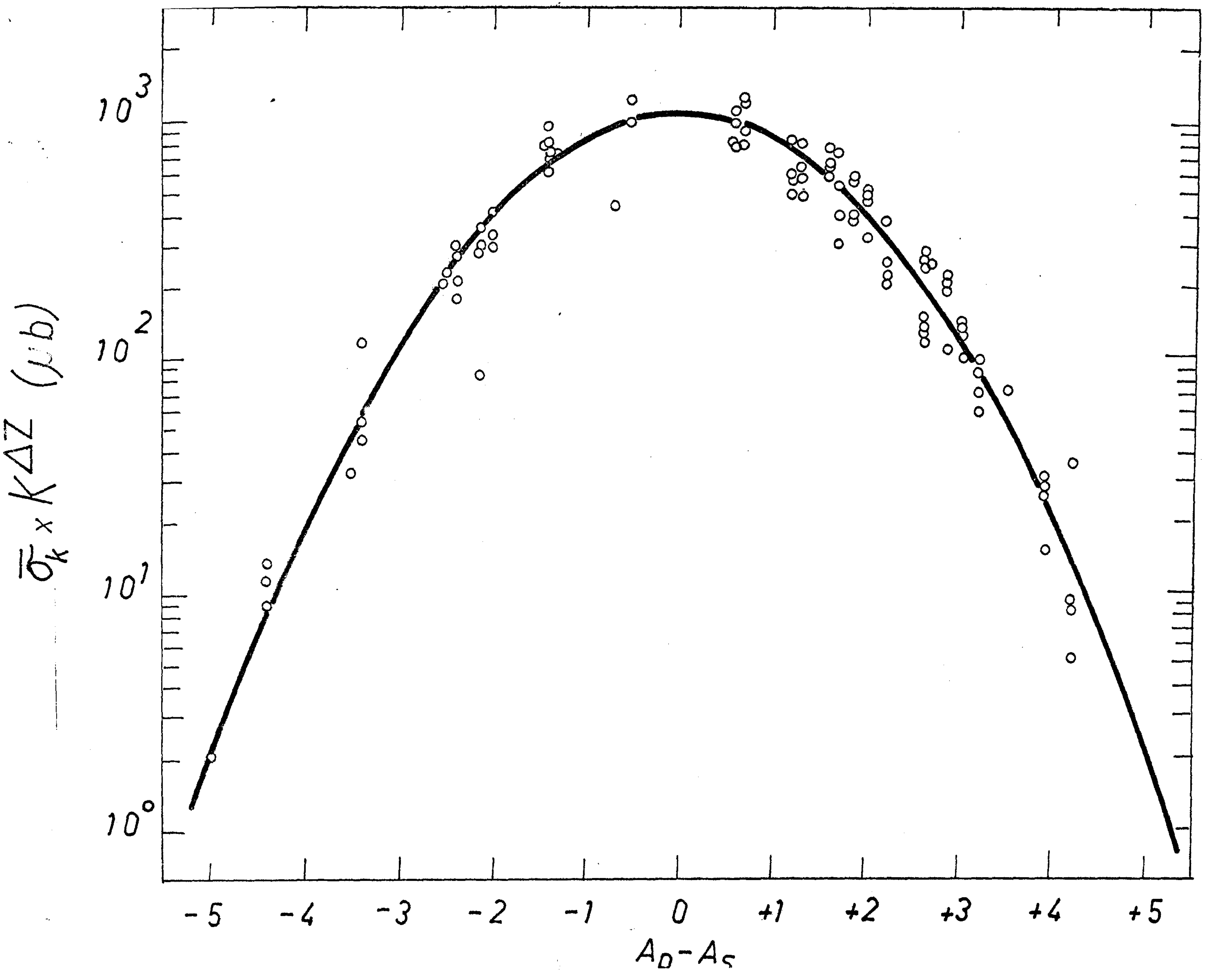
(b) Values calculated from $A_s = SZ - TZ^2$, with S = 2.09 and T = 0.0033, quoted in ref.(4).

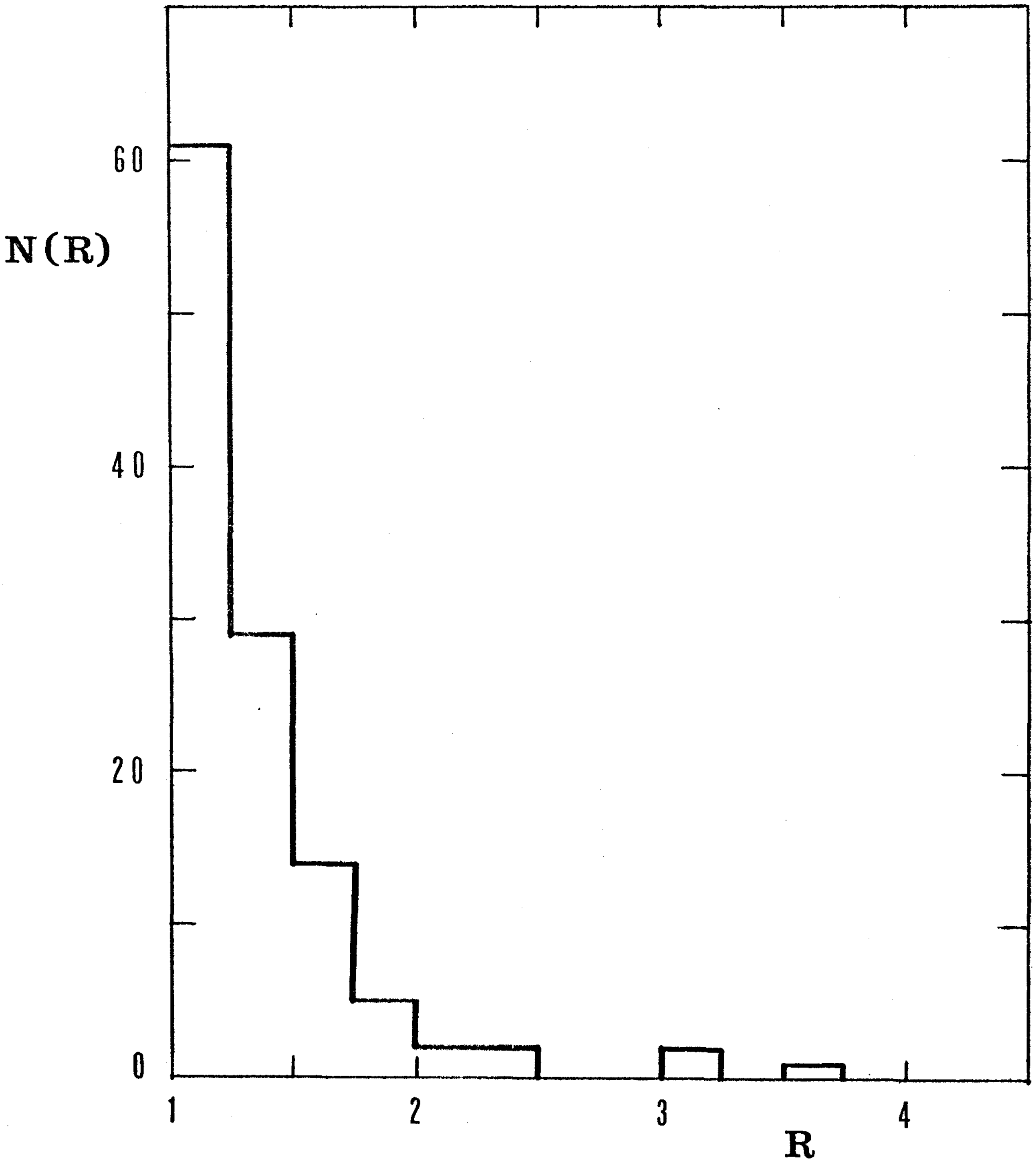
Figure captions

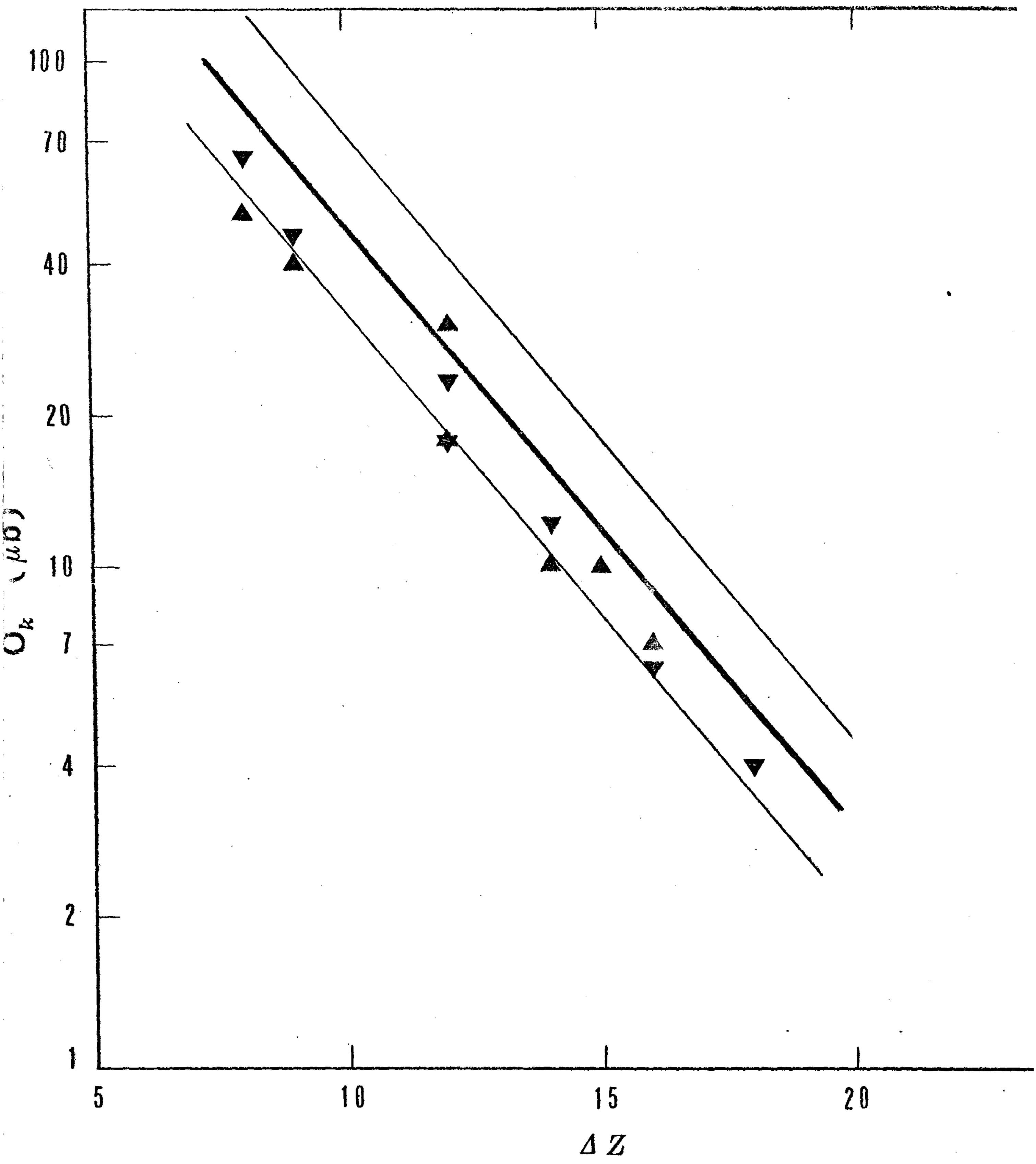
Fig. 1. - Trend of $\sigma^{*} = \bar{\sigma}_{\kappa}(K)^{\Delta Z}$ as a function of $(A_p - A_s)$. The circles represent experimental values of σ^{*} obained from the yields reported in ref. (^{1}). The curve is the best fit of the experimental points; its analytical expression is given by eq. (10).

Fig. 2. - Comparison between experimental and calculated cross sections. The quantity R represents the ratio $\bar{\sigma}_{\kappa,exp}/\bar{\sigma}_{\kappa,calc}$ (if $\bar{\sigma}_{\kappa,exp} \geq \bar{\sigma}_{\kappa,calc}$) or $\bar{\sigma}_{\kappa,calc}/\bar{\sigma}_{\kappa,exp}$ (if $\bar{\sigma}_{\kappa,exp} < \bar{\sigma}_{\kappa,calc}$). The histogram dispays the frequency N(R) relative to each fixed range of R values. The abscissa has been divided in steps of 0.25 units.

Fig. 3. - Experimental and calculated values of the mean cross section of ^{24}Na photoproduction from ^{39}K, ^{40}Ca, ^{51}V, ^{55}Mn, ^{nat}Fe, ^{59}Co, and ^{nat}Cu. Triangles: ref. (^{1,10}); reversed triangles: ref. (^{11}); star: ref. (^{9}). The straight line gives the trend obtained from eq. (11). The two lines above and below it have been obtained by multiplying and dividing, respectively, the calculated values by the factor 1.5, within which lie 80% of the experimental points analysed in the present work (see text and fig. 2).


$\overline{\sigma}_k \times K^{\Delta Z}$ (μb)
10^3
10^2
10^1
10^0
-5
-4
-3
-2
-1
0
+1
+2
+3
+4
+5
$A_D - A_S$


N(R)
60
40
20
0
1
2
3
4
R


σ_k (μb)
100
70
40
20
10
7
4
2
1
5
10
15
20
ΔZ