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Abstract

Superfield formulation of S. Weinberg's tadpole method to compute effective potential in supersymmetric theories is illustrated by considering the general renormalizable action involving only chiral scalar superfields. Unconstrained superfield potentials are introduced to simplify the "effective" superfield propagator which is derived in a compact form.

Key-words: Supersymmetry; Effective potential

SUPERFIELD TADPOLE METHOD FOR SUSY EFFECTIVE POTENTIAL

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The methods of Coleman and Weinberg[1], S. Weinberg[2] and Jackiw[3] are usually employed for effective potential computation in conventional field theory. They may also be used for SUSY theories[4] written in component form. However, the superfield[5] formulation has now been developed[6] sufficiently and it is more efficient to exploit the supersymmetry and work with supergraphs especially for a manageable calculations in higher loops. The tadpole method first noticed by Weinberg[2] requires simply the evaluation[7] of one-point functions of the "shifted theory" to the desired number of loops. It gives directly the first partial derivatives of the effective potential needed to discuss the spontaneous symmetry breaking or imposing renormalization constraints.

We present here a superfield formulation of the tadpole method for the general supersymmetric renormalizable action involving only chiral scalar superfields. It can be extended to theories with gauge superfields. However, the usual complication of non-diagonal propagators is present even for SUSY theories. The superfield formulation allows us further simplifications. We may introduce unconstrained "superfield potentials"[8] for chiral superfields. The corresponding effective superpropagators can be put in a compact form (Eq. (6)) which results in a simpler computation.

The action involving n chiral scalar superfields is [9]

$$\int d^8z\, \overline{\Phi}_i \Phi_i + [\int d^6s\, W(\Phi) + \text{h.c.}] \tag{1}$$

where $W(\Phi)=\lambda_i\Phi_i+\frac{1}{2}m_{ij}\Phi_i\Phi_j+\frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k$ is the superpotential. The shifted theory[§] is obtained by $\Phi_i\to\Phi_i+C_i(\theta)$, $\overline{\Phi}_i\to\overline{\Phi}_i+\overline{C}_i(\overline{\theta})$ where $C_i=a_i+f_i\theta^2$, $\overline{C}_i=\overline{a}_i+\overline{f}_i\overline{\theta}^2$ are constant chiral superfields with vanishing spinor components. The free action determing the effective superpropagators of the shifted theory is found to be

$$\frac{1}{2}\int d^8z\, (S,S^\dagger)\, A \begin{pmatrix} S \\ S^\dagger \end{pmatrix} + \int d^8z\, (J,\overline{J}) \begin{pmatrix} S \\ S^\dagger \end{pmatrix} \tag{2}$$

where $S,S^\dagger$ are superfield potentials(8) defined by $\Phi_i=-\frac{1}{4}\overline{D}^2S_i$, $\overline{\Phi}_i=-\frac{1}{4}D^2S_i^\dagger$ and an external source term is added. The operator A is given by

$$A = \frac{1}{16}\begin{pmatrix} -4C(\theta)\overline{D}^2 & \overline{D}^2D^2I_n \\ D^2\overline{D}^2I_n & -4\overline{C}(\overline{\theta})D^2 \end{pmatrix} \tag{3}$$

where $C(\theta)=M+f\,\theta^2$, $\overline{C}(\overline{\theta})=\overline{M}+\overline{f}\,\overline{\theta}^2$ are given in terms of matrices M and f defined by $M_{ij}=\frac{\partial^2W(a)}{\partial a_i\partial a_j}$ and $f_{ij}=(\frac{\partial^3W}{\partial a_i\partial a_j\partial a_k})f_k=2g_{ijk}f_k$. Taking into account of

$$\begin{pmatrix} P_2 & 0 \\ 0 & P_1 \end{pmatrix} A = A \quad ; \quad \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} A = 0 \tag{4}$$

and Euler-Lagrange equations we conclude that the superpropagators may be defined by[††]

[§] In gauge theories the real scalar superfield is likewise shifted by a constant real superfield with vanishing vector and spinor components.

[††] The causal boundry conditions are understood. An analogous discussion may also be made for photon propagator.

$$A\begin{pmatrix}\Delta^{SS} & \Delta^{SS^\dagger}\\ \Delta^{S^\dagger S} & \Delta^{S^\dagger S^\dagger}\end{pmatrix} = i\begin{pmatrix}P_2 & 0\\ 0 & P_1\end{pmatrix}\delta^8(z-z') \qquad (5)$$

The identities $P_1D^2=D^2P_2=D^2$ etc. permit us to assume Δ^{SS}, $\Delta^{SS^\dagger}$ to be antichiral while $\Delta^{S^\dagger S}$, $\Delta^{S^\dagger S^\dagger}$ chiral superfields with respect to the supercoordinate z. It follows then from Eq. (5)

$$\Delta^{S^\dagger S^\dagger} = \frac{C(\theta)}{4\Box}\bar{D}^2\Delta^{SS^\dagger} \qquad (6a)$$

$$\Delta^{SS^\dagger}(z,z') = i\, P_1[\Box-\bar{C}\,P_1C]^{-1}\,\delta^8(z-z') \qquad (6b)$$

with analogous expressions for other elements[†]. A compact form expliciting the pole structure is easily derived

$$\Delta^{SS^\dagger} = i\, e^{-i\theta\sigma.\partial\bar{\theta}}\, e^{-i\theta'\sigma.\partial\bar{\theta}'}[A\bar{\theta}^2\theta'^2 + B + C\bar{\theta}^2 + E\theta'^2]\,\delta^4(x-x')$$
$$+ i\, P_1(\Box-\bar{M}M)^{-1}\,\delta^8(z-z') \qquad (7)$$

where $(\partial_\ell \leftrightarrow ik_\ell)$

$$A = +[k^2+\bar{M}M]^{-1} - [k^2+\bar{M}M-\bar{f}(k^2+M\bar{M})^{-1}f]^{-1}$$
$$k^2B = \bar{M}(k^2+M\bar{M})^{-1}f\,C$$
$$k^2C = [\bar{f}(k^2+M\bar{M})^{-1}f-k^2-\bar{M}M]^{-1}\,\bar{f}\,M(k^2+\bar{M}M)^{-1}$$
$$k^2E = (k^2+\bar{M}M)^{-1}\,\bar{M}\,f[\bar{f}(k^2+M\bar{M})^{-1}f-k^2-\bar{M}M)^{-1} \qquad (8)$$

Only the last term in Eq. (7) survives for $f_i=0$ giving[(8)] essentially the simplified chiral superfield propagators found by Grisaru, Roček and Siegel[(6)] making use of the generating func-

[†] We note that the method of Ref. 9 is not applicable in the present case. Eq. (6), of course, agrees with the special case treated in Ref. 6.

tional represented by a functional integral over constrained superfields Φ with chiral external sources.

The interaction part of the shifted theory is

$$\int d^6 s [\lambda_i \Phi_i + m_{ij} C_i \Phi_j + g_{ijk} C_i C_j \Phi_k] + \int d^8 z \, \overline{C}_i \Phi_i$$

$$+ \frac{1}{3} g_{\ell jk} \int d^8 z \, \Phi_\ell S_j (-\frac{1}{4}\overline{D}^2) S_k + \text{h.c.} \tag{9}$$

In the zero loop approximation the one-point function (tadpole) is obtained to be

$$\Gamma_0^{(1)} = \int d^2\theta \{\lambda_i + m_{ij} C_j(\theta) + g_{ijk} C_j C_k + \int d^2\overline{\theta}\, \overline{C}_i(\overline{\theta})\} \tilde{\Phi}_i(0,\theta,\overline{\theta}) + \text{h.c.} \tag{10}$$

where a tilde denotes Fourier transform and $\tilde{\Phi}_i(0,\theta,\overline{\theta}) = [\tilde{A}_i(0) + \sqrt{2}\,\theta\,\tilde{\psi}_i(0) + \theta^2 \tilde{F}_i(0)]$. We obtain on evaluating the integral

$$\Gamma_0^{(1)} = (\overline{f}_i + \frac{\partial W}{\partial a_i}) \tilde{F}_i(0) + M_{ij} f_j \tilde{A}_i(0) + \text{h.c.} \tag{11}$$

The tadpole contributions of component fields may be read off as the coefficients of $\tilde{F}_i(0)$, $\tilde{A}_i(0)$ etc. Hence we obtain[2][7]

$$-\frac{\partial V_0}{\partial f_i} = (\overline{f}_i + \frac{\partial W}{\partial a_i}) \quad ; \quad -\frac{\partial V_0}{\partial a_i} = M_{ij} f_j \tag{12}$$

plus their complex conjugate equations. Here $V_0(a,f;\overline{a},\overline{f})$ is the zero loop effective potential with spinor components set to zero and $a \to A$, $f \to F$ is understood. Integration of partial differential equations in Eq. (12) gives the tree level result

$$V_0 = -[\bar{f}_i f_i + f_i \frac{\partial W}{\partial a_i} + \bar{f}_i \frac{\partial \bar{W}}{\partial \bar{a}_i}] \qquad (13)$$

which reduces to $V_0 = |F_i|^2$ on using the auxiliary field equations of motion, $\partial V_0/\partial f_i = 0$. Eq. (12) implies that if det $M \neq 0$ supersymmetry is not broken at the tree level.

The computation of one loop effective potential requires the evaluation of one tadpole supergraph, say, for $\bar{\Phi}$. We may read it from Eq. (9)

$$\Gamma_1^{(1)} = i\int d^4\theta\, \tilde{\bar{\Phi}}_\ell(0,\theta,\bar{\theta})\,[\mathrm{Tr}\{g_\ell[-\frac{1}{4}D^2\Delta^{S^\dagger S^\dagger}(z,z')]\}]_{z=z'} \qquad (14)$$

where $(g_\ell)_{ij} = g_{\ell ij}$. The explicit expression in Eq. (7) makes the θ-integration very simple and we obtain

$$-\frac{\partial V_1}{\partial \bar{f}_\ell} = \mathrm{tr}\ g_\ell (k^2+M\bar{M})^{-1} f\,[k^2+\bar{M}M-\bar{f}(k^2+M\bar{M})^{-1}f]^{-1}$$

$$-\frac{\partial V_1}{\partial \bar{a}_\ell} = \mathrm{tr}\ g_\ell M\{(k^2+\bar{M}M)^{-1} - [k^2+\bar{M}M - \bar{f}(k^2+M\bar{M})^{-1}f]^{-1}\}$$

$$+\ \mathrm{tr}\ M g_\ell\{(k^2+M\bar{M})^{-1} - [k^2+M\bar{M} - f(k^2+\bar{M}M)^{-1}f]^{-1}\} \qquad (15)$$

plus their complex conjugates and where $\mathrm{tr} \equiv \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}$. For supersymmetric minima (tree level) they vanish. We may integrate Eq. (15) using the nice properties of "Trℓn" operation to obtain

$$V_1 = \frac{1}{2}\ \mathrm{tr}\ \ell n[I-(k^2+\bar{M}M)^{-1}\bar{f}(k^2+M\bar{M})^{-1}f] \qquad (16)$$

which agrees with the result obtained by other methods[10]. For

Wess-Zumino model Eq. (15) and (16) take simple forms[11]. Having at hand the explicit form of effective superpropagators Eqs. (6) and (7) higher loop calculations† may be done in the same straight forward manner. We remark also that the extension of the usual theory of effective action in terms of classical superfields is easily made. However, the procedure adopted in our context is more transparent.

<u>Appendix</u>: Collected here are some relations used often in deriving Eqs. (7) and (15)

$$A(I-BA)^{-1} = (I-AB)^{-1}A \quad ; \quad A^{-1}(I-AB)^{-1} = A^{-1} + B(I-AB)^{-1}$$

$$\ell n(I+M) = \sum_{1}^{\infty} \frac{(-1)^{n-1}}{n} M^n \quad ; \quad (I+M)^{-1} = \sum_{0}^{\infty} (-1)^n M^n$$

$$\mathrm{Tr}\, \ell n(I+AB) = \mathrm{Tr}\, \ell n(I+BA)$$

$$\partial \mathrm{Tr}\, \ell n(I+AB) = \mathrm{Tr}[(\partial A)(I+BA)^{-1} B + (\partial B)(I+AB)^{-1} A]$$

$$\overline{D}^2 (\overline{\theta}-\overline{\theta}')^2 \delta^4(x-x') = -4\, e^{i\theta\sigma.\partial(\overline{\theta}-\overline{\theta}')} \delta^4(x-x')$$

$$\overline{D}^2\, e^{-i\theta\sigma.\partial\overline{\theta}} \delta^4(x-x') = -4\, \theta^2 \Box\, \delta^4(x-x')$$

$$\Box P_1 \delta^8(z-z') = e^{-i[\theta\sigma.\partial\overline{\theta} + \theta'\sigma.\partial\overline{\theta}' - 2\theta'\sigma.\partial\overline{\theta}]} \delta^4(x-x')$$

$$D^2 (\theta-\theta')^2 \delta^4(x-x') = -4\, e^{-i(\theta-\theta')\sigma.\partial\overline{\theta}} \delta^4(x-x')$$

$$D^2\, e^{i\theta\sigma.\partial\overline{\theta}} \delta^4(x-x') = -4\overline{\theta}^2 \Box\, \delta^4(x-x')$$

$$\Box P_2 \delta^8(z-z') = e^{-i[\theta\sigma.\partial\overline{\theta} + \theta'\sigma.\partial\overline{\theta}' - 2\theta\sigma.\partial\overline{\theta}']} \delta^4(x-x')$$

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† Calculations for Wess-Zumino model in 2-loops and in gauge theories will be reported elsewhere.

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