

# Quark–clustering effects in neutrino-nucleon deep inelastic scattering

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## Abstract

The deep inelastic scattering of neutrinos on unpolarized nucleons is considered in a generalized parton model which takes into account quark–clustering effects, modeled by diquarks. The diquark contributions are expected to be significant at small  $Q^2$  values and to vanish asymptotically, and describe phenomenologically higher-twist corrections. The most general case is studied, which includes both scalar and pseudo–vector diquarks inside the nucleons, as well as the contribution of scalar-vector and vector-scalar transitions. The resulting scaling violations are briefly discussed.

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The role of possible spin-0 and spin-1 constituents of nucleons in deep inelastic scattering (DIS) has been studied since the advent of the parton model [1, 2]. In the framework of the pure quark model, such constituents can arise in a natural way as two-quark bound states, the scalar and (pseudo-)vector diquarks. It is now becoming increasingly clear that the concept of diquark is important for understanding baryon structure and several intermediate-energy particle reactions [3]; the diquark approach is among the few attempts to offer a systematic phenomenological description of non perturbative and higher-twist corrections. It is then clear that it is of particular interest to improve our knowledge about the role of diquarks in deep inelastic scattering induced both by electrons and neutrinos.

In the early '80 diquark contributions to deep inelastic nucleon structure functions were analysed and compared with the existing data [4, 5, 6, 7]. This led to a picture of the nucleon containing almost pointlike scalar diquarks and heavier, more extended, vector diquarks [6, 7]. Such analyses were carried out in the case of electromagnetic and weak deep inelastic scattering on unpolarized nucleons, but within somehow simplified versions of the diquark model.

The diquark contributions to the electromagnetic unpolarized and polarized nucleon structure functions, in the most general case of scalar and vector diquarks, allowing for a vector anomalous magnetic moment and for scalar–vector and vector–scalar diquark transitions, were studied in Ref. [8]. Based on those results, and considering also the diquark inelastic contributions, the observed higher-twist effects in  $F_2$  have recently been shown [9] to be well described by the diquark model with a set of parameters rather close to those found in previous analyses; within the same model, higher twist corrections to Bjorken sum rule were predicted in Ref. [10].

In the case of neutrino DIS, all the analyses performed so far in the framework of the diquark model [5, 6, 7] have been quite incomplete. In fact, the spin of diquarks is usually ignored for simplicity, or treated heuristically, and also, in some instance [5], the scalar–vector and vector–scalar transitions are neglected. Thus a systematic and complete study of diquark effects on weak deep inelastic scattering is called for.

This paper is aimed at calculating the most general elastic contributions of diquarks to the unpolarized deep inelastic scattering induced by neutrinos, following the scheme of Ref. [8].

To start with, let us recall the expression of the inclusive cross-section for both neutral

current ( $NC$ ) and charged current ( $CC$ ) DIS reactions [11]:

$$\frac{d^2\sigma_{NC(CC)}}{d\Omega dE'} = \frac{1}{2m_N} \left( \frac{G_F}{2\pi} \right)^2 \left( \frac{M_{Z(W)}^2}{Q^2 + M_{Z(W)}^2} \right)^2 \frac{E'}{E} L_{\alpha\beta}(\nu) W_{NC(CC)}^{\alpha\beta} \quad (1)$$

where  $m_N$ ,  $G_F$ ,  $M_{Z(W)}$  are, respectively, the nucleon mass, the Fermi weak coupling constant, the intermediate boson  $Z^0$  ( $W^\pm$ ) mass. The leptonic tensor  $L_{\alpha\beta}$  and the hadronic tensor  $W^{\alpha\beta}$  are given by

$$L_{\alpha\beta}(\nu) = l_\alpha l'_\beta + l'_\alpha l_\beta - g_{\alpha\beta}(l \cdot l') + i\epsilon_{\alpha\beta\gamma\delta} l^\gamma l'^\delta \quad (2)$$

$$\begin{aligned} \frac{1}{2m_N} W_{\alpha\beta} = & -g_{\alpha\beta} W_1 + \frac{p_\alpha p_\beta}{m_N^2} W_2 - \frac{i\epsilon_{\alpha\beta\gamma\delta} p^\gamma q^\delta}{2m_N^2} W_3 + \frac{q_\alpha q_\beta}{m_N^2} W_4 \\ & + \frac{p_\alpha q_\beta + p_\beta q_\alpha}{2m_N^2} W_5 + i \frac{p_\alpha q_\beta - p_\beta q_\alpha}{2m_N^2} W_6 \end{aligned} \quad (3)$$

where  $l$ ,  $l'$  and  $p$  are, respectively, the four-momenta of the incoming neutrino, of the outgoing lepton and of the nucleon;  $q$  is the momentum transfer. It is well known that the terms involving  $W_{4,5,6}$  disappear as result of the contraction of these tensors and the cross section then reads, in the laboratory frame

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{G_F^2}{2\pi^2} \left( \frac{M^2}{M^2 + Q^2} \right)^2 E'^2 \left[ 2W_1 \sin^2(\theta/2) + W_2 \cos^2(\theta/2) - W_3 \frac{E + E'}{m_N} \sin^2(\theta/2) \right], \quad (4)$$

where  $M = M_W$  for  $CC$  reactions, and  $M = M_Z$  for  $NC$  reactions.

In the parton model the virtual boson interaction with the nucleon is replaced by the incoherent sum of the virtual boson interactions with all nucleon constituents, supposed to be free. Neglecting the Fermi motion of the constituents inside the nucleon we have, in the unpolarized case [11]

$$W^{\mu\nu}(N) = \frac{1}{4m_N \nu x} \sum_{j,s,S} n_j(x, s; S) W^{\mu\nu}(j, j'), \quad (5)$$

where  $\nu = E - E'$  and  $n_j(x, s; S)$  is the number density of partons of type  $j$ , covariant spin  $s$  and four-momentum  $k = xp$ , inside a nucleon of four-momentum  $p$  and spin  $S$ .  $W^{\mu\nu}(j, j')$  is the partonic tensor which describes the elastic interaction between the virtual boson and the parton  $j$  ( $W^*j \rightarrow j'$ ).

Let us now compute explicitly the tensor  $W^{\mu\nu}(j, j')$  in the case of scalar diquarks ( $j = j' = S$ ), vector diquarks ( $j = j' = V$ ), scalar-vector ( $j = S, j' = V$ ) and vector-scalar ( $j = V, j' = S$ ) transitions.

First we focus on scalar diquarks (whose flavor content is  $ud$ ). When a scalar diquark interacts with a charged vector boson it undergoes a transition to a vector diquark, with flavor  $uu$  or  $dd$ <sup>1</sup>. We shall study later on the scalar–vector transitions. For the moment we consider a purely scalar interaction which can only occur in neutral current scattering.

The most general coupling of a scalar diquark to the  $Z^0$  is of a vector–like nature, as in the electromagnetic case, and reads

$$S^\mu = b_0(2k + q)^\mu, \quad (6)$$

where  $k$  is the diquark momentum and  $b_0$  is the weak scalar form factor of the diquark, revealing its composite nature. From the partonic tensor

$$W^{\mu\nu}(S, S) = S^{\mu*} S^\nu, \quad (7)$$

one can extract the two nucleon structure functions

$$F_1^{(S)} \equiv m_N W_1^{(S)} = 0 \quad (8)$$

$$F_2^{(S)} \equiv \nu W_2^{(S)} = xS(x)b_0^2 \quad (9)$$

where  $S(x)$  is the number density of scalar diquarks with momentum  $k = xp$ . One obviously has also  $F_3^{(S)} = -\nu W_3^{(S)} = 0$  since there is no axial term in the scalar–diquark–vector–boson coupling.

Let us consider the case of vector diquarks. Both the charged current and the neutral current contribute to  $W^{\mu\nu}(V, V)$ . However in the two cases diquarks with different flavor contents are selected. The  $W^+$  boson couples only to a vector diquark of the type  $V(ud)$  in the proton, the  $W^-$  boson couples to  $V(ud)$  and  $V(uu)$  in the proton, the  $Z^0$  boson can obviously couple to any diquark.

We start from the most general parity non–conserving coupling of a virtual intermediate boson with a spin-1 massive particle [12, 13]

$$V^\mu(\lambda_1, \lambda_2) = \left[ V^{\eta\mu\rho} + A^{\eta\mu\rho} \right] \epsilon_\rho^*(\lambda_1) \epsilon_\eta(\lambda_2) \quad (10)$$

where the vector coupling is given by

$$V^{\eta\mu\rho} = b_1(2k + q)^\mu g^{\rho\eta} - b_2 \left[ (k + q)^\eta g^{\rho\mu} + k^\rho g^{\eta\mu} \right] + b_3(2k + q)^\mu (k + q)^\eta k^\rho \quad (11)$$

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<sup>1</sup>We neglect possible transitions to scalar diquarks in excited orbital states.

and the axial one by

$$A^{\eta\mu\rho} = a_1(2k + q)_\sigma \epsilon^{\sigma\eta\mu\rho} + a_2 q_\sigma (2k + q)_\delta \epsilon^{\eta\rho\sigma\delta} (2k + q)^\mu . \quad (12)$$

Here  $k$  is the diquark momentum,  $q$  the momentum transfer and the  $\epsilon(\lambda)$ 's are the polarization vectors of vector diquarks with helicity  $\lambda$ . The form factors  $a_i(Q^2)$  and  $b_i(Q^2)$  are real functions due to time reversal invariance [12].

From eq. (10) we construct the partonic tensor

$$W^{\mu\nu}(V, V) = \sum_{\lambda_1, \lambda_2} V^{\mu*}(\lambda_1, \lambda_2) V^\nu(\lambda_1, \lambda_2) \quad (13)$$

or

$$W^{\mu\nu}(V, V) = [V^{\eta\mu\rho} + A^{\eta\mu\rho}][V^{\alpha\nu\beta} + A^{\alpha\nu\beta}] \left[ \sum_{\lambda_2} \epsilon_\eta^*(\lambda_2) \epsilon_\alpha(\lambda_2) \right] \left[ \sum_{\lambda_1} \epsilon_\beta^*(\lambda_1) \epsilon_\rho(\lambda_1) \right] . \quad (14)$$

By resorting to the quark–diquark wave function of the nucleon [2] one easily finds [8]  $\sum_S n_V(x, s; S) = 2/3 V(x)$  for any value of  $s$ , where  $V(x)$  is the density number of vector diquarks with momentum  $k = xp$ . Making use of the relation ( $m_D$  is the diquark mass)

$$\sum_\lambda \epsilon_\mu^*(k, \lambda) \epsilon_\nu(k, \lambda) \rightarrow -g_{\mu\nu} + \frac{1}{m_D^2} k_\mu k_\nu \quad (15)$$

in Eq. (14) and contracting all Lorentz indices, we obtain the explicit expression of  $W^{\mu\nu}(V, V)$ , which, inserted into Eq. (5), yields, by comparing with Eq. (3), the vector diquark contribution to the nucleon structure functions

$$F_1^{(V)} = \frac{1}{3} V(x) \left[ \left( 1 + \frac{Q^2}{4m_N^2 x^2} \right) b_2^2 + 2 \left( 1 + \frac{2m_N^2 x^2}{Q^2} + \frac{Q^2}{8m_N^2 x^2} \right) a_1^2 \right] , \quad (16)$$

$$\begin{aligned} F_2^{(V)} &= \frac{1}{3} x V(x) \left\{ 2 \left( b_1^2 + \frac{Q^2}{4m_N^2 x^2} b_2^2 \right) \right. \\ &+ \left[ \left( 1 + \frac{Q^2}{2m_N^2 x^2} \right) b_1 - \frac{Q^2}{2m_N^2 x^2} b_2 + Q^2 \left( 1 + \frac{Q^2}{4m_N^2 x^2} \right) b_3 \right]^2 \\ &+ \left. 2 \left( 1 + \frac{Q^2}{4m_N^2 x^2} \right) a_1^2 + 8m_N^2 x^2 Q^2 \left( 1 + \frac{Q^2}{4m_N^2 x^2} \right) a_2^2 \right\} , \quad (17) \end{aligned}$$

$$F_3^{(V)} = 0 , \quad (18)$$

where  $V(x) = V_{ud}(x)$  for  $W^+$  mediated (i.e. neutrino induced) DIS off protons, and  $V(x) = V_{ud}(x) + V_{uu}(x)$  for  $Z^0$  and  $W^-$  mediated DIS. The diquark densities  $S(x)$ ,  $V_{uu}(x)$ ,  $V_{ud}(x)$  are normalized in such a way that  $\int_0^1 [S(x) + V_{uu}(x) + V_{ud}(x)] dx = 1$ .

Note that vector diquarks do not contribute to  $F_3$  because the antisymmetric terms that originate from the contractions of the  $VA$  and  $AV$  currents in eq. (14) cancel out when we sum over all polarization states.

The contribution of the transition between scalar and vector diquarks is computed along the same lines followed for vector diquarks, starting from the most general three-particle coupling involving a vector boson, a spin-0 and a spin-1 diquark [12]:

$$T_\nu^{(S \rightarrow V)} = \left[ c_1 g_{\gamma\nu} + c_2 k_\gamma k_\nu + c_3 \epsilon_{\alpha\beta\gamma\nu} q^\alpha k^\beta \right] \epsilon^{*\gamma} \quad (19)$$

$$T_\nu^{(V \rightarrow S)} = \left[ c_1 g_{\gamma\nu} + c_2 (k+q)_\gamma (k+q)_\nu + c_3 \epsilon_{\alpha\beta\gamma\nu} q^\alpha k^\beta \right] \epsilon^\gamma \quad (20)$$

The transition form factors  $c_{1,2,3}$  must also be real. The contribution of the transition processes to the structure functions is

$$F_1^{(T)} = D^{(T)}(x) \left[ \frac{1}{2Q^2} c_1^2 + \frac{m_N^2 x^2}{2} \left( 1 + \frac{Q^2}{4m_N^2 x^2} \right) c_3^2 \right], \quad (21)$$

$$F_2^{(T)} = x D^{(T)}(x) \left[ \frac{1}{4m_N^2 x^2} c_1^2 + \frac{Q^2}{4m_N^2 x^2} c_1 c_2 + \frac{Q^2}{4} \left( 1 + \frac{Q^2}{4m_N^2 x^2} \right) c_2^2 + \frac{Q^2}{4} c_3^2 \right], \quad (22)$$

$$F_3^{(T)} = 0. \quad (23)$$

In the above equations, if  $T = S \rightarrow V$ ,  $D^{(T)}(x) = S(x)$  for  $W^\pm$  and  $Z^0$  scattering; if  $T = V \rightarrow S$ ,  $D^{(T)}(x) = (1/3) V_{ud}(x)$  for  $Z^0$  scattering,  $D^{(T)}(x) = (1/3) V_{uu}(x)$  for  $W^-$  scattering off protons, and there is no contribution from  $W^+$  scattering off protons. The functions  $F_3^{(S \rightarrow V)}$  and  $F_3^{(V \rightarrow S)}$  vanish for the same reason as in the vector case.

It is straightforward to see that the results of the diquark model for the electromagnetic deep inelastic scattering (Eqs. (1.19) of ref. [8]) are recovered by setting  $a_1 = a_2 = c_1 = c_2 = 0$  in Eqs. (9, 16, 17, 21, 22).

A glance at our results for the structure functions shows that, as in the electromagnetic case [8], pointlike diquarks would give rise to very strong scaling violations, not observed in the data. Of course, since diquarks are bound states of two quarks and not pointlike objects, any realistic comparison with experimental data should take into account their form factors.

If we assume, as it is natural, that all form factors which appear in the vector current have the same large- $Q^2$  behavior as in the electromagnetic case [9], namely that they

decrease, at least, as

$$\begin{aligned}
 b_0 &\sim \frac{1}{Q^2} \\
 b_{1,2} &\sim \frac{1}{Q^4} \\
 b_3 &\sim \frac{1}{Q^6} \\
 c_3 &\sim \frac{1}{Q^3}
 \end{aligned}
 \tag{24}$$

it follows that the dominant diquark contribution due to vector couplings is only proportional to  $1/Q^4$ . However, depending on the large- $Q^2$  behaviour of the form factors  $a_1$  and  $a_3$  which appear in the axial current, we might have more sizeable contributions. If we set, generically

$$a_1 \sim \frac{1}{Q^n}, \quad a_2 \sim \frac{1}{Q^k}, \quad c_1 \sim \frac{1}{Q^l}, \quad c_2 \sim \frac{1}{Q^m},
 \tag{25}$$

then we would get powerlike scaling violation of the type  $1/Q^2$  if  $n = 2$  or  $k = 3$ , in vector–vector reactions, or  $m = 3$  in vector–scalar and scalar–vector transitions.

It is well known that, in the framework of the usual parton model, neutrino-initiated deep inelastic scattering experiments are a fundamental tool to determine the momentum distributions of quarks with different flavours and to distinguish between quark and anti-quark contributions. Within the extended parton model used here it is straightforward to isolate the contribution of the scalar–vector diquark transition, which cannot be done with only electromagnetic interactions. Indeed, the difference between the neutral and charged current structure functions,  $F_{1,NC} - F_{1,CC}$ , given by (for  $\bar{\nu}$ -proton or  $\nu$ -neutron DIS)

$$F_{1,NC} - F_{1,CC} = \frac{1}{3} [V_{ud}(x) - V_{uu}(x)] \left[ \frac{1}{2Q^2} c_1^2 + \frac{m_N^2 x^2}{2} \left( 1 + \frac{Q^2}{4m_N^2 x^2} \right) c_3^2 \right],
 \tag{26}$$

can, at least in principle, be used to impose constraints on the  $Q^2$  dependence of the transition form factors. Since the distributions  $V_{ud}(x)$  and  $V_{uu}(x)$  were already fixed by fitting the higher twist effects in the unpolarized electromagnetic structure function  $F_2$  [9], this expression can also be used to test the  $x$  dependence predicted by the diquark model.

A detailed study of these higher-twist terms and their possible evaluation are at present impossible, due to lack of experimental data. In this paper we have developed a general and physically motivated phenomenological framework, within which it is possible to interpret and understand higher-twist effects in neutrino-initiated deep inelastic processes.

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