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Fluctuations in a Primordial Anisotropic Era

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ABSTRACT

The primordial Universe is treated in terms of a non-perfect fluid configuration endowed with an anisotropic expansion. The deGennes-Landau mechanism of phase transition acts as a very efficient process to provide the elimination of the previous anisotropy and to set the universe in the current isotropic FRW stage. The entropy produced, as a consequence of the phase transition, depends on the strenght of the previous shear. We suggest the hypothesis that the germinal perturbations that will grow into the observed system of galaxies occur in the anisotropic era. We present a model to deal with this idea that provides a power spectrum of fluctuations of the form $\delta_k^2 \sim 1/(a+bk^2)$. We compare this prediction of our model to the current knowledge on the galaxy formation process.

Key-words: Cosmology; Anisotropy; Galaxy formation.

1 Introduction

The standard cosmological model treats the matter content of the Universe in its very condensed prior era as an ideal gas. This oversimplification is roughly responsible for a great part of our modern vision of the global properties of spacetime in the actual expanding Friedmann-Robertson-Walker (FRW) universe. However, it is also a scenario with several difficulties.

Among all problems that are usually attributed to this standard model [1] there are two that ask for an immediate attack:

- The existence of a singularity on the FRW cosmological metric.
- The high degree of isotropy of the microwave background radiation (MBR).

It is not our purpose here to comment further on the question of primordial singularity[1] but instead we will limit our exam here only to the above second question.

It is a rather general belief that the high degree of isotropy shown at cosmological dimensions deserves indeed a further explanation. In the standard view, this problem appears as a consequence of a causal trouble, once the existence of a horizon present in the FRW geometry inhibits the free exchange of information throughout all the spacetime at the period in which the cosmic radiation starts to become free of spurious interaction with the rest of all existing matter. The most fashionable solution to this, in the last years, the so called inflationary proposal, circunvents this difficulty by the introduction of an efficient mechanism which allows the Universe to pass quickly from a very small volume to a larger one in a very short time, in comparison with the growing of the cosmical horizon. By arguments that are now well known this, in turn, avoids the above causal difficulty. We would like to stress that in the inflationary scenario there is no real description of a true mechanism of isotropization. Although the solution of the horizon problem is certainly a necessary condition of homogeneization, it is far from being sufficient. In the present paper we would like to extend the usual framework and analyse the main consequences of assuming the presence of a non-negligible anisotropy in prior epochs. Thus, the main problem becomes to find an efficient mechanism to extinguish this primordial shear[2].

To circunvent such difficulty, in the last decade, the idea was set up to modify the simple description that treated the primordial matter in terms of a perfect fluid by the alternative assumption that it should be represented instead in terms of quantum fields. In the present paper we will examine another description of the matter content of the Universe in its early highly concentrated era. We shall see that, as a natural consequence of our approach, not only we can provide a solution to the problem of finding a isotropization mechanism but further than this, a well-desired by-product of this solution is achieved: a proposal of a new mechanism for the formation of inhomogeneities in an otherwise homogeneous non-isotropic primordial background.

There has been plenty of arguments in the literature that support the existence of a

[1]The interested reader may consult for instance the analysis on this in [2].

[2]We remind that some previous proposals in the early 60's [3] to treat anisotropic universes have stumbled precisely in the difficulty of finding such mechanism of isotropization.

period in which anisotropic stresses had an important role in the cosmic past[3]. This means that the associated matter-energy distribution responsible for the curvature of the global spacetime should not be represented by the simple form of a perfect fluid configuration. Unfortunately, it became almost a common sense among physicists to accept that any non-perfect fluid model is a rather strange treatment of the cosmic content; although it should be precisely the opposite that should cause surprise and mistrust. Indeed, the reduction to a perfect fluid, of the myriads of interactions of particles and fields in a tremendously high background curvature, in the very hot early era, is temerary; and it seems nothing but a miracle that such a gross simplification resisted such a long period without further criticism. It seems worth to note that even the alternative treatment to describe the main sources of the curvature of spacetime in terms of quantum or classical (scalar) fields (as in the first versions of the inflationary scenario) admits an equivalent representation as a non-perfect fluid endowed with viscosity that yields the presence in the stress-energy tensor of a tensorial pressure $\Pi_{\mu\nu}$.

It has been a common procedure in the first works on this subject to make the simple assumption that the dynamical quantities (e.g. $\Pi_{\mu\nu}$) are linearly related to the associated kinematical structures, e.g., the shear tensor $\sigma_{\mu\nu}$. A typical case was exhibited in the pioneer work of Misner [3]. However it was soon recognized that such simplification was of no use in solving the main problems, e.g. the isotropization of the Universe. Thus, one should look for a more general description [4, 5]. In the next sections we will show how one can proceed in this direction.

Our analysis here is based on a previous paper [6] in which a mechanism of cosmological phase transition was presented. Although the basic structure of such phenomenon is taken from the treatment of the nematic-isotropic states of matter in the standard deGennes-Landau model, there is an important novelty here: the control parameter, in the new version, is not the temperature T, but it depends on a global property of the expansion[4], identified to the Hubble parameter Θ . At first sight this could appear as a minor modification, once a lot of permissible geometries display a direct relationship between T and Θ. However, the quantity Θ belongs to the kinematical framework of the cosmic fluid, and as such it must obey certain conservation laws that restrain, in principle, its arbitrariness and impose some constraints in any evolving theory. This, on the other hand, should not be treated as a drawback of the uses of Θ as the controller of the distinct phases of the fluid. The reason is simple: contrary to Landau model in which the dependence of the phase transition on T is put by hand, and is supported by its consequences and not by first principles, here in our mechanism the Θ-dependence is a direct consequence of the Stokesian nature of the cosmic fluid.

The major distinction of our scenario from the previous ones concerns precisely the treatment of the matter in the prior epoch. In section 1.1 we describe the cosmical matter

[3]Almost all analysis which claims to be a deep investigation of the exceptional state that represents the first few moments near the point of maximum condensation of the universe (singular or not) have one point in common: they agree that the use of a perfect fluid configuration to characterize that epoch is nothing but a toy model. One is allowed to use it just for convenience of treatment; but it remains a welcome fact that it yields a good first order approximation of the global properties of the universe.

[4]This should not be confused with the fact that the expansion is not isotropic: the quantity that characterizes the anisotropic expansion is the shear that will be identified later on to the order parameter; the scalar of expansion Θ is an independent variable.

as a non-perfect fluid. Then we review the phenomenon of phase transition which allows a very efficient mechanism of isotropization. The dependence of the control parameter on the kinematics poses some problems when dealing with the fluctuations. This is treated in the subsequent section, where we present the idea that perturbations ocurring in the previous anisotropic era could be the cause of the ulterior formation of large scales inhomogeneities which are the origin of the creation of structures like the actual system of galaxies. We present a toy model to show how such perturbation could be treated. We use a simple scheme of a nematic liquid-crystal to evaluate the basic shear fluctuations. As a consequence of this we show that the density of contrast decays proportionally to $e^{-\frac{r}{\xi}}$, in which ξ is the correlation length. This yields for the power spectrum of the density fluctuation the form $\delta_k^2 \sim (A + e_1 k^2)^{-1}$. We end with section 5 in which various steps of the Cosmological Program that we analyse here are described.

1.1 Cosmic Fluid Configuration

At the very condensed era the Universe develops viscous processes. This has been considered by many authors in a rich variety of distinct situations. Its origin may be ascribed either to direct matter-to-matter interaction (as in the Misner model [3] which treats the interaction of neutrino with matter as a viscous process) or by creation of particles due to the gravitational cosmological field (Zeldovich) [7]. The basic underlying symmetry of the geometry, related to the cosmic fluid, is usually taken to be of one of the two types:

- Homogeneous and isotropic;
- Homogeneous and anisotropic.

The first case, that constitutes one of the corner-stones of the standard model, requires precisely a sort of very special initial condition that we are trying to avoid. Thus we start our analysis on the hypothesis that the second case occurred in our universe.

There have appeared many distinct treatments dealing with this case [8]. However, all these proposals suffer from the same disease: they do not provide an efficient isotropization mechanism. This has even led some authors to turn the analysis of this problem to a geometrical framework [9] with generic statements that led isotropization to be considered as a miracle once they argued that the spectrum of anisotropic spatially homogeneous geometries ending in a Friedmannian stage has null measure. Others, in an opposite position, have claimed that isotropization is a rather trivial phenomena that can be achieved by the effects of an arbitrary cosmological constant [10].

Here we want to continue our recent proposal to deal with this problem in a new way. We start by the assumption that there has been an epoch in which the cosmic matter behaves as a non-perfect fluid to which we ascribe the property that its associated velocity field has a non-null shear. We shall see in a subsequent section that such cosmic fluid can undergo phase transitions and pass in a continuous way from a less symmetric phase to a more symmetric one; that is, it can remove its anisotropies during the cosmic evolution [11, 12][5].

[5] This behaviour is very similar to a liquid crystal.

The idea that the primordial Universe passed an anisotropic phase led naturally to the following question: is there any observable consequence of such primordial anisotropy that could be used, even indirectly, as a proof of the existence of such an era? As far as the previous analysis is concerned the negative answer to this question diminished the interest on such prior proposals. Here we would like to argue differently, with a positive answer to this question. Concomitantly to providing a proper mechanism of isotropization, we will present a new proposal that states that germinal perturbations in the cosmic fluid, seed of galatic structures, occur precisely in this anisotropic era. We will present a simple toy-model in order to show how this mechanism works.

We start with the natural question: What can be said about the kinematical properties of the matter comoving observers? Every one agrees in one point: that it seems reasonable to make the hypothesis that there is no primordial vorticity, once its conservation law troughout the evolution would be manifested into today's galaxy motion. The standard hypothesis is made that we can establish a global Gaussian system of coordinates, by the definition of a cosmical time. Then, one is led to a very general description of the matter, both kinematically and dynamically. This can be implemented by taking for the derivative $V_{\mu;\nu}$ and for the anisotropic pressure $\Pi_{\mu\nu}$ the generic expressions with which we will deal with next. We set

$$V_{\mu;\nu} = \sigma_{\mu\nu} + \frac{1}{3}\Theta h_{\mu\nu} \tag{1}$$

where the quantity $h_{\mu\nu}$ is the projector on the 3-dimensional rest-space of the comoving observer V^{μ}:

$$h_{\mu\nu} = g_{\mu\nu} - V_{\mu}V_{\nu} \tag{2}$$

while $V^{\mu}V_{\mu} = 1$; the Hubble parameter Θ and the symmetric traceless shear $\sigma_{\mu\nu}$ are defined in the usual way:

$$\Theta \equiv V^{\mu}{}_{;\mu}$$

and

$$\sigma^{\alpha\beta} = \frac{1}{2}\, h^{\mu\alpha}h^{\beta\lambda}\, V_{(\mu;\lambda)} - \frac{1}{3}\ \Theta h^{\alpha\beta}.$$

We take the stress-energy tensor of the cosmic fluid to be

$$T_{\mu\nu} = \rho\, V_{\mu}V_{\nu} - p\, h_{\mu\nu} + \Pi_{\mu\nu} \tag{3}$$

We will consider the standard Stokesian fluid in which the anisotropic pressure depends only on the kinematical quantities Θ and σ_{ij}. Since $\Pi_{\mu\nu}$ is symmetric and traceless it can be written in the form:

$$\Pi_{\mu\nu} = f_1\, h_{\mu\nu} + f_2\, \sigma_{\mu\nu} + f_3\, \sigma_{\mu\alpha}\, \sigma^{\alpha}_{\nu}. \tag{4}$$

The functions f_i are given by the series

$$\begin{aligned} f_i &= f_{i0} + f_{i1}\, I_1 + [f_{i2}\,(I_1)^2 + \tilde{f}_{i2}\, I_2] \\ &+ [f_{i3}\,(I_1)^3 + \tilde{f}_{i3}\, I_1\, I_2 + \tilde{\tilde{f}}_{i3}\, I_3] + \dots \end{aligned} \tag{5}$$

The property that $\Pi_{\mu\nu}$ is traceless reduce the arbitrariness of the coefficients, once they must satisfy the constraint

$$3f_1 + \sigma^2 f_3 = 0. \tag{6}$$

Let us choose the four-velocity $V^\mu = \delta^\mu_0$ and denote the spatial components by Latin indices running $(1,2,3)$. The quantities I_k are the canonical invariants of the $3x3$ matrix Θ^i_j defined by

$$\Theta^i_k \equiv \sigma^i_k + \frac{1}{3}\,\Theta\,\delta^i_k \tag{7}$$

and are written as

$$I_1 = Tr\,\Theta^i_k = \Theta \tag{8}$$

$$I_2 = Tr\,[\Theta^i_k\,\Theta^k_l] = \sigma^i_k\,\sigma^k_i + \frac{1}{3}\,\Theta^2 \tag{9}$$

$$I_3 = Tr\,[\Theta^i_k\,\Theta^k_l\,\Theta^l_m] = \sigma^i_k\,\sigma^k_l\,\sigma^l_i + \Theta\,\sigma^i_l\,\sigma^l_i + \frac{1}{9}\,\Theta^3 \tag{10}$$

Thus we can rewrite eq.(4) in the form

$$\Pi_{\mu\nu} = f_2\,\sigma_{\mu\nu} + f_1\,[\sigma_{\mu\alpha}\,\sigma_\nu{}^\alpha - \frac{1}{3}\,\sigma^2\,h_{\mu\nu}] \tag{11}$$

Let us restrict our analysis here to a simple case. That is, in the general form introduced above in eq.(11), we will make the choice that f_1 is a constant and f_2 contains only terms linear and quadratic in the invariants I_k, that is,

$$\Pi_{\mu\nu} = [\alpha - a^2\Theta + \beta\,\sigma^2]\,\sigma_{\mu\nu} + \gamma\,[\sigma_{\mu\alpha}\sigma_\nu{}^\alpha - \frac{1}{3}\,\sigma^2\,h_{\mu\nu}] \tag{12}$$

in which α, a, β, and γ are constants. Just for convenience we will redefine the constant α as being $a^2\Theta^*$.

Let us point out that this limitation in the expansion of the anisotropic stress up to the third order in the parameters Θ and $\sigma_{\mu\nu}$ is nothing but a matter of simplification of the present exposition, and it is possible to proceed further on with the present model for a more general dependence. Besides, it has been shown [6] that, as far as the mechanism of phase transition is concerned, this limitation is not restrictive.[6]

One should wonder if the structure of the fluid that we are concerning here is not a very intricate one. That this is not the case can be shown by an analysis that, for instance, compares it with the structure of the energy-momentum tensor of a scalar field. For this, in the general case, e.g. without symmetries, we must deal with an infinite series for the corresponding expansion of the stress-anisotropic tensor as in eq(11).

We are now almost prepared to go into the main result concerning the isotropization mechanism of the above fluid configuration. However, before going into the details of this, let us make some very brief comments on the standard phase transition mechanism as it appeared in Cosmology during the last decade.

[6] We limit our presentation here to the case in which the primordial anisotropy is uni-axial. In order to take into account bi-axial phases, we must go further on in the series and consider terms up to the sixth power.

2 Phase Transition in the Early Universe

One of the most fashionable approaches in the last decade in Cosmology was the use of standard Landau phase transition mechanism in order to analyse some specific configurations of the early universe. This appeared as related to the possible distinct behaviour of matter that is described in terms of a single scalar field Φ. Indeed, it has been shown by methods of field theory that under certain special circunstances the energy momentum tensor of Φ can be associated to a cosmological constant. This was the result of the application of the socalled spontaneous breakdown of symmetry in the cosmical frame. The origin of this procedure goes back to the suggestion to treat the effects of the surroundings on Φ through a potential that becomes a temperature-dependent quantity. The scalar field itself becomes an order parameter and the temperature T the controller. In a non-static, e.g. expanding universe, it implies that at the conditions of thermal equilibrium, T is a function only of the cosmological time t. However, in the standard uses of this mechanism of phase transition, such time dependence was not explicitly taken into account. This was not a serious drawback since it was always assumed that the whole cosmical structure was homogeneous and isotropic, which implies that the temperature T is a well behaved function only of the scale factor Θ. Now, if we move to the considerations of a more general behaviour for the cosmic fluid, e.g. as in the inclusion of anisotropies or in the presence of inhomogeneities, we must go beyond this simple analysis.

The mechanism of phase transition, in the phenomenological treatment of Landau, depends not only on the existence of a parameter of order (to make possible the distinction between the different phases), but on an associated controller, which in most cases is identified to the temperature.

In our present investigation, we shall turn our attention to the generalization of the phase transition mechanism developped by deGennes to deal with structures like, for instance, liquid-crystals. One has to investigate the possible states of matter by taking as the order parameter a tensorial quantity that characterizes the isotropy/anisotropy of the space. Following a procedure similar to the one used for the scalar case, deGennes introduces a distortion parameter, say Q_{ij}, which is responsible for the characterization of the symmetry of the states. Then a modification on the free-energy F as a functional of Q_{ij} is made. The theory follows along the same lines as in the traditional Landau case, by looking for the particular values of Q_{ij} that minimize the free-energy. Let us follow the same lines as it has been employed in the standard treatment of phase transition in the cosmological framework up to just one unique additional condition of not requiring the further symmetry that the space is isotropic. In this vein, we are led to apply directly the deGennes-Landau's approach to deal with the primordial fluid. We shall see that this treatment of the cosmic fluid can be responsible for a continuous change of the symmetry of the Universe: from a prior anisotropic to an ulterior isotropic stage.

For a huge structure, as is the case when treating the whole Universe, the tensorial order parameter should be directly related to the shear of the cosmic fluid. The theory does not need to make Landau assumption that there exists an equilibrium temperature that controls the evolution. Indeed, it has been considered a very attractive model to use an intrinsic geometric quantity as the true control-parameter. The natural model [6] deals with the (Hubble) expansion factor Θ in place of the temperature. This is not astonishing and could have been already used in all previous cosmological program of phase transition, since in the old standard model Θ is a regular well-behaved function of T.

We now will examine the consequences of the condition upon which the self-gravitating cosmological fluid looks for the states that minimize its free-energy [17]. We set:

$$F = F_o(P,T) + \Delta F_{grav} \tag{13}$$

in which $\Delta F_{grav} = m^2 \Pi_{\mu\nu}\sigma^{\mu\nu}$. The analysis of the extrema of such free-energy yields the results summarized in a previous lemma [6] that for completeness, we will enunciate in the following subsection.

2.1 Generalized deGennes-Landau Cosmological Lemma

There are three critical values for the Hubble parameter: Θ_c, Θ^* and Θ_t, that separate the differents stages of the Universe, increasing the global disorder as follows:

- $\Theta < \Theta_c$: The most favourable state is an isotropic phase (I);
- $\Theta_c < \Theta < \Theta_t$: The most favourable state is the isotropic phase (I) but there is a local minimum corresponding to a small anisotropy (U);
- $\Theta_t < \Theta < \Theta^*$: The most favourable state is the anisotropic phase (U) but there is a local minimum corresponding to an isotropic (less favourable) phase (I);
- $\Theta^* < \Theta$: Corresponds to an anisotropic phase (U).

The constants Θ_c and Θ_t can be written in terms of the variables that appear in the expansion of the anisotropic tensor:

$$\Theta_c = \Theta^* - \frac{3\gamma^2}{64a^2\beta}.$$

and

$$\Theta_t = \Theta^* - \frac{\gamma^2}{24a^2\beta}.$$

The usual kind of substances dealt by deGennes are thermotropic: the temperature is the control-parameter. In the case we examine here, one can say that we treat the primordial Universe in analogy to such thermotropic fluids, through the direct dependence of our control-parameter, the Hubble constant Θ, only on the temperature.

It should also be a matter of interest to analyse the evolution of the entropy of the Universe induced by such phase transition. Assuming that the dependence of the control parameter (the Hubble expansion Θ) on the temperature is a regular monotonic function, as it occurs for instance in the standard FRW model, we conclude that the variation of the entropy, $\Delta S = -\frac{\partial F}{\partial T}$, is proportional to the value of square of the total shear σ^2 at the moment of transition to the isotropic era. Thus it follows that the contribution to the total entropy of the phase transition, from the anisotropic primordial Universe to the current isotropic FRW geometry depends on the initial value of the shear. Based on the efficiency of the deGennes-Landau mechanism described above we can contemplate the situation in which the actual high value of the entropy of the Universe could be related to the largeness of the primordial anisotropy. The scenario which we have sketched here seems to support such suggestion.

In the above description of the phase transition we do not take into account any spatial inhomogeneity. The Landau-DeGennes theory can go on without this knowledge. This seems a rather marvellous property of the theory, once it allows us to deal separately with the problems of isotropization and of the formation of structures. However, it is not out of reason to speculate if both questions should not be related. Let us examine how this unification could be treated. Before this, we would like to point out to the reader that we can now indeed enter in the analysis of the observable consequences of the existence of such era only because we have at our disposal a very efficient mechanism to remove the prior anisotropy.

3 Structure Formation

The observed pattern of inhomogeneities present in our Universe is generally admitted to be a consequence of a particular set of germinal perturbations in the otherwise spatially homogeneous and isotropic FRW background. The crucial problem that remains unsolved is modelling the origin of the initial spectrum of these perturbations. Both the standard model and the more recent inflationary scenario assume that this perturbation occurs in an epoch in which any eventual primordial anisotropy had already vanished.

How could this process be modified in our present proposal? We will try to answer this question by considering the very natural hypothesis that takes the origin of the fluctuations present in the latter isotropic and homogeneous era (the FRW geometry) as reminiscent of shear fluctuations, occurring at very high temperature **before** the annihilation of the primordial anisotropy. To analyse this idea further and in order to have some knowledge that could guide us on this, let us see what occurs in the analogous situation on the conventional theory of a liquid crystal in our laboratory.[7]

Our question is thus the following: under what conditions can small fluctuations pass from the ordered to the disordered phase with a significant probability? That is, how can matter, in the homogenous and isotropic phase inherit space-time dependent fluctuations from primordial anisotropic perturbations?

The answer to this question can be found by an examination of the similar behaviour of liquid-crystals. This treatment is unified by means of the phenomenological criterium set up by Ginzburg which, in our cosmical application, can be expressed in the following way. Let $\Delta\mathcal{F}$ be the difference of the minima of the energy densities associated to the anisotropic (U) and isotropic (I) states for a given temperature T. The criterium asserts that if in a volume χ^3 it is true that

$$\Delta\mathcal{F}\chi^3 \ll k_B T$$

[7]We deal here with a simplified model; it seems worth thus to follow the standard procedure in conventional cosmology, by imitating well-known terrestrial phenomena.

then the fluctuations can pass from the ordered less symmetric (more ordered) phase to the subsequent isotropic (more disordered) cosmological phase.

At this point one should be careful with the domain of validity of this inequality. We will assume throughout this work that this condition remains valid all the time. Only in this case we can envisage to associate the presence of inomogeneities in the FRW era to a previous anisotropic perturbation.

In our present case the temperature, that is, the control-parameter, is transformed into the scale factor Θ. In order to apply the above criterium to our scenario we must guarantee both: (i) that Θ can be maintained as the control-parameter (that is, it does not become a spatial-dependent quantity); and (ii) that the passage to the isotropic phase occurs in the epoch in which the corresponding value for Θ is sufficiently high in order to surpass any quantity associated to a measure of the barrier separating the isotropic and the anisotropic phases. In other words, once the actual order parameter (the scale factor) is a regular function of the temperature, we see that if the fluctuations occur very early, in a very hot era, then the above criterium could be used, allowing the fluctuations ocurring in the anisotropic era to be at the origin of the ulterior process of galaxy formation.

The question then turns to the knowledge of the inhomogeneities present at the anisotropic phase. Let us explain how this can occur. Before this, however, and to simplify here our task, let us guarantee that we can keep using Θ as the true controller.

3.1 The Control Parameter Problem

The usual structure of phase transition in the terrestrial laboratory is controlled by the temperature T. In the above analogy on Cosmology for the treatment of the cosmical fluid that we are employing, the corresponding control-parameter is the Hubble expansion Θ defined by the comoving observers. The internal structure of the geometrical framework of Einstein's theory of gravity connects the behaviour of this parameter to the evolution of the remaining kinematical quantities, e.g., the shear; and to the matter content on the Universe. Thus before considering the scheme of fluctuations that generates the spectrum of density perturbation we would like to simplify our treatment by allowing Θ to keep its basic controller property. For this to be possible, one needs — as it is the case for the temperature in the standard treatment of fluctuations in usual nematic substances, e.g. liquid-crystals — to separate the quantity Θ from the general perturbations. In other words, we would like to allow the shear to becomes a space-dependent quantity and the corresponding general perturbation of the matter, without having any spatial dependence of the parameter of control Θ[8]. Only in this case our treatment can keep a complete equivalence with the conventional perturbation scheme on the laboratory[9].

[8]Recently [23] more realistic description of the phenomenon of phase transition employed in cosmology, taking into account the evolution of the background geometry, has been examined. However, let us point out here that this dependence of the control parameter on the global time is not completelly general. Some aditional hypothesis on the homogeneity structure of the space is imposed. In our treatment here we circunvent this problem by limiting the possible processes to those in which the control parameter is homogeneous.

[9]We note that in the general case, i.e., when Θ has generic fluctuations, a more sophisticated procedure must be used. We postpone the analysis of this generalization to a future work.

The kinematical quantities associated to the fluid have, besides the evolution equations (e. g. Raychaudhuri's equation), some constraints that relate their corresponding spatial dependences. For our purposes, the equation that concern us here is given by[10]:

$$\frac{2}{3}\Theta_{,\mu}\, h^{\mu}_{\lambda} - (\sigma^{\alpha\beta} + \omega^{\alpha\beta})_{;\alpha}\, h_{\beta\lambda} - a^{\alpha}(\sigma_{\alpha\lambda} + \omega_{\alpha\lambda}) = R^{\mu\alpha}\, V_{\mu} h_{\alpha\lambda} \tag{14}$$

in which $a^{\mu} \equiv V^{\mu}{}_{;\alpha} V^{\alpha}$ is the acceleration. The fluid has no non-gravitational interaction and no heat flux; we thus set $a^{\mu} = 0$ and $q^{\mu} = 0$. Besides, these quantities will not be activated during the perturbation. It then follows from eq.(14) and using Einstein's equations of General Relativity that any spatial dependence of Θ is associated to the spatial divergence of the shear. From what has been said above the spectrum of permissible perturbation will be limited to the divergence-free perturbed shear[11].

This solves our difficulty and allows us to keep the quantity Θ as a good control-parameter[12].

3.2 Fluctuations in the Anisotropic Era

In the usual treatment of generation of large-scale structures in a spatially homogeneous Universe [14] the scale factor Θ does not have its spatial independence preserved under an arbitrary change of the metric and the matter content. Indeed, in the FRW background, the perturbation of the matter density $\delta\rho$ is given in terms of the evolution of the scale factor by the equation

$$\delta\dot{\Theta} + \frac{2}{3}\Theta\, \delta\Theta = \frac{1 - 3\lambda}{2}\, \delta\rho \tag{15}$$

in which a dot means derivative with respect to the global Gaussian time.

This property is a direct consequence of the usual hypothesis that the germinal perturbation that will grow into galaxies occurs in an epoch in which the Universe had already dissipated any eventual irregularity, like the shear, for instance. In the present model we propose an alternative hypothesis which states that this perturbation occurs in an earlier time, in which the cosmic fluid was still in the more ordered configuration, when the anisotropy had not disappeared completely and the geometry was not yet in its FRW form.

In this case the equation that controls the behaviour of the density perturbation $\delta\rho$ is no more given by eq. (15) but instead is controlled by its generalized form

$$\delta\dot{\Theta} + \frac{2}{3}\bar{\Theta}\, \delta\Theta + \bar{\sigma}_{\mu\nu}\, \delta\sigma^{\mu\nu} = \frac{1 - 3\lambda}{2}\, \delta\rho \tag{16}$$

[10]In order to obtain this equation one must use the non-commutativity of the covariant derivative and eq.(1).

[11]We remark that the possibility of non-trivial perturbation of the density of matter in the case of divergence-free shear, depends on the existence of a non-vanishing shear in the background. Indeed, it is a consequence of the conservation of the matter that, for instance, in FRW isotropic background, there is no possibility of evolving perturbation which has a divergence-free shear. The easiest way to show this, in FRW background, is to develop the perturbations in terms of the spherical harmonics basis [8].

[12]Let us emphasize that this requirement is by no means an essential requirement for our model, but just a simplification of the problem that concerns the analysis of the evolution of the perturbations in the cosmical framework.

in which $\bar{\Theta}$ and $\bar{\sigma}_{\mu\nu}$ represent the unperturbed values.

At this point we face a fundamental question: What can be said for the fluctuated shear $\delta\sigma_{\mu\nu}$? Or, in other words, what is the pattern of the germinal fluctuation of the shear that will cause perturbations on the matter density ?

Based on the considerations on phase transition that we analysed above let us follow Landau's treatment and use the standard Fourier decomposition for the density of free-energy $\mathcal{F}$ to include the perturbed terms. In the lowest-order approximation we have

$$\Delta F_{grav} = \int \mathcal{F}(\sigma_{ij}(x^k))d^3x \tag{17}$$

Once $\mathcal{F}$ is a scalar, besides the previous invariants given by the algebraic quantities $Tr\,\sigma^2$ and $Tr\,\sigma^3$ that appears in eq (13) we must consider the additional derivatives terms: $\sigma_{ij;k}\,\sigma^{ij;k}$ and $\sigma^{ij}{}_{;j}\,\sigma^k{}_{i;k}$. We can then write

$$\mathcal{F} = \Phi\, Tr\,\sigma^2 + \gamma Tr\,\sigma^3 + e_1\,\sigma_{ij;k}\,\sigma^{ij;k} + e_2(\sigma^{ij}{}_{;j})^2 \tag{18}$$

in which Φ and γ can be obtained from the homogeneous case. From our previous hypothesis, the term on divergence of σ_{ij}, that is $e_2(\sigma^{ij}{}_{;j})^2$ does not exist, once (for the reasons we have pointed out before) we restrict our analysis only to divergence-free perturbations. The fundamental states U and I are still obtained as the minima of the unperturbed background (or by taking the limit $e_1 \to 0$). This means that the spatial dependence of the flutuation of the shear does not modify the fundamental structure of the equilibrium states which is obtained by taking the thermal average in the perturbed scheme, that is, in the thermodynamic limit $V \to \infty$ (see [17]).

To proceed within our scenario let us present a toy model which provides a calculable spectrum for the density of contrast

$$\frac{\delta\rho}{\bar{\rho}}$$

.

Taking the average of the perturbed Raychaudhuri's equation we obtain

$$D + < \bar{\sigma}^{\mu\nu}\,\delta\sigma_{\mu\nu} >= \tilde{\lambda} < \delta\rho > \tag{19}$$

in which $\tilde{\lambda} \equiv \frac{1-3\lambda}{2}$ and D is a spatial constant which results from to the vanishing of the space fluctuation of the Hubble parameter. A further simplification of the model sets two hypothesis on the way this mean is to be calculated:

- Take the thermal averages for the shear;
- Use the Gaussian approximation to evaluate the two-point correlation function for the shear.

The fluctuations that satisfy these conditions have been examined in deGennes'work [12, 17]. We define

$$\delta\Psi \equiv \bar{\sigma}^{\mu\nu}\delta\sigma_{\mu\nu}$$

then

$$< \delta\Psi > + D = \tilde{\lambda} < \delta\rho > \tag{20}$$

It then follows, for the two-point correlation function

$$< \delta\Psi(x^i)\, \delta\Psi(x^i + r^i) > + D^2 = \tilde{\lambda}^2 < \delta\rho(x^i)\, \delta\rho(x^i + r^i) > \tag{21}$$

The right-hand side is nothing but the product of $\tilde{\lambda}^2$ and $\bar{\rho}^2$ with the galaxy-galaxy correlation function $\xi(r^i)$ [16].

We are concerned with the spectrum of galaxy perturbation δ_k which is nothing but the Fourier transformation of $\delta\rho/\bar{\rho}$. Hence it depends on the two-point correlation function of the shear. Applying Ginzburg's criterium we should evaluate the two-point correlation function of the shear in order to obtain the corresponding spectrum of the density fluctuations. For simplicity, it is worth to work in the associated phase space. We set

$$\sigma_{ij}\,(k) = \frac{1}{V} \int d^3x \sigma_{ij}\,(x) exp(-ikx)$$

Since we limit our analysis here only to the uniaxial case we can use standard parametrization. Then, the average matrix of shear takes the form

$$\begin{pmatrix} -\frac{1}{2}\bar{\Sigma} & 0 & 0 \\ 0 & -\frac{1}{2}\bar{\Sigma} & 0 \\ 0 & 0 & \bar{\Sigma} \end{pmatrix}. \tag{22}$$

So, the free energy, that is given by equation (17), becomes:

$$F = \frac{1}{2(2\pi)^3} \int d^3k(\Phi + e_1 k^2)\, Tr\,(\sigma_{ij})^2 \tag{23}$$

The mean energy is given by:

$$\bar{F} = -k_B T\, lim_{V\to\infty} \frac{lnZ}{V} \tag{24}$$

where,

$$Z = \int [D\delta\Psi] exp(-\beta F),$$

and $\delta\Psi$ is nothing but the quantity Σ in the above parametrization.

Hence, the two-point correlation function is:

$$< \delta\Psi(k)\delta\Psi(k') >= Z^{-1} \int [D\delta\Psi]\, \delta\Psi(k)\, \delta\Psi(k')\, exp(-\beta F) \tag{25}$$

Following a standard procedure (see Appendix B) to evaluate this quantity, we obtain:

$$< \delta\Psi(k)\, \delta\Psi(k') >= \frac{1}{\Phi + e_1 k^2} \tag{26}$$

It then follows, for the power spectrum δ_k^2 the form

$$\delta_k^2 \sim \frac{1}{\Phi + e_1 k^2}$$

This accomplishes our task here to obtain the form of the spectrum of the matter perturbation generated by the shear fluctuation. Let us make now some further comments on this.

First of all we should say that although we have made all the above calculations for the case in which there is a plane of isotropy, that is, for the uniaxial case, the power spectrum that we have obtained is almost independent of this[13]. The reason for this independence is a direct consequence of the gaussian nature of the approximation that we have employed in our calculation.

The form of the spectrum that we obtained has a very important advantage over most of the concurrents: in the standard power law spectra, for instance, there appear divergences either on large or on small scales, if they are extended over the infinite domain. This is not the case for the spectra we derived above which is bounded in the entire range.

4 The Power Spectrum

The model that we are presenting here is an example of a scenario concerning the effects of a prior anisotropic era in the cosmic past. It seems worth thus to analyse some of its main consequences that could be submited to the observational test[14]. Just to simplify our exposition we will compare our results with the theoretical predictions of a typical inflationary scenario.

The inflationary model provides a power spectrum for the perturbation which attracted the interest of theorists. Letting aside the many difficulties from the fundamental point of view that still pervades such scenario, the main drawback of it, from an observational point of view, as it has been pointed out by many authors [21], is precisely the difficulty of conciliating the scale invariance property of such model to the observed power spectrum of galaxy clustering determined from the CfA Redshift Survey, for instance. This observation shows an evolution for the spectrum that can be understood only if one accepts the idea that the fluctuations in the gravitational potential of a density distribution becomes dependent on the scale of the perturbation. In other words, it becomes difficult to conciliate the scale independence of standard inflation with observation. A possible solution to this could be, as suggested by Jones, to fit Harrison Zel'dovich spectrum to the corresponding observed large scale part and accepts a factor two of disagreement in the small scale amplitudes. This, of course, is somehow a compromise solution which becomes possible only due to our ignorance of the galaxy formation process, as pointed out in [21].

[13]We will analyse the bi-axial case in a subsequent work.

[14]In this section we will follow the notation and the main ideas on the status of observational cosmology as described by Jones [21].

4.1 The Large Scale Limit and the Angular Dependence of Temperature Fluctuations

The behaviour of the large scale fluctuations predicted in the anisotropic model that we are presenting here is similar to a white noise disturbance. Indeed, from the expression of δ_k^2 above, this spectrum becomes constant for small values of k, that is, for large scale of perturbations.

Through an analysis of the Sachs-Wolfe effect, it then follows a direct relationship between temperature fluctuations and the spectrum of density fluctuation. In the case of very large scales a direct examination [21] shows that it is possible to distinguish the behaviour of the fluctuation of the temperature on angular scales. The rms value of the temperature fluctuation is given by

$$\frac{\Delta T}{T} \sim \theta^{\frac{1}{2}(1-n)}.$$

It then follows from this expression that the model which we present here shows an explicit angular dependence for the amplitude of the temperature fluctuations which is given by

$$\frac{\Delta T}{T} \sim \theta^{\frac{1}{2}}.$$

This is a well desired result once it points out on the possibility to decide between cosmological scenarios from observations.

Finally, let us add that a direct inspection of the spectrum of the galaxy clustering shows that it is possible to fit it over the observed range with our analitical function for δ_k. We will come back to this elsewhere.

5 Conclusion

The Cosmological Program that we are considering in the present paper can be summarized by the following steps:

- The global structure of the Universe can be represented by the highly symmetric configuration of FRW geometry.
- The corresponding material content of the Universe is described by a perfect fluid.
- These two above properties of both geometry and matter of the Universe were not valid during all its history.
- In the past, very near the point of maximum global condensation (denoted as **primordial era**) the Universe passed a phase in which the overall process of expansion was less regular and a non-negligeable anisotropy of the spatially homogeneous cosmic fluid existed.
- This phase did not admit the representation of the cosmic matter content in terms of such simple perfect fluid description: viscous processes existed.

- The elimination of the primordial irregularities (like the primordial shear, for instance) were made precisely through viscous processes.
- It is possible to use the standard deGennes-Landau theory of phase transition to model such process of regularization.
- Small fluctuations on the spatially homogeneous structure that ocurred in the primordial era can survive the phase transition as it occurs in standard fluids that satisfy Ginzburg's criterium of perturbation.
- The actual system of galaxies and clusters that we observe today are nothing but a residual consequence of those primordial fluctuations in the anisotropic era.

The present paper gives a simple model of this program.

6 Appendix A

In Landau's theory the structure of the phase transition is contained in the polynomial dependence of the free energy on an order-parameter σ, e.g.

$$F(p,T,\sigma) = F_0(p,T) + n\sigma + a\sigma^2 + b\sigma^3 + c\sigma^4 + \cdots .$$

The possibility of an equilibrium state $\sigma = 0$ imposes that $n = 0$. Besides, the existence of different phases characterized by a control-parameter, say the Temperature T, is contained in the assumption that the coefficient a can be written as $a = a_0(T - T_c)$. In the case of a tensor parameter, $\sigma_{\mu\nu}$ like the one employed in this paper to define the liquid-crystal behaviour, the external influence trough, for instance, a magnetic field yields a modification of the above series by the additional term $\Delta F \sim H_\mu H_\nu \sigma^{\mu\nu}$. This, in turn can be equivalently written in terms of the electromagnetic energy-momentum tensor $T_{\mu\nu}$, once it follows that $\Delta F \sim T_{\mu\nu}\sigma^{\mu\nu}$ as $\sigma_{\mu\nu}$ is symmetric and traceless. Using Einstein's General Relativity, we write

$$\Delta F \sim R_{\mu\nu}\sigma^{\mu\nu}$$

which is at the origin of our equation (13) from the text. The use of this expression in the case of a self-gravitating Stokesian fluid implies that Landau scheme of phase transition is automatically sets into work. The role of the control-parameter is played by the Hubble expansion Θ, as in reference [6].

7 Appendix B

Just for completeness let us present here a rough calculation of the Gaussian two-point correlation function. We set

$$< \delta\Psi^*(q_0)\, \delta\Psi(q_0') >= Z^{-1} \int [D\delta\Psi] \delta\Psi^*(q)\, \delta\Psi(q')\, exp(-\beta F) \tag{27}$$

Where the free energy is given by:

$$F = \int \Psi^*(q)\, \mathbf{T}(\mathbf{q}, \mathbf{q}')\, \Psi(q') d^3q\, d^3q' \tag{28}$$

In order to solve eq (27) we follow the common trick of defining a functional $W[p]$:

$$\begin{aligned} W[p] &= \int [D\Psi]\, exp[-\frac{\beta}{2} \int \Psi^*(q)\, \mathbf{T}(q, q')\, \Psi(q') d^3q\, d^3q'] \\ &. \quad exp[\int p(q)\, \Psi(q)\, d^3q] \end{aligned} \tag{29}$$

Hence the equation (27) becomes:

$$< \delta\Psi^*(q_0)\, \delta\Psi(q_0') >= \frac{1}{W[p]}\, lim_{p\to 0}\, \frac{\delta^2 W[p]}{\delta p(q_0)\delta p(q_0')} \tag{30}$$

In order to evaluate $W[p]$ we transform the integral into a sum. Thus, it yields that $W[p]$ is nothing but a Gaussian integral, which can be evaluated in a straightforward way. The result is

$$W[p] = \frac{2\pi}{\beta\epsilon^6}\, (det\, \mathbf{T})^{\frac{1}{2}}\, \exp[\frac{\beta\epsilon^6}{2} \sum_{i,j} p_i\, T_{ij}^{-1}\, p_j] \tag{31}$$

Coming back to the integral form and making the functional derivative on p we have

$$W[p] = N exp(\frac{\beta}{2} \int p(q)\, \mathbf{T}^{-1}(q, q')\, p(q') d^3q d^3q') \tag{32}$$

That is

$$\frac{\delta^2 W[p]}{\delta p(q_0)\delta p(q_0')} = \beta \mathbf{T}^{-1} W[p] + (\beta \int d^3q' \mathbf{T}^{-1}(q_0, q') p(q'))^2\, W[p] \tag{33}$$

Hence, using equation (30) we have:

$$< \delta\Psi^*(q_0)\, \delta\Psi(q_0') >= \beta\, \mathbf{T}^{-1}(q_0, q_0') \tag{34}$$

This result independs of the particular form of the operator $\mathbf{T}$.

From our hypothesis of the toy model we are considering, we can evaluate this for a simple liquid-crystal configuration. Let us take the known example of a calculation of $< \delta\Psi(x^i)\delta\Psi(x^i + r^i) >$ in the special case in which the elastic constant e_2 vanishes[15]. Assuming that the background has a planar anisotropy, the knowledge of $\delta\Psi$ depends only on one single component of the perturbed shear. Thus, without generality loss, we set

$$\bar{\sigma}^{ij} \delta\sigma_{ij} = -3\bar{\Sigma}\Psi_{per} \tag{35}$$

[15]This corresponds to our case in which the divergence of the shear is taken to vanishes.

in which Ψ_{per} represents the $(0-0)$ component of the perturbed shear and the average shear is given by

$$\begin{pmatrix} -\frac{1}{2}\bar{\Sigma} & 0 & 0 \\ 0 & -\frac{1}{2}\bar{\Sigma} & 0 \\ 0 & 0 & \bar{\Sigma} \end{pmatrix}. \tag{36}$$

The calculation of the Fourier transform of Ψ_{per} is known in the literature [17] and yields:

$$\Psi^2_{per}(k) = \frac{B}{A + e_1 k^2} \tag{37}$$

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