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Fundamental Physical Constants and Their Stability

by

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Abstract

The choice, nature, classification and precision of determination of fundamental physical constants are described. Problem of temporal variations is also discussed. Need for further determinations of absolute measurements of G and time variations of the gravitational constant is pointed out.

Key-words: Constants; Variations; Unified theories.

1. In any physical theory we meet with constants which characterize the stability properties of different types of matter: of objects, processes, classes of processes and so on. These constants are important because they arise independently in different situations and have the same value, at any rate within accuracies we gained nowadays. That is why they are called fundamental physical constants (FPC) [1]. To define strictly this notion is not possible. It is because the constants, mainly dimensional, are present in definite physical theories. In the process of scientific progress some theories are replaced by more general ones with their own constants, some relations between old and new constants arise. So, we may talk not about absolute choice of FPC, but only about the choice corresponding to the present state of the physical sciences.

Really, quite recently (before the creation of the electroweak interaction theory and some Grand Unification Models it was considered that this *choice* is the followings:

$$c,\ \hbar,\ \alpha,\ G_F,\ g_s,\ m_p (or\ m_e),\ G,\ H,\ \rho,\ \Lambda,\ k,\ I,$$

where α, G_F, g_s and G are constants of electromagnetic, weak, strong and gravitational interactions, H, ρ and Λ are cosmological parameters (Hubble constant, mean density of the Universe and cosmological constant), k and I are the Boltzmann constant and the mechanical equivalent of heat which play the role of conversion factors between temperature from one side, energy and mechanical units from another side. After adoption in 1983 of a new definition of the meter ($\lambda = ct\ or\ \ell = ct$) this role is partially played also by the speed of light c. It is now also a conversion factor between units of time (frequency) and length, it is defined with absolute (null) accuracy.

Now, when the theory of electroweak interactions has a firm experimental basis and we have some good models of strong interactions the more prefarable choice is as follows:

$$\hbar,\ (c),\ e,\ m_e,\ \theta_w,\ G_F,\ \theta_c,\ \Lambda_{QCD},\ G,\ H,\ \rho,\ \Lambda,\ k,\ I$$

and, possibly, three angles of Kobayashi-Maskawa - θ_2, θ_3 and δ. Here θ_w is the Weinberg angle, θ_c is the Cabibbo angle and Λ_{QCD} is a cut-off parameter of quantum chromodynamics. Of course, if the theory of four known now interactions will be created then we probably will have another choice. As we see the macroconstants remain the same though in some unified models, i.e. in multidimensional ones, they may be related in some manner (see below).

All these constants are known with different *accuracies*. The most precisely defined constant was and remain the speed of light c: its accuracy was 10^{-10} and now it is defined with null accuracy. Atomic constants, $e, \hbar, m$ and others are defined with accuracies $10^{-6} \div 10^{-7}$, G-with the accuracy 10^{-4}, θ_w-with accuracy 10%; the accuracy of H is also 10% though several groups give values differing by the factor of 2. Even worse situation is now with other cosmological parameters (FPC): mean density estimations vary within an order of magnitude; for Λ we have limits above and below, in particular zero value is also acceptable.

As to the *nature* of FPC, we may mention several approaches. One of the first hypotheses belongs to J.A. Wheeler: in each cycle of the Universe evolution FPC arise anew along with physical laws which govern its evolution. Thus, the nature of FPC and physical laws is connected with the origin and evolution of our Universe.

Less global approach to the nature of dimensional constants suggests that they are needed to make physical relations dimensionless or they are measures of asimptotic states. Really, the speed of light appears in relativistic theories in factors like v/c, at the same time velocities of usual bodies are less than c, so it plays also the role of an asymptotic limit. The same sense have some other FPC: $\hbar$ is the minimal quantum of action, e is the minimal observable charge (if we do not take into account quarks which are not observable in a free state) etc.

Finally, FPC or their combinations may be considered as natural scales defining basic units. If earlier basic units were chosen more or less arbitrarily, i.e. the second, meter and kilogram, than now first two are based on stable (quantum) phenomena. Their stability is ensured by well established physical laws which include FPC.

Exact knowledge of FPC and precision measurements are necessary for testing main physical theories, extention of our knowledge of nature and, in the long run, for practical applications of fundamental theories. Within this, such theoretical problems arise: 1) development of models for confrontation of a theory with experiment in critical situations (i.e. for verification of *GR*, *QED*, *QCD* or *GUT*), 2) setting limits for spacial and temporal variations of FPC.

As to *classification* of FPC we may set them now into four groups due to

their generality:
1) Universal constants such as $\hbar$ which divides all phenomena into quantum and nonquantum (micro and macro worlds) and to a certain extent c, which divides all motions into relativistic and nonrelativistic, 2) constants of interactions like α, θ_w, Λ_{QCD} and G; 3) constants of elementary constituencies of matter like m_e, m_w, m_x, etc., and 4) transformation multipliers such as k, I and partially c. Of course, this division into classes is not absolute. Many constants shifted from one class to another. For example, e was a charge of a particular object-electron, class 3, then it became a characteristic of a class 2 (electromagnetic interaction, $\alpha = \frac{e^2}{\hbar c}$ in combination with $\hbar$ and c), speed of light c was nearly in all classes: from 3 it moved into 1, then also into 4. Some of the constants ceased to be fundamental (i.e. densities, magnetic moments, etc.) as they are calculated via other FPC.

As to the *number* of FPC, there are two opposite tendencies: number of "old" FPC is usually diminishing when a new, more general theory is created, but at the same time new fields of science arise, new processes are discovered in which new constants appear. So, in the long run we may come to some minimal choice which is characterized by one or several FPC, may be, connected with the so called Planck parameters-combinations of c, $\hbar$ and G:

$$L = \left(\frac{\hbar G}{c^3}\right)^{1/2} \sim 10^{-33} cm, \quad m_L = (c\hbar/2G)^{1/2} \sim 10^{-5} g \ , \\ \tau_L = L/c \sim 10^{-43} s.$$

The role of these parameters is important as m_L characterizes the energy of unification of four known fundamental interactions: strong, weak, electromagnetic and gravitational ones and L is a scale where classical notions of space-time loose their meaning.

2. The problem of the gravitational constant G measurement and stability is a part of a very much developing field, called gravitational-relativistic metrology. It appeared due to the growth of a measuring technique precision, spread of measurements over large scales and tendency to the unification of fundamental physical interaction (see [2]).

Absolute value measurements of G. There are several laboratory determinations of G with precisions of 10^{-3} and only 4 at the level of 10^{-4}. They are (in $10^{-11} m^3 kg^{-1} s^{-2}$);

1. Facy, Pontikis, France, 1972 - 6,6714 ± 0,0006

2. Sagitov et al., USSR, 1979 - 6,6745 ± 0,0008
3. Luther, Towler, USA, 1982 - 6,6726 ± 0,0005
4. Karagioz, USSR, 1988 - 6,6731 ± 0,0004

From this table it is seen that first three experiments contradict each other (they do not overlap within their accuracies). And only the fourth experiment is in accord with the third one.

The official CODATA value of 1986

$$G = (6,67259 \pm 0,00085) \cdot 10^{-11} \cdot m^3 \cdot kg^{-1} \cdot s^{-2}$$

is based on the Luther and Towler determination. One should make a conclusion that the problem is still open and we need further experiments on the absolute value of G. Many groups are preparing and doing them using different types of technique, among them is the Karagioz group (Russia) which has the installation operating already for two years continuously [3].

There exist also some satellite determinations of G (namely $G \cdot M_{earth}$) at the level of 10^{-8} and several geophysical determinations in mines. The last give usually much higher G values than the laboratory ones.

The precise knowledge of G is necessary for the evaluation of mass of the Earth, planets, their mean density and in the end for the construction of Earth models; for transition from mechanical to electromagnetic units and back; for evaluation of other constants through relations between them given by unified theories; for finding new possible types of interactions and geophysical effects.

The knowledge of constants values has not only a fundamental meaning but also the metrological one. Modern system of standards is based mainly on stable physical phenomena. So, the stability of constants plays a crucial role. As all physical laws were established and tested during last 2-3 centuries in experiments on the Earth and in the near space, i.e. at a rather short space and time intervals in comparison with the radius and age of the Universe the possibility of slow *variations* of constants (i.e. with the rate of the evolution of the Universe) cannot be excluded a priori.

So, the supposition about the absolute stability of constants is an extrapolation and each time we must test it.

3.The problem of variations of FPC arose with the attempts of explanation of relations between micro and macroworld fenomena. Dirac was the first to introduce [4] the so called "Large Numbers Hypothesis" which relates some known very big (or very small) numbers with the dimensionless age of the Universe $T \sim 10^{40}$ (age of the Universe in seconds 10^{17}, divided by the characteristic elementary particle time 10^{-23} seconds). He suggested that the ratio of the gravitational to strong interactions strengths, $Gmp^2/\hbar c \sim 10^{-40}$, is inversely proportional to the age of the Universe: $Gmp^2/\hbar c \sim T^{-1}$. Then, as the age varies some constants or their combinations must vary also. Atomic constants seemed to Dirac more stable so he've chosen the variation of G as T^{-1}.

After original Dirac *hypothesis* some new ones appeared and also some generalized *theories* of gravitation admitting the variations of an effective gravitational coupling. We may single out two stages in the development of this field:

1. Study of theories and hypotheses with variations of FPC, their predictions and confrontation with experiments (1937-1977).
2. Creation of theories admitting variations of an effective gravitational constant in a particular system of units, analyses of experimental and observational data within these theories [5-7] (1977-present).

Within the development of the first stage from the analysis of the whole set of existed astronomical, astrophysical, geophysical and laboratory data the conclusion was made [6,1] that variations of atomic constants are excluded, but variations of the effective gravitational constant in atomic system of units do not contradict available experimental data at the level $10^{-11} \div 10^{-12} year^{-1}$. Moreover, in [5-7] the conception was worked out that variations of constants are not absolute but depend on the system of measurements (choice of standards, units and devices using this or that fundamental interaction). Each fundamental interaction through dynamics, described by the corresponding theory, defines the system of units and the system of basic standards.

Now we review shortly some hypotheses on variations of FPC and experimental tests [1]:

Following Dyson (1972) we may introduce dimensionless combinations of micro and macroconstants:

$$\alpha = e^2/\hbar c = 7,3.10^{-3} \ , \quad \gamma = Gm^2/\hbar c = 5.10^{-39} \ ,$$

$$\beta = G_F\, m^2 c/\hbar^3 = 9.10^6 \ , \quad \delta = H\hbar/mc^2 = 10^{-42} \ ,$$
$$\varepsilon = \rho G/H^2 = 2.10^{-3} \ , \quad t = T/(e^2/mc^3) \approx 10^{40}$$

We see that α, β and ε are of order 1 and γ and δ are of the order 10^{-40}. Nearly all existing hypotheses on variations of FPC may be represented as:

Hyposesis 1 (standard):
α, β, γ are constant, $\delta \sim t^{-1}, \varepsilon \sim t$.

Here we have no variations of G and δ and ε are defined via cosmological solutions.

Hyposesis 2 (Dirac):
α, β, ε are constant, $\gamma \sim t^{-1}$, $\delta \sim t^{-1}$.

Then $\dot{G}\,/G = 5.10^{-11} year^{-1}$ if the age of the Universe is taken as $T = 2.10^{10}$ years.

Hyposesis 3 (Gamow):
$\gamma/\alpha = Gm^2/e^2 \sim 10^{-37}$, so e^2 or α are varied, but not G, $\beta, \gamma, \varepsilon =$ const, $\alpha \sim t^{-1}$, $\delta \sim t^{-1}$.

Then $\dot{\alpha}\,/\alpha = 10^{-10} year^{-1}$.

Hyposesis 4 (Teller): trying to account also for deviations of α from 1 he suggested $\alpha^{-1} = \ell n \gamma^{-1}$.

Then β, ε are constants, $\gamma \sim t^{-1}$, $\alpha \sim (\ell n t)^{-1}$, $\delta \sim t^{-1}$

$$\dot{\alpha}\,/\alpha = 5.10^{-13} year^{-1}$$

The same relation for α and γ was used also by Landau, DeWitt, Staniucovich, Terasawa and others, but in different approaches in comparison with Teller.

Some other variants may be also possible, e.g. Brans-Dicke theory with $G \sim t^{-r}$, $\rho \sim t^{r-2}$, $r = [2 + \frac{3w}{2}]^{-1}$, the combination of Gamov's approach and Brans-Dicke's etc. [1].

4. There are different astronomical, geophysical and laboratory *data* on possible variations of FPC.

astrophysical data:

a) from comparison of fine structure ($\sim \alpha^2$) and relativistic fine structure ($\sim \alpha^4$) shifts in spectra of radiogalaxies Bahcall and Schmidt (1967) obtained

$$|\dot{\alpha} / \alpha | \leq 2.10^{-12} year^{-1}$$

b) comparing lines in optical ($\sim Ry = me^4/\hbar^2$) and radio bands of the same sources in galaxies Baum and Florentin-Nielsen got the estimate

$$|\dot{\alpha} / \alpha | \leq 10^{-13} year^{-1},$$

and for extragalactic objects

$$| \alpha/\alpha | \leq 10^{-14} year^{-1}$$

c) from observations of superfine structure in H-absorption lines of the distant radiosource Wolf et al. (1976) obtained that

$$| \alpha^2 (m_e/m_p) g_p | < 2.10^{-14} \ ;$$

from these data it is seen that hyposesis 3 and 4 are excluded. The same conclusion is done on the bases of *geophysical data*. Really,

a) α-decay of $U_{238} \rightarrow Pb_{208}$. Knowing abundancies of U_{238} and P_{238} in rocks and independently the age of these rocks the limit

$$|\dot{\alpha} / \alpha | \leq 2.10^{-13} year^{-1}$$

was obtained,

b) from spontaneous fission of U_{238} such estimation was done: $|\dot{\alpha} / \alpha | \leq 2, 3.10^{-13} year^{-1}$.

c) finally, from β-decay of Re_{187} to Os_{187}

$$|\dot{\alpha} / \alpha | \leq 5.10^{-15} year^{-1}$$

was obtained.

We must point out that all astronomical and geophysical estimations are strongly model-dependant. So, of course, it is always desirable to have *laboratory tests* of variations of FPC.

a) such a test was first done by the Russian group in the Committee for Standards (Kolosnitsyn, 1975). Comparing rates of two different types of clocks, one based on the Cs standard and another on the beam molecular generator they found that $|\dot{\alpha}\ /\alpha\ | \leq 10^{-10} year^{-1}$.

b) from similar comparison of Cs standard and $SCCG$ (Super Conducting Cavity Generator) clocks rates Turner (1976) obtained the limit

$$|\dot{\alpha}\ /\alpha\ | \leq 4.1 \cdot 10^{-12} year^{-1}$$

All these limits were placed on the fine structure constant variations. From the analysis of decay rates of K_{40} and Re_{187} the limit on the possible variations of the weak interaction constant was obtained (see approach for variations of β, e.g. in [9]).

$$|\dot{\beta}\ /\beta\ | \leq 10^{-10} year^{-1}.$$

But the most strict data were obtained by A. Schlyachter (USSR) from the analysis of the ancient natural nuclear reactor data in Gabon, Oklo, because the event took place 2.10^9 years ago. They are the following:

$$|\dot{G}_s\ /G_s\ | < 5.10^{-19} year^{-1}, |\dot{\alpha}\ /\alpha\ | < 10^{-17} year^{-1}$$
$$|\dot{G}_F\ /G_F\ | < 2.10^{-12} year^{-1}$$

So, we really see that all existing hyposeses with variations of atomic constants are excluded.

5. Now we still have no unified theory of all four interactions. There is a good theory of electroweak interactions, models of GUT which include the strong interaction and also some attempts to create a theory of everything (TOE). As we have no such a theory it is possible to construct systems of measurements based on any of these four interactions. But practically it is done now on the basis of the mostly worked out theory – on electrodynamics (more precisely on QED). Of course, it may be done also on the basis of the gravitational interaction (as it was partially earlier). Then, different units of basic physical quantities arise based on dynamics of the given interaction, i.e. the atomic (electromagnetic) second, defined via frequency of atomic transitions or the gravitational second defined by the mean Earth motion around the Sun (ephemeris time).

It doesn't follow from anything that these two seconds are always synchronized in time and space. So, in principal they may evolve relatively each other, for example with the rate of the evolution of the Universe or some other rate.

That is why in general variations of the gravitational constant are possible in atomic system of units ($c, \hbar, m$ are constant) and masses of all particles - in gravitational system of units (G, $\hbar$, c are constants by definition). Practically we can test only the first variant as modern basic standards are defined in atomic system of measurements. Possible variations of FPC had to be tested experimentally but for this it is necessary to develop corresponding theories admitting such variations and their definite effects.

Mathematically these systems of measurement may be realized as two conformally related metric forms. Arbitrary conformal transformations give us a transition to an arbitrary system of measurements.

One of the ways to describe variable gravitational coupling is the introduction of a *scalar field* as an additional variable of the gravitational interaction. It may be done by different means (e.g. Jordan, Brans-Dicke, Canuto and others). We prefare the variant of gravitational theory with conformal scalar field (Higgs-type field [9]) where Eintein's general relativity may be considered as a result of spontaneous symmetry breaking of the conformal symmetry (Domokos, 1976). In our variant spontaneous symmetry breaking of the global gauge invariance leads to nonsingular cosmology [10]. Besides, we may get variations of the effective gravitational constant in the atomic system of units when m, c, $\hbar$ are constant and variations of all masses in the gravitational system of units (G, c, $\hbar$ are constant). It is done on the basis of approximate [11] and exact cosmological solutions with local inhomogenity [12].

The effective gravitational constant is calculated using equations of motions. Postnewtonian expansion is also used in order to confront the theory with existing experimental data. Among postnewtonian parameters the parameter f describing variations of G is included. It is defined as

$$\frac{1}{GM}\frac{d(GM)}{dt} = fH \ . \qquad \textbf{(0.1)}$$

According to Hellings data [13] from the Viking mission

$$\tilde{\gamma} -1 = (-1,2 \pm 1,6)\cdot 10^{3}, \quad f = (4 \pm 8)\cdot 10^{-2} \qquad (0.2)$$

In the theory with conformal Higgs field [11] we obtained the following relation between f and $\tilde{\gamma}$:

$$f = 4(\tilde{\gamma} -1)\ . \qquad (0.3)$$

Using Hellings data for $\tilde{\gamma}$ we may calculate in our variant f and compare it with f from [13]. Then we get $f = (-9,6 \pm 12,8)\cdot 10^{-3}$ which agrees with (0.2) within its accuracy.

We used here only Hellings data of variations of G. But the situation with experiment and observations is not so simple. Along with [13] there are some other data [1]:

1. From the growth of corals, pulsar spin down, etc. on the level $|\dot{G}/G| < 10^{-10} \div 10^{-11} year^{-1}$.

2. Van Flandern's positive data from the analysis of a lunar mean motion around the Earth and ancient eclipses data (1976, 1981):

   $$|\dot{G}/G| = (6 \pm 2)10^{-11} y^{-1}.$$

3. Reasenberg's estimates of the same Viking mission as in [13] (1987):

   $$|\dot{G}/G| < (0 \pm 2)\cdot 10^{-11} y^{-1}$$

4. Hellings result in the same form is

   $$|\dot{G}/G| < (2 \pm 4)\cdot 10^{-12} y^{-1}$$

5. Recent result from nucleosythesis (Acceta et al. 1992)

   $$|\dot{G}/G| < (\pm 0,9)\cdot 10^{-12} y^{-1}$$

As we see there is a vivid contradiction in these results, so, of course, further experiments are necessary for solving the problem of temporal G variations. The most promising are the planned future missions to Mars (1994).

According to Hellings estimations [13] after several years of observations of spacecrafts on and around the Mars one may have the improvement of the order of magnitude in a testing of $\dot{G}/G$.

As we saw different theoretical schemes lead to temporal variations of the effective gravitational constant:

1. Empirical models and theories of Dirac's type, were G is replaced by $G(t)$.
2. Numerous scalar-tensor theories of Jordan-Brans-Dicke type where G depending on the scalar field $\sigma(t)$ appears.
3. Gravitational theories with the conformal scalar field arising in different approaches [6,7,14,15].
4. Multidimensional unified theories in which there are dilaton fields and effective scalar fields appear in our 4-dimensional spacetime from additional dimensions [16]. They may help also in solving the problem of changing cosmological constant from Planckian to present values.

As it was shown in [16,17] temporal variation of FPC are connected with each other in *multidimensional models* of unification of interactions. So, experimental tests on $\dot{\alpha}/\alpha$ may be at the same time be used for estimation of $\dot{G}/G$ and vice versa. Moreover, variations of G are related also to the cosmological parameters ρ, Ω and q that gives opportunities of raising the precision of their determination.

As variations of FPC are closely connected with the behaviour of internal scale factors it is a direct probe of properties of extra dimensions and corresponding theories [18-20].

Other windows for testing hidden dimensions are opening when one is studying multidimensional models in spherically-symmetrical case. Then, as we may see, some deviations from the Newton and Coulomb laws are possible [21-27].

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