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by

Juan A. Mignaco, Itzhak Roditi

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

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We present a slightly modified approach to the application of the principle of minimal sensitivity (PMS) used to improve results in perturbation theory. Calculations are shown to be easier and essentially give the same results.

Since the important work by Stevenson [1], following the pioneering work of Grunberg and of Celmaster and coworkers [2], the analysis of perturbative results improved by the application of the renormalization group (RG) [3] received a lot of attention. Stevenson adapted a criterion known as Principle of Minimal Sensitivity (PMS) to require that results at a given order of perturbation theory be calculated for a renormalization scheme (RS) such that the change at next order will be minimal, that is, the difference between the exact result (RG invariant) of a physical quantity and its perturbative approximation is chosen to be the smallest and least sensible to RS changes.

In this note, we want to advocate the principle of minimal sensitivity in to what we believe to be its most "economical" form, which we shall call $\overline{\text{PMS}}$. The economy is related to the somehow easier way results can be obtained as compared to Stevenson's original work [1] and several other developments made a posteriori [4].

In short, PMS uses the expansion up to a given order of the interesting quantities, imposes the automatic vanishing of terms up to the same order and from the remainder extracts information on the optimal values of the couplant and the parameters characterizing the optimal RS (up to that order). The equations are compatible with the consistency conditions for the problem. To be precise, consider (as Stevenson [1]) a physical quantity R. The consistency conditions are

$$\frac{\partial R}{\partial \tau} \equiv \left(\frac{\partial}{\partial \tau}\Big|_a + \hat{\beta}(a)\frac{\partial}{\partial a}\right)R = 0 \qquad (1a)$$

$$\frac{\partial R}{\partial c_j} \equiv \left(\frac{\partial}{\partial c_j}\bigg|_a + \beta_j(a)\frac{\partial}{\partial a}\right)R = 0 \tag{1b}$$

where τ, a, c_j, $\hat{\beta}$, β_j and the following W_i^j are the same as in Stevenson [1].

Now, assume that R has a perturbative expansion

$$R = a^N(1 + r_1 a + r_2 a^2 + \ldots) \tag{2}$$

as well as

$$\hat{\beta} = -a^2(1 + ca + c_2 a^2 + c_2 a^3 + \ldots) \tag{3}$$

and

$$\hat{\beta}_j = \frac{\partial a}{\partial c_j} = \frac{1}{j-1}\, a^{j+1}\left[1 + W_i^j a + W_2^j a^2 + \ldots\right] \quad (j = 2,3,\ldots) \tag{4}$$

with

$$W_i^j = \frac{-1}{(i+j-1)} \sum_{k=1}^{i} (i + j - 1 - 2k)\, c_k W_{i-k}^j \qquad (i = 1,2,\ldots) \tag{5}$$

$$c_1 \equiv c, \quad W_o^j \equiv 1$$

The consistency conditions, order by order in powers of the couplant‡ are:

‡Notice that here the powers a^{N+i} are related to the powers a^{N+i-1} of the physical quantity R.

$$a^{N+1}\left(\frac{\partial r_1}{\partial \tau} - N\right) + a^{N+2}\left(\frac{\partial r_2}{\partial \tau} - Nc - (N+1)r_1\right) +$$

$$+ a^{N+3}\left(\frac{\partial r_3}{\partial \tau} - Nc_2 - (N+1)r_1 c - (N+2)r_2\right) + \ldots = 0$$

$$a^{N+1}\frac{\partial r_1}{\partial c_2} + a^{N+2}\left(\frac{\partial r_2}{\partial c_2} + N\right) + a^{N+1}\left(\frac{\partial r_3}{\partial c_2} + NW_1^2 + (N+1)r_1\right) + \ldots = 0 \tag{6}$$

$$a^{N+1}\frac{\partial r_1}{\partial c_3} + a^{N+2}\frac{\partial r_2}{\partial c_3} + a^{N+3}\left(\frac{\partial r_3}{\partial c_2} + \frac{N}{2}\right) + \ldots = 0$$

From them we see, for instance, that the first term must vanish identically at lowest order for R and β. The condition PMS uses to go a little further is

$$Nc + (N+1)r_1 + (N+1)r_1 ca = 0 \tag{7}$$

which is just a mixture of the terms appearing at orders a^{N+2} and a^{N+3} involving only c and r_1, the quantities being used at lowest orders of R and β. Notice that one must solve for $\bar{r}_1$ as well as for $\bar{a}$. PMS in this sense goes beyond perturbative expansions. If, on the other hand we want to mantain ourselves within the perturbative framework we can start the improvement by imposing that only the terms belonging to order a^{N+2} should vanish

$$Nc + (N+1)r_1 = 0 \tag{8}$$

This is equivalent to say that the corrections to R due to RS changes begin at order a^{N+3}, that is $\left.\frac{\partial R}{\partial \tau}\right|_{\tau=\bar{\tau}} = O(a^{N+3})$

($\bar{\tau}$ being the particular value obtained below, see eqs. (10) and (15)). The extension of this procedure yields, at any order, characterized by the variation of the parameters $\tau, c_2, \ldots, c_k$:

$$\sum_{m=0} (N+m) r_m c_{k-m} = 0 \tag{9a}$$

$$- \frac{1}{j-1} \sum_{m=0}^{k+1-j} (N+m) r_m W^j_{k+1-j-m} = 0 \qquad (J = 2, \ldots, k) \tag{9b}$$

To supplement these equations we have the relations for the renormalization group invariants [1,2,4], $\rho_1, \ldots, \rho_k$. Solving the equations we end with the special values $\bar{\tau}, \bar{c}_2, \ldots \bar{c}_k$, $\bar{k}_2, \ldots \bar{r}_k$ needed. Next we solve for $\bar{a}$ using:

$$\bar{\tau} = (\bar{a})^{-1} + c\,\ell n\left(\frac{c\bar{a}}{1+c\bar{a}}\right) + \int_0^a \frac{(\bar{c}_2 + \ldots + \bar{c}_k x^{k-2})}{(1+cx)(1+cx+\ldots+\bar{c}_k x^k)} dx \tag{10}$$

obtaining:

$$R^{(k+1)}_{\overline{PMS}} = \bar{a}^N (1 + \bar{r}_1 \bar{a} + \ldots + \bar{r}_k \bar{a}^k) \tag{11}$$

The difference with the standard PMS prescription is that while in it relations involving implicitly $\bar{r}_i$'s and $\bar{a}$ are to be solved, here, in $\overline{PMS}$, one obtains values for $\bar{r}_i$'s that can be used for the $\bar{\tau}$ and $\bar{c}_i$'s. This makes $\overline{PMS}$ look rather similar to the Grunberg FAC criterion [2] ($\bar{r}_1 = \ldots = \bar{r}_k = 0$). With FAC,

$$R^{(k+1)}_{FAC} = \bar{a}^N \tag{12}$$

To illustrate this point, let us take the case of $R^{(2)}$ with $\bar{r}_1$ as given by eq. (8),

$$\bar{r}_1 = \frac{-N}{N+1} c \qquad (13)$$

The connection with the RG invariant ρ_1 is

$$\rho_1 = \bar{\tau} - \frac{\bar{r}_1}{N} \qquad (14)$$

giving

$$\bar{\tau} = \rho_1 - \frac{c}{N+1} \qquad (15)$$

With this value of $\bar{\tau}$, one solves for $\bar{a}$ (eq.(10)) and substitutes in R; the pertinent values for N = 1 are listed in table 1. It is evident, as Stevenson [1] himself pointed, that the difference between expressions for $\overline{\text{PMS}}$ and PMS is of order a^{N+k+2} (for $R^{(k+1)}$) since the start, and this fact appears in the numbers quoted (see table 1), but it is precisely our point here that handling eqs. (9) and (10) is simpler than the original PMS prescription. It is worth to emphasize here that the procedure we have just described is different from Stevenson's [1] improvement formula for we do not need to use any particular scheme to obtain the results.

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Table 1

Values of $R^{(2)}$ for the e^+e^- annihilation ratio [5] $\sum Q_j^2\ [1+R^{(2)}]$ with three flavors ($c = \frac{16}{9}$) and $\rho_1 = 10$ ($\frac{Q}{\Lambda_{MS}} = 35.287586$) within the various shemes. We also quote the respective values of couplant a and of r_1.

$R^{(2)} = a(1 + r_1 a)$			
	$R^{(2)}$	a	r_1
FAC	0.072118	0.072118	0
PMS	0.072396	0.077035	-0.781818
$\overline{\text{PMS}}$	0.072391	0.077767	-8/9