ISSN 0029 - 3865

CBPF-NF-046/84

$\lambda\phi^4$ IN ν DIMENSIONS

by

C.G. Bollini and J.J. Giambiagi

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

$\lambda\phi^4$ IN ν DIMENSIONS

Carlos Guido Bollini and J.J. Giambiagi

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

ABSTRACT

A unified treatment of the $\lambda\phi^4$ theory in any number of dimensions (ν) is given, within the scheme of dimensional regularization. Use is made of the Gaussian approximation and a comparison with the one loop effective potential is given in different dimensions.

For fixed renormalized quantities, there are cases (in particular for $\nu \Rightarrow 4$) for which the bare parameters go to zero. Nevertheless the corrections (Gaussian or perturbative) are such that the final results are finite.

Key-words: Field theory; Renormalization; Dimensional regularization; "$\lambda\phi^4$" theory.

§ I INTRODUCTION

One of the simplest theories that can be analized is the scalar one with $\lambda\phi^4$ coupling. Nevertheless, questions about its existence have been raised time and again [1] and even now the situation seems to be rather inconclusive.

Apparently, there is a point where the situation seems to be conclusive, namely: a well defined limiting process has to be used to be able to obtain "a theory".

Extensive use has been made lately of the "Gaussian method" [2] which is claimed to be better than the loop expansion [3], and has been applied to the case under discussion [4] and also for the analysis of symmetry breaking [5], a case previously discussed in reference [6].

What we want to do is to introduce dimensional regularization [7] to deal with the divergent integrals which appear. In this way we will also be able to consider the Gaussian effective potential for $\lambda\phi^4$ theory in any number of dimensions. We can see then in a compact way how the behaviour of the theory depends on the number of dimensions.

The same procedure is followed with the one-loop potential, which is written in any number of dimensions and compared with the Gaussian one.

In this analysis we adopt the point of view, already used in previous references, that the bare constants are adjusted so as to obtain the predetermined renormalized variables, m_R and λ_R.

In § II we write the Gaussian effective potential as a function of the number of dimensions ν. Following the path of ref. [4] we deduce the relations between m_R, λ_R and m_B, λ_B as well as the conditions to minimize the potential.

In § III we discuss the cases $\nu = 2n$.

In § IV we go to the specific cases $\nu = 1,2,3$ and 4.

In § V We repeat all these calculations with the one loop effective potential and finally in § VI we discuss the results.

§ II

We take the expression for the Gaussian effective potential from ref. [4] form (7)

$$V = I_1 + \frac{1}{2}(m_B^2 - \Omega^2) I_o + \frac{1}{2} m_B^2 \phi^2 + \lambda_B \phi^4 + 6\lambda_B I_o \phi^2 + 3\lambda_B I_o^2 \qquad (1)$$

where

$$I_n(\Omega) = \int \frac{d^3k\,(\omega_k)^{2n-1}}{(2\pi)^3 2} \qquad \omega_k = (\vec{k}^2 + \Omega^2)^{1/2} \qquad (2)$$

These integrals diverge for $n \geq -1$.

In order to handle these divergences we define:

$$I_n(\Omega) = \int \frac{d^{\nu-1}k}{2(2\pi)^{\nu-1}} \omega_k^{2n-1} = \frac{2\Pi^{\frac{\nu}{2}}}{\Gamma(\frac{\nu}{2})} \int_o^\infty \frac{dk\, k^{\nu-2}}{2(2\pi)^{\nu-1}} (\omega_k)^{2n-1} \qquad (3)$$

$$I_n = A_n \Omega^{2n+\nu-2}, \quad A_n = \frac{\Gamma(1-n-\frac{\nu}{2})}{2^\nu \Pi^{\frac{\nu}{2}-\frac{1}{2}}\, \Gamma(\frac{1}{2}-n)}, \quad A_o = \frac{\Gamma(1-\frac{\nu}{2})}{2^\nu \Pi^{\frac{\nu}{2}}} \qquad (4)$$

Note, in particular:

$$\nu A_1 = A_o \tag{5}$$

(1) represents the effective potential with the value (4) for the integrals in dimension ν.

We observe that (1) is well defined for ν = odd, while it has poles for ν = even.

The value of Ω which minimizes (1) is :(equating the derivative to zero and using (5))

$$\Omega^2 = m_B^2 + 12\lambda_B A_o \Omega^{\nu-2} + 12\lambda_B \phi^2 \tag{6}$$

Defining $\left.\frac{d^2V}{d\phi^2}\right|_{\phi=0} = m_R^2$ (remembering that $\frac{\partial V}{\partial\Omega} = 0$) and taking into account (6) we obtain:

$$m_B^2 + 12\lambda_B A_o \Omega_o^{\nu-2} = m_R^2 = \Omega_o^2 \tag{7}$$

where Ω_o^2 is the value of Ω^2 for $\phi = 0$. From (7),

$$m_B^2 = m_R^2 - 12\lambda_B A_o m_R^{\nu-2} \tag{8}$$

Replacing in (6)

$$(m_R^2 - \Omega^2) + 12\lambda_B A_o(\Omega^{\nu-2} - m_R^{\nu-2}) + 12\lambda_B \phi^2 = 0 \tag{9}$$

Define:

$$\lambda_R = \frac{1}{4!}\left.\frac{d^4V}{d\phi^4}\right|_{\phi=0} \qquad \text{with the result :} \tag{10}$$

$$\lambda_R = \frac{3\lambda_B}{1 - 6\lambda_B(\nu-2)A_o m_R^{\nu-4}} - 2\lambda_B \tag{11}$$

Solving this algebraic eq. gives:

$$\lambda_B = -\frac{1+6\lambda_R(\nu-2)A_o m_R^{\nu-4}}{24(\nu-2)A_o m_R^{\nu-4}} \pm \sqrt{\left[\frac{1+6\lambda_R(\nu-2)A_o m_R^{\nu-4}}{24(\nu-2)A_o m_R^{\nu-4}}\right]^2 + \frac{\lambda_R}{12(\nu-2)A_o m_R^{\nu-4}}} \tag{12}$$

we go back to (1) using (8) and substracting the value of V for $\phi = 0$. We obtain

$$p = V - V\Big|_{\phi=0} = \frac{A_o}{\nu}(\Omega^\nu - m_R^\nu) + \frac{1}{2}(m_R^2 - \Omega^2)A_o\Omega^{\nu-2} + \frac{1}{2}m_R^2\phi^2 + \lambda_B\phi^4$$
$$+ 3\lambda_B\left[A_o(\Omega^{\nu-2} - m_R^{\nu-2})\right]^2 - 6\lambda_B A_o(m_R^{\nu-2} - \Omega^{\nu-2})\phi^2 \tag{13}$$

By the use of (9) we can elliminate Ω^ν from this last eq. the result being:

$$p = -\frac{(\Omega^2 - m_R^2)^2}{48\lambda_B} + \frac{\Omega^2(\Omega^2 - m_R^2)}{12\nu\lambda_B} - 2\lambda_B\phi^4 + \left(\frac{1}{\nu} - \frac{1}{2}\right)(\Omega^2 - m_R^2)A_o m_R^{\nu-2} + \left(\frac{1}{2} - \frac{1}{\nu}\right)\Omega^2\phi^2 \tag{14}$$

Dividing by m_R^4 and introducing the definitions

$$x = \frac{\Omega^2}{m_R^2}\ , \quad \bar{\phi}^2 = \frac{\phi^2}{m_R^2} \quad \bar{p} = \frac{p}{m_R^4} \tag{15}$$

$$\bar{p} = -\frac{(x-1)^2}{48\lambda_B} + \frac{x(x-1)}{12\nu\lambda_B} - 2\lambda_B\bar{\phi}^4 + \left(\frac{1}{2} - \frac{1}{\nu}\right)(1-x)A_o m_R^{\nu-4} + \left(\frac{1}{2} - \frac{1}{\nu}\right)x\phi^2 \tag{16}$$

Dividing (9) by m_R^2 and using (15)

$$(1-x) + 12\lambda_B\bar{\phi}^2 + 12\lambda_B A_o(x^{\frac{\nu-2}{2}} - 1)m_R^{\nu-4} = 0 \tag{17}$$

This last eq. fixes x as a function of $\bar{\phi}$ and using this x in (16) we obtain the effective potential as a function of $\bar{\phi}$.

In all these eqs. the number of dimension ν appears as a parameter, covering in a compact way all integer dimensions.

§ II EFECTIVE POTENTIAL AS A FUNCTION OF ν

Our purpose is to study these pair of eqs. as a function of ν ((16) and (17)).

The first observation to be made is that for non-integer values of ν, everything is finite. As A_0 has poles only at $\nu=2n$, all coefficients appearing in (16) and (17) are well defined. To solve (17) we assume for small ϕ and $\nu \neq 2n$

$x = c(1 + a\bar{\phi}^2 + b\bar{\phi}^4)$. From here

$$x^{\frac{\nu-2}{2}} = c^{\frac{\nu-2}{2}} \left(1 + \frac{(\nu-2)}{2}(a\bar{\phi}^2 + b\bar{\phi}^4) + \frac{1}{8}(\nu-2)(\nu-4)a^2\bar{\phi}^4\right) \tag{18}$$

Replacing in (17) and equating to zero the coefficients of powers of $\bar{\phi}$ we obtain a solution

$$a = \frac{12\lambda_B}{1 - 12\lambda_B A_0 m_R^{\nu-4} \frac{\nu-2}{2}}$$

$$12\lambda_B A_0 m_R^{\nu-4}\left(\frac{\nu-2}{2}\right) \neq 1$$

$$b = \frac{12\lambda_B A_0 m_R^{\nu-4} \frac{1}{8}(\nu-2)(\nu-4)a^2}{1 - 12\lambda_B A_0 m_R^{\nu-4} \frac{\nu-2}{2}} \tag{19}$$

$c = 1$

The effective potential (16) is then (up to ϕ^4 term):

$$\bar{p} = \frac{1}{2}\,\bar{\phi}^2 + \lambda_R\,\phi^4 \tag{20}$$

As it should be for self-consistency.

The solution (18), (19) is only valid for small $\bar{\phi}^2$. For large values of $\bar{\phi}^2$, (17) tells us that x is also large ($x \gg 1$). If we then compare x with $x^{\frac{\nu-2}{2}}$ we see that, asymptotically, for $\nu > 4$ the latter dominates, while for $\nu < 4$ the former is dominant. So we have, from (17):

$$\text{for } \nu > 4 \quad x^{\frac{\nu-2}{2}} \rightarrow -\frac{1}{A_o m_R^{\nu-4}}\,\phi^2 \quad , \tag{21}$$

which means that

$$x \rightarrow \phi^{\frac{4}{\nu-2}} = \phi^{\alpha} \quad \text{with } \alpha < 2. \tag{22}$$

For this reason the term $-2\lambda_B\phi^4$ dominates in (16) and

$$\bar{p} \rightarrow -2\lambda_B\phi^4 \tag{23}$$

On the other hand, for $\nu < 4$, (17) gives

$$x \rightarrow 12\lambda_B\phi^2 \tag{24}$$

and from (16)

$$\bar{p} \rightarrow \lambda_B\phi^4 \tag{25}$$

We want to make a comment.

The theory "$\lambda\phi^4$" is considered renormalizable, by power counting, for $\nu \leq 4$ in which case (25) shows that the Gaussian effective potential goes like $\lambda_B \bar{\phi}^4$, exactly like the original potential.

Assuming $\lambda_B > 0$ we see that the theory makes sense for $\nu < 4$ (ν not being a pole).

For $\nu > 4$ the effective potential does not have a lower bound and the theory is known to be nonrenormalizable. (See (23)).

§ 3 $\nu \to 2n$, $n > 2$

The previous discussion refers to the case where A_0 had no pole. This is not the case when ν is an even integer.

From form (12) we see that when $\nu \to$ to a pole of A_0, one of the roots is finite and the other goes to zero; i.e.:

$$\lambda_B \to -\frac{\lambda_R}{2} \tag{26}$$

$$\lambda_B \cong \left(1 - \frac{12\varepsilon}{\lambda_R}\right) 4\varepsilon \tag{27}$$

where

$$\varepsilon = \frac{1}{24(\nu-2)m_R^{\nu-4} A_0} \tag{28}$$

where it is evident that ε goes to zero like $(\nu - 2n)$ near a pole of A_0. Let us first discuss the finite case (26).

From (4) we see that A_0 has a pole at $\nu = 2n$ with residue

$$\operatorname*{Res}_{\nu\to 2n} A_o = \frac{(-1)^n 2}{(4\pi)^n (n-1)!} \tag{29}$$

From (17) we deduce that for $\nu \to 2n$, $x \to 1$ in the following way:

$$x = 1 + (\nu - 2n) K\bar{\phi}^2 \quad , \tag{30}$$

K being a constant to be determined. So:

$$x^{\frac{\nu-2}{2}} \cong 1 + (n-1)(\nu - 2n) K\phi^2$$

and replacing in (17) we obtain:

$$K_n = \frac{(-1)^{n-1} (4\pi)^n (n-2)!}{2m_R^{2(n-2)}} \tag{31}$$

Observe that (31) coincides with the value for (a) from (19). Note also that b goes to zero like $(\nu - 2n)^2$. Replacing (30) in (16) we get.

$$\bar{p} = \frac{1}{2}\,\bar{\phi}^2 - 2\lambda_B \bar{\phi}^4 = \frac{1}{2}\,\bar{\phi}^2 + \lambda_R \phi^4 \tag{32}$$

We see from (32) that having started with $\lambda_B \ngtr 0$ we ended with $\lambda_R < 0$, a negative coefficient for the ϕ^4 term. So, the effective potential fails to give a ground state to the theory.

Now we go over to the case $\lambda_B \to 0$ (cf (27)). If we first take eq. (19) we note that although a has a well defined limit, this is not the case for b, which is seen to have a pole that invalidates (18).

From (27) and the definition of A_o (cf (4)), we deduce

$$12\lambda_B A_o m_R^{\nu-4} = \frac{2}{\nu-2}\left[1 + \frac{1}{4\lambda_R} K_n(\nu - 2n)\right] \tag{33}$$

Taking (33) into account, (17) gives, in lowest order:

$$1 - x + \frac{1}{n-1}(x^{n-1} - 1) = 0 \tag{34}$$

which, for $n > 2$, has $x = 1$ as double root. This is also the only positive root of (34). We can then expand $x^{\frac{\nu-2}{2}}$ as a function of the small parameter $x - 1$. We have

$$x^{\frac{\nu-2}{2}} = [1 + (x-1)]^{\frac{\nu-2}{2}} = 1 + \frac{\nu-2}{2}(x-1) + \frac{1}{8}(\nu-2)(\nu-4)(x-1)^2 + \ldots \tag{35}$$

Replacing (33) and (35) in (17) we get:

$$1 - x + 12\lambda_B\phi^2 + \left[1 + \frac{K_n}{4\lambda_R}(\nu-2n)\right]\left[x - 1 + \frac{1}{4}(\nu-4)(x-1)^2\right] = 0 \tag{36}$$

In lowest order:

$$12\lambda_B\phi^2 + \frac{\nu-4}{4}(x - 1)^2 = 0$$

$$(x - 1)^2 = - \frac{24\lambda_B}{n-2}\phi^2 \tag{37}$$

Which shows that $x = 1 + O(\varepsilon^{1/2})$ and also that is complex (unphysical), for any $n > 2$. This should be related to the fact that in this case the theory is not renormalizable.

§ 4 SPECIAL CASES

Of the lower dimensional cases, $\nu = 1$ and $\nu = 3$ are well defined and without problems. In particular eq. (17) reduces to a cubic (resp. quadratic) equation in $x^{1/2}$ with finite coefficients [3]. When solved for x, the effective potential can be explicitly calculated (cf (16)) and the general properties pointed out in § 2 can be checked.

For $\nu = 2$, although A_0 has a pole, the product $(\nu - 2) A_0$ is finite and equal to $-\frac{1}{2\pi}$.

Then, if we take into account that

$$x^{\frac{\nu-2}{2}} - 1 \rightarrow \frac{\nu-2}{2} \ln x \quad , \tag{38}$$

We see that eq. 17, reduces, for $\nu = 2$, to:

$$1 - x + 12\lambda_B \phi^2 - \frac{3\lambda_B}{\pi m_R^2} \ln x = 0 \tag{39}$$

For small ϕ^2 we assume

$$x = 1 + a\phi^2 + b\phi^4 \ldots$$

For which eq. (39) gives:

$$a = \frac{12\lambda_B}{1 + 3\frac{\lambda_B}{\pi m_R^2}} \quad ; \quad b = \frac{3}{2} \frac{a^2 \lambda_B}{\pi m_R^2 + 3\lambda_B} \tag{40}$$

(40) are easily seen to coincide with (19) for $\nu = 2$.

The effective potential eq. (16) is also seen to be finite for $\nu \to 2$.

Now we go over to the particular case

$$\nu \Rightarrow 4 \quad , \quad \lambda_B \to 0$$

If we start from (17)

$$(1-x) + 12\lambda_B\phi^2 + 12\lambda_B A_o m_R^{\nu-4}\left(x^{\frac{\nu-2}{2}} - 1\right) = 0 \tag{17}$$

We see that to the lowest order it is identically satisfied, as (33), which is valid in this case, implies

$$12\lambda_B A_o m_R^{\nu-4} \to 1.$$

Developping (17) near $\nu = 4$, rearranging terms using 33 and $x^{\frac{\nu-2}{2}} - 1 = x - 1 + x\ell nx(\frac{\nu-4}{2})$ we get:

$$(1-x)\left(1 + \frac{(4\pi)^2}{4\lambda_R}\right) + 16\pi^2\phi^2 + x\ell nx = 0 \tag{41}$$

which coincides with form (30) of ref. [4].

For the effective potential we get for $\nu \to 4$

$$\bar{p} = \frac{(x-1)^2}{128\pi^2} + \frac{(x-1)}{16\lambda_R} + \frac{1}{4}x\phi^2 \tag{42}$$

which coincides with form (29) of same ref. when (41) is taken into account.

If we look at (8)

$$m_B^2 = m_R^2 - 12\lambda_B A_o m_R^{\nu-2} \tag{8}$$

and use (33) in lowest order (this is true for $\nu \to 2n$)

$$m_B^2 = \frac{\nu-4}{\nu-2}\, m_R^2 = \frac{n-2}{n-1}\, m_R^2 \qquad n \geq 2 \tag{46}$$

We see then that in four dimensions $(n = 2)$, $m_B \Rightarrow 0$.

We see also that $\lambda\phi^4$ is a very peculiar theory, both λ_B and m_B are infinitesimal constants and the theory must be computed at $\nu \neq 4$ and only afterwards, the limit $\nu \to 4$ must be taken. It is in this sense that the theory seems to exists.

§ 5 ONE LOOP EFFECTIVE POTENTIAL

With the aim of comparison we will take the one loop effective potential given in ref. (8) as a function of the number of dimensions.

$$V = \frac{1}{2}\, m_B^2\phi^2 + \lambda_B\phi^4 - \frac{\Gamma(-\frac{\nu}{2})}{2(4\pi)^{\nu/2}}(m_B^2)^{\frac{\nu}{2}}\left(1 + \frac{12\lambda_B}{m_B^2}\,\phi^2\right)^{\frac{\nu}{2}} \tag{44}$$

Developping the last term for small ϕ^2 (up to ϕ^4)

$$V = -\frac{\Gamma(-\frac{\nu}{2})}{2(4\pi)^{\nu/2}}(m_B^2)^{\frac{\nu}{2}} + \frac{1}{2}\, m_R^2\phi^2 + \lambda_R\phi^4 \tag{45}$$

with

$$m_R^2 = m_B^2 + \frac{\Gamma(1-\frac{\nu}{2})}{(4\pi)^{\frac{\nu}{2}}}\, m_B^{\nu-2}\, 12\lambda_B \tag{46}$$

$$\lambda_R = \lambda_B - \frac{\Gamma(2-\frac{\nu}{2})}{(4\pi)^{\frac{\nu}{2}}} m_B^{\nu-4} 36\lambda_B^2 \tag{47}$$

Comparing resp. with (8) and (11) we see that they coincide up to first and second order in λ_B, resp.

On the other hand, asimptotically in ϕ, the loop potential is proportional to ϕ^ν. This means that: for $\nu < 4$ the term $\lambda_B\phi^4$ dominates, as for the Gaussian approximation. On the other hand, for $\nu > 4$ the one-loop correction is unbounded .

For $\nu \to 2n$ the Γ-functions in (46) and (47) have poles. Then for m_R^2 and λ_R to be finite it is necessary to have $\lambda_B \to 0$ and $m_R^2 \to 0$, (from (46) (47)). It is then easy to see that we should have

$$m_B^\nu \to \frac{(4\pi)^{\frac{\nu}{2}}(\frac{\nu}{2}-1)m_R^4}{4\Gamma(1-\frac{\nu}{2})\lambda_R} \tag{48}$$

$$\frac{\lambda_B}{m_B^2} \to \frac{1}{3(\frac{\nu}{2}-1)}\frac{\lambda_R}{m_R^2} \tag{49}$$

$$\text{i.e. } m_B^\nu = \frac{(-1)^{n-1}(4\pi)^n m_R^4(\nu-2n)}{4(n-2)!\,\lambda_R} \tag{50}$$

for $\nu \to 2n$.

The zero loop potential then goes to zero, but the one loop potential gives a finite result, namely:

$$V = \frac{\left(1+\frac{4}{n-1}\frac{\lambda_R}{m_R^2}\phi^2\right)^n m_R^4}{8n!\,(n-2)!\,\lambda_R} \tag{51}$$

In particular, for $\nu = 4$ $(n=2)$ we get

$$V = \frac{m_R^4}{16\lambda_R} + \frac{1}{2} m_R^2\phi^2 + \lambda_R\phi^4 \quad . \tag{53}$$

It is interesting to observe that while the original potential goes to zero, the one loop correction gives rise to the same potential in term of the renormalized quantities.

Also from (49) (50) for $\nu \to 4$

$$m_B^2 \simeq \frac{2\pi m_R^2}{\lambda_R^{1/2}} (4-\nu)^{1/2} \tag{54}$$

$$\lambda_B \simeq \frac{2\pi}{3} \lambda_R^{1/2} (4-\nu)^{1/2} \tag{55}$$

(54) and (55) shows that we must approach $\nu = 4$ from below.

For the case $\nu \to 2$, we note that for (46) and (47) to give finite m_R^2 and λ_R it is necessary that we have

$$m_B^2 \to m_R^2 - \frac{1}{(1-\frac{\nu}{2})} \frac{3\lambda_B}{\pi} \quad , \quad \lambda_B \to \lambda_R$$

The first coincide with (8) exactly while the second one coincides with (11) in lowest order.

§ 6 DISCUSSION

In the first part of the paper we discussed the Gaussian approximation within the scheme of dimensional regularization. This allowed us to treat the $\lambda\phi^4$ theory in a unified way for any number of dimensions. For any ν we observe that the asymptotic behaviour of the effective potential is of the form.

$+ \lambda_B \phi^4$ if $\nu < 4$ or

$- 2\lambda_B \phi^4$ if $\nu > 4$

Showing how the renormalizability reflects in the Gaussian approximation.

For odd powers $\nu = 2n+1$ the effective potential is finite and the asymptotic behaviour is the one pointed out above.

For $\nu = 1,2,3,$ everything is finite and well behaved.

For $\nu = 4$ we essentially reobtain the results of ref. [4] with the addition that the bare mass goes to zero with the bare coupling constant.

In the second part we analyse the one-loop approximation with results that are similar to the previous one. For example the asymptotic behaviour, where only for $\nu < 4$ it is of the form $\lambda_B \bar{\phi}^4$ while for $\nu > 4$ it goes like ϕ^ν.

The cases in which the bare coupling constant goes to zero are examples of "evanescent couplings" which were previously introduced to obtain the triangle axial anomaly [9].

In perturbation theory one can start (at $\nu \neq 4$) with a coupling which is zero when read at dimension four but which tends to zero for $\nu \to 4$ in such a way that when multiplied with a divergent integral ($\sim \frac{1}{\nu-4}$) coming from perturbative corrections, the result is finite. For instance a divergent integral ($\sim \frac{1}{\nu-4}$) when multiplied with λ^2 ($\lambda \sim \sqrt{4-\nu}$, see form (55)) gives a finite result. We can then see that "the limit of the theory" for $\nu \to 4$ is not equal "to the theory of the limit". The latter being a free particle theory.

In the Gaussian approximation something similar occurs. The case $\lambda_B \to 0$ is also "evanescent" but when multiplied with the divergent quantity A_0 it gives a finite result (see for example (33)).

AKCNOWLEDGEMENTS

We are indebted to Professors P.M. Stevenson, J.A. Mignaco and C.A. Aragão de Carvalho for stimulating discussions on the subject.

REFERENCES

1. K.G. Wilson, Phys. Rev. D6 (1972) 419
   J. Fröhlich, Nucl. Phys. B200 (FS4) (1982)
   C. Aragão de Carvalho, C.S. Carracciolo and J. Fröhlich Nucl. Physics B215 (FS7) (1983) 209
2. J.M. Cornwall, R. Jackiw and E. Tamboulis-Phys. Rev. (1974) 2428
   T. Barnes and G.I. Ghandour, Phys. Rev. D22 (1980) 924
   W.A. Bardeen and M. Moshe, Phys. Rev. D28 (1983) 1378
3. P.M. Stevenson: Preprint MAD/TH/163 (1984)
4. P.M. Stevenson, Zeit. f. Physik C Vol.24 87 (1984)
5. M. Consoli & A. Ciaucitto, Preprint CERN/Th 3951/84
6. S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888,
7. G.'tHooft, M. Veltman: Nucl. Phys. B50 (1972) 318
   G.M. Cicuta, E. Montaldi: Lett. Nuovo Cim. 4, 329 (1972)
   C.G. Bollini, J.J. Giambiagi: Phys. Lett. 40B (1972) 566 Nuovo Cim. 12B (1972)20
8. C.G. Bollini and J.J. Giambiagi, Phys. Letters: 134B, 436 (1984)
9. C.G. Bollini and J.J. Giambiagi, Acta Phys. Austr. 38, 211 (1973)
   C.G. Bollini and J.J. Giambiagi, Nucl. Phys. B, 97 522 (1975)