ISSN 0029-3865

CBPF-NF-046/83

THE TWELVE COLOURFUL STONES AS BUILDING BLOCKS

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Abstract

Based on twelve colourful stones and adopting SU(3) as symmetry group, we build up candidates for physical states. Quarks and leptons are generated as composite structures.

I. INTRODUCTION

The general idea here is to start nature process basing only on colour. In the Big Bang conception, the Universe has an alfa point determined by temperature parameter. It tells that the u niverse after a high temperature state starts cooling. Our approach would be that after a state that depends only on colour, it moves by creating singlets. The belief is that the Universe dynamics is carried out by a tendency for generating singlets that will be in continuous transformations. This evolution will represent rearrangements of the colourful structures. The tables build up in the text show a part of these possible singlets states.

In this work we are assuming a theory for the initial colour ful state. It is called the twelve colourful stones [1]. It is based on the existence of three colours in two families. The intuition for such choice is based in two facts. For the first, observe the measurements of the total cross section for the an nihilation of electron-positron pairs in colliding beam experi ments [2]. The data up to C.M. energy 35.8 GeV are an indication for three colours. Thus we have called stones the particles whose dynamics is based only on the concepts of colour and gauge. However because of colour confinement we can not relate the nature of these stones. Observe now the measurable nature. It reveals two basic entities: the fermionic and the bosonic. We consider this two structures aspect as a second fact to guide us to investigate the colourful world also with two families. They were called by yang and yin. The yang structure is correlated with a spin zero field ϕ^i and the yin with a spin half field ψ^j. As each family has three colours with the respective anticolour we have twelve colourful stones. Physically it will be important to mix these families. In order to do this it is necessary to define them with the same three colours. Then there is just one group involved. Therefore the singlets can be build up either by elements of the same family or mixing them. The group used in this work is $SU(3)_C$.

In parts II and III, the states are systematically described by composition of stones. In part IV, quarks and leptons are generated with bosonic and fermionic possibilities. In part V, the conclusion is presented asking for a unity between the yang and yin stones.

II. COLOURFUL STONES AS BASIC TRIPLETS

Consider the yin case. The vector components ψ_i for the fundamental representation 3 are the colourful states called yin stones those for the conjugate representation $\overline{3}$ are called anti-yin stones. More explicitly,

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \qquad \overline{\psi} = \begin{pmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \end{pmatrix}$$

with the rotation

$$\psi' = U\,\psi$$

$$\text{where } U = e^{i\omega^a t_a}\ , \quad a = 1,\ldots 8 \tag{1}$$

Similarly for the yang case. The notation is as in Fig. 1. III is the fundamental representation for yang stones. The representations of $SU(3)_C$ can be build up by taking tensor products of independent representations as $3 \otimes \overline{3}$, $III \otimes \overline{III}$, $III \otimes \overline{3}$, etc. The irreducible representation is realized in the space of the tensor $T^{\beta_1 \ldots \beta_q}_{\alpha_1 \ldots \alpha_p}$ whose independent components may be regarded as a multiplet of dimension D(p,q). These particles will be composed of

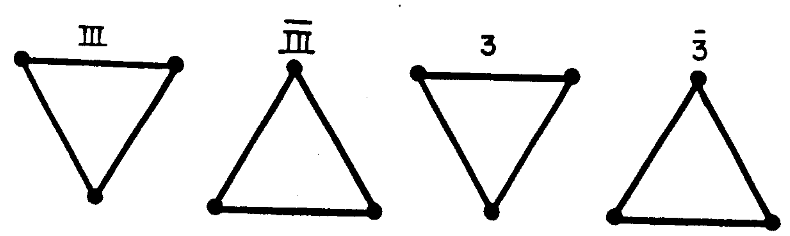

Fig. 1. Yang-yin triplets are characterized by the roman and arabic numbers respectively.

stones (yang-yin) and/or anti-stones (yang-yin).

In addition to $SU(3)_C$ the lagrangian density [1] is also invariant under independent phase changes of ϕ and ψ. The transformation form a group U(1) x U(1). We can identify the generator of one of the U(1) group as yang number Y, and call the generator of the other U(1) group as yin number y. Thus each state build up by the stones will be characterized by a Y/y number. Therefore two colourful stone numbers can be assigned to all particles. The antiparticle is defined with opposite numbers. The conservation law is that Y and y are separately conserved. In Appendix A is given a sample.

III. THE PRODUCT OF REPRESENTATIONS

Basing on the fundamental representations 3, $\overline{3}$, III and $\overline{\text{III}}$, as in Fig. 1, we are going to study the multiplets. The structures to be generated will be less fundamental as the number of factors in the representation product increases. Essentially, the possibilities will be either to generate colourful or colourless structures. The stones have three ways to be arranged. They are the yang-yang, yin-yin and yang-yin compositions. Each Y/y state has three kinds of partners. They are the opposite, complementary and reflexive states. The first is defined by having opposite yang and yin numbers. In the second, either the yang or the yin is opposite. In the last, these numbers are reflected. The states containing the same number of yang and yin stones will be arranged in sets. Each set is build up by basic states. This means that all other states of that set will be just the opposite and the complements of these basic states.

Consider a case with n stones. The number of states generated by the yang or yin composition is n+1. There are n+1 sets. For an n even, the number of bosonic set is $\frac{n}{2}+1$. To the odd case is $\frac{n+1}{2}$. Suppose a set in the yang yin composition with m yin stones. The number of states will be $(n-m+1)(m+1)$. When the number of both yin and yang is even, there will be $(\frac{m}{2}+1)(\frac{n-m}{2}+1)$ basic states in a set. If m is odd and n-m even, will be $(\frac{n-m}{2}+1)$ $(\frac{m+1}{2})$. Similarly for the other case.

The hadrons have been described by three quarks. Considering this, we are going to make a study up to six stones. In order to summarize the text, Tables up to three stones arrangement will be included.

Two Stones Composition

Observing first the cases where the fields ϕ_i and ψ_j are mixed, we have

$$\chi_i = f_i^{jk} \phi_j \psi_k \qquad (2)$$

and

$$\eta = \phi_i \psi^i \qquad (3)$$

Equation (2) defines the quark. The colours of the fields are arranged through the constant f_{ijk}. The form of this constant will depend on the irreducible representation where the quarks are defined. Equation (3) defines the lepton. Observe that there do not exist other possibility to mix the two fields different from (2) and (3). Thus from the twelve colourful stones building blocks it is not possible to generate a third kind of elementary structure with spin half.

(2) and (3) in Table I are in the structures (g) and (h). These also appear the respective antiparticles through (i) and (j). However from (a)-(f) there are other candidates to fundamental particles. Such structures are bosonic. The expressions will be similar to (2) and (3) but with equal fields. They would produce respectively four and two kinds (one colourful and another colourless) of bosonic elementary structures.

Three Stones Composition

Let us study it as a particular case of the n stones generalization. The yang or yin composition has four states. There are four sets, where two are bosonic. Consider the set III on Table II. It has just one yin stone. Thus, the number of states will be six. The number of basic states in this set is two. They are 1/2 and 1/0.

In total, Table II generates twenty states. It contains eight types of singlets. (a), (d), (o), (t) are bosons. (e), (h), (i), (n) are fermions. The yang yin crossing generates two sets. One with spin half and the other with spin zero or one.

Four Stones Composition

There are five sets. The yang or yin composition has five states. Consider the set with two yin stones. It has nine states. The number of basic states in it is four. They are 2/2, 2/0 0/2 and 0/0. The other states are complements. In total, there are eight colourless states. Each one contains two singlets. Therefore, we have as potentially experimental facts, eight fermionic structures and eight bosonic.

Five Stones Composition

It has fifty six states. Six from yang composition, six from yin. It contains twenty kinds of colourless states. Each one has three types of singlets. We could associate in the yin composition, each singlet with each type of spin.

Six Stones Composition

It has seventy one states, where twenty four are colourless. There are two kinds of colourless states. One has five singlets and the other six. The first has six bosonic and four fermionic structures. The other number of this structures are eight and six respectively. In total, the number of measurable fermions is different from the bosons.

IV. QUARKS AND LEPTONS

As quarks, we define the simplest colourful structure, irrespective of the spin. They can be either the triplets or the sextet in Table I. Quarks contents of 3, $\overline{3}$, 6 and $\overline{6}$ are in Figs. 2 and 3. Each figures will represent a meaning in the moment that the Y/y quantum numbers are associate to them. They are 2/0, 1/1, 0/2 and the respective antiparticles. The fermionic quark field is

$$\chi_i = f_i^{[jk]} \; \phi_{[j} \, \psi_{k]} \tag{4}$$

where $i = 1,2,3$. This means three quarks with different colours. The constant $f_i^{[jk]}$ is the antisymmetric tensor ε_{ijk}. It yields,

$$\overline{\chi}^i \chi_i = \phi_j^{\,*} \phi^j \, \overline{\psi}_k \psi^k - \phi_\ell^{\,*} \phi^j \, \overline{\psi}_j \psi^\ell \tag{5}$$

(5) is gauge invariant under (1)

For the sextet case,

$$\chi_i = f_i^{(jk)} \; \phi_{(j} \, \psi_{k)} \tag{6}$$

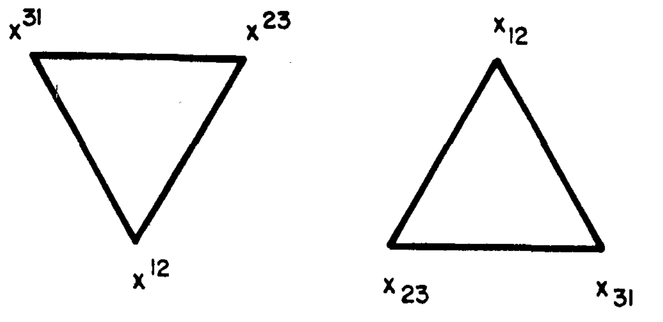

Fig. 2. Quarks as members of a triplet with a corresponding Y/y number. χ_i represents a colourful composite field. It appears three different natures for quarks.

where i=1,...,6. The symmetric $f_{i(jk)}$ satisfy the relation

$$f_{i(jk)}\, f_{i(lm)} = \delta_{j\ell}\,\delta_{km} + \delta_{jm}\,\delta_{k\ell} \tag{7}$$

Similarly to (5) the composition in the sextet will be invariant. In Appendix B the generators for the sextet are calculated.

Following the eightfold way [3], we define

$$Q^c = I_3^c + \frac{Y^c}{2} \tag{8}$$

with the difference that I_3^c and Y^c are colour dependent. They can not be interpreted with observables directly. In this way a colourful state is defined by a Y/y number and Q^c. For instance, the spin half quark has nine states. Adding to it the bosonic structures there are ninety possible states.

We interpret the lepton as the singlets in Table I. Because stability the electron is chosen as the most basic lepton. Therefore, we have to define it from the Table I. It would be the state 1/-1. Then, looking in Table I the positron and two bosonic partners would appear.

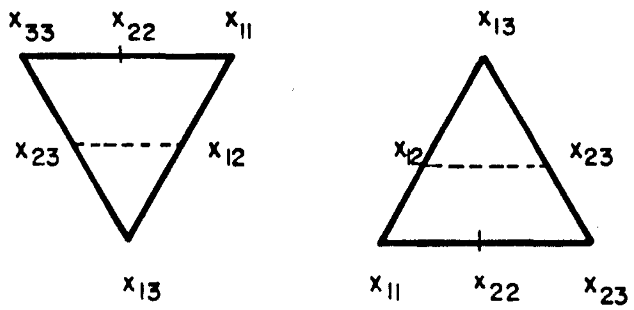

Fig. 3. Quarks as sextet members.

V. CONCLUSION

In some sense, depending on accelarator generation, physics will be offering some candidate to building block. The present experimental spectrum suggests quarks and leptons as non-composite particles. However, they have distinct properties. Thus, it becomes important to clarify what are the conditions for elementarity. Since long time ago, it is being required the presence of an indivisible structure. This thought is not clear in quark physics, but there appear two special properties: the confinement and the asymptotic freedom. The experimental evidence for the fact that a particle can move almost freely in a structure brings, to our mind, a new insight for the concept of elementarity. Another criterium to be added would be the concept of minimum block. This block would be the basis for a chain of more complex structures. Thus a problem for quarks and leptons is that they represent two differents types of minimum blocks. The dynamics of a theory can also be another way of establishing elementarity. It is based on some fundamental parameters. The trend is to create a basic dynamics that would work with a minimum numbers of parameters. An elementary theory would have to be governed by this basic dynamics.

In this way, we shall quote below conditions that we propose for a structure to be classified as an elementary one.

(i) Be characterized by just one fundamental parameter. It must have an experimental reality.

(ii) Represent a unique building block.

(iii) Have the asymptotic Freedom property.

The twelve colourful stones would be consistent with such properties in the following way,

(i) The only fundamental parameter is colour. It remains to be shown in future a dependence of mass and charge with colour.

(ii) The building block is the so called yang-yin structure. From it, we can generate quarks and leptons according to (2) and (3). Thus, a further yang yin unity is required.

(iii) The possible gauge invariant Lagrangian for the gauge fields have the three-gauge-boson-vertex. Therefore, we expect the property of asymptotic freedom.

With the advent of quark physics, the concept of colour has started being revealed. The question is whether it is an entity just correlated with quarks or a general element of nature. The meaning of the tables in text is that all particles would have an origin in colour. We did a study based on $SU(3)_C$ group. These tables constitute the engineering for building up structures flowing in the colourful river. These states have opposite, complements and reflexive forms. They put us in contact with physics in the moment that the candidates to observables states, the colour singlets can systematically be generated. The basis of them is the presence of two distinct families (spin half and spin zero) transforming under the fundamental representation of the same group.

The use of a symmetry group here was done in an opposite path to the eight fold way. There, starting from observables as isotopic spin and hypercharge, we were led to an SU(3) flavour group. Here, the physical states are not intended to be classified into multiplets, but rather as singlets of the symmetry group. It is left as a posteriori the identification of these states.

Finally, we would observe that the twelve colourful stones approach is being based only on an internal colour group. It brought a new way to build up the bosonic and fermionic colourless world. The origin of them is in two kinds (yang and yin) of building blocks. However, this fact has not been deeply exploited in the lack of a connection between the colour and the space time structure. In the moment this connection is established we will have achieved a full unification in the colourful world. This naturally motivates the introduction of supersymmetry. The emerged physical states were up to now unconnected. This colour unity will bring a correlation between these observable states.

ACKNOWLEDGEMENT

We are very grateful to A. Antunes, M. Doria and J. Tiomno for discussions. This work would like to acknowledge and be a part in the effort of the people in Trieste to make of that Centre a Third World Oasis.

APPENDIX A

Considering that ϕ and ψ are independent fields the yang and yin numbers conserve separately. This defines a particle with the structure

$$\text{PARTICLE} \equiv \text{YANG NUMBER / YIN NUMBER} \tag{A1}$$

Under this assignment, the simplest compositions were studied in Table I. They were identified as quarks and leptons. Choosing the electrons as basis, define

$$e^- \equiv 1/-1 \quad , \quad e^+ \equiv -1/1 \tag{A2}$$

Stable particles as ν, μ, τ can not have their Y/y number determined from the electron case. Thus in principle they will be defined as

$$\nu \equiv x/-x \tag{A3}$$

Let us analyse the reaction

$$e^+ + e^- \to q + \bar{q} \tag{A4}$$

under Y/y number,

$$-1/1 + 1/-1 \to 1/1 + -1/-1 \tag{A5}$$

(A5) trivially realizes the prescription that the yang and yin numbers conserve independently.

APPENDIX B: Generators in the Sixth Representation

The quarks generated from the twelve colourful stones can be triplets and sextets multiplets. Thus becomes necessary to calculate the generators for the 6 and $\overline{6}$ representations. The states are parametrized as in Fig. Bl. The I, U and U spins are defined as in the Gellman matrices. Defining,

$$
\begin{aligned}
I_-\psi_1 &= \sqrt{2}\ \psi_2 \qquad & I_-\psi_4 &= 0 \\
I_-\psi_2 &= \sqrt{2}\ \psi_3 \qquad & I_-\psi_5 &= 0 \\
I_-\psi_3 &= 0 \qquad & I_-\psi_6 &= \psi_4
\end{aligned}
\qquad \text{(B1)}
$$

$$
\lambda_1 = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & 0 & 0 & 0 & 0 \\ \sqrt{2} & 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}
$$

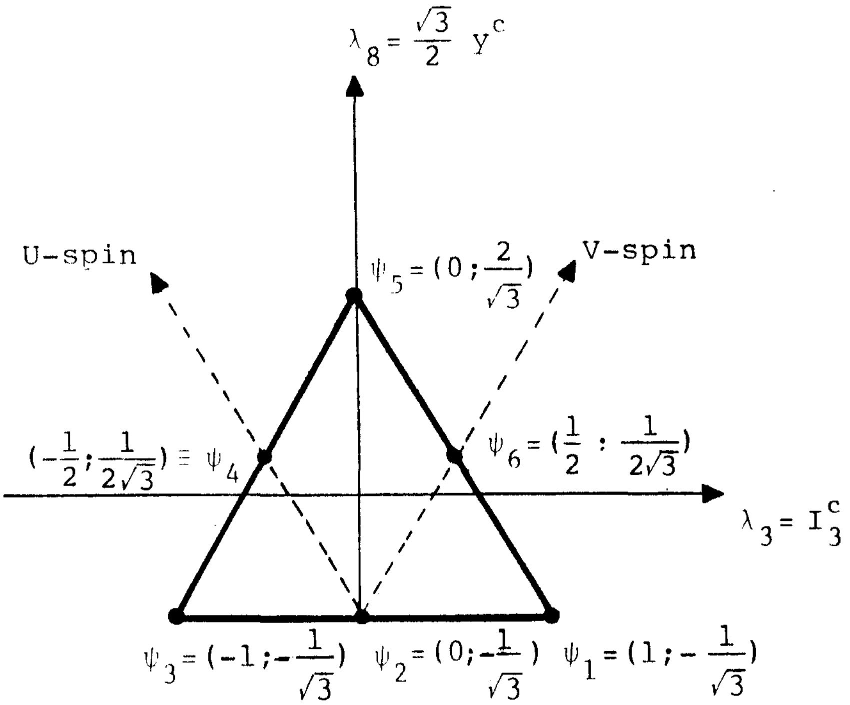

Fig. Bl. Quarks in the sixth representation.

$$\lambda_2 = -\frac{i}{2}\begin{pmatrix} 0 & \sqrt{2} & 0 & 0 & 0 & 0 \\ -\sqrt{2} & 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\lambda_4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = -\frac{i}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & -\sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_7 = -\frac{i}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\sqrt{2} \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{B2}$$

The generators for $\overline{6}$ will be the complex conjugate of (B2).

TABLE I. Two Stones Product Representation

Yang Composition (Set I)	Yang/Yin Number	Spin	Tensor
(a) III x III = $\bar{3} \oplus 6$ (*)	2/0	0	$\phi_{[\alpha\beta]}$; $\phi_{(\alpha\beta)}$
(b) III x $\overline{\text{III}}$ = $1 \oplus 8$	0/0	0	$\phi_\alpha \phi^\alpha$; $\phi_\alpha \phi^\beta - \frac{1}{3}$ Tr $\phi_\alpha \phi^\alpha$
(c) $\overline{\text{III}}$ x $\overline{\text{III}}$ = $3 \oplus \bar{6}$	-2/0	0	$\phi^{[\alpha\beta]}$; $\phi^{(\alpha\beta)}$
Yin Composition (Set II)			
(d) 3 x 3 = (a)	0/2	0,1	$\psi_{[\alpha\beta]}$; $\psi_{(\alpha\beta)}$
(e) 3 x $\bar{3}$ = (b)	0/0	0,1	$\psi_\alpha \psi^\alpha$; $\psi_\alpha \psi^\beta - \frac{1}{3}$ Tr $\psi_\alpha \psi^\alpha$
(f) $\bar{3}$ x $\bar{3}$ = (c)	0/-2	0,1	$\psi^{[\alpha\beta]}$; $\psi^{(\alpha\beta)}$
Yang-Yin Crossing (Set III)			
(g) III x 3 = (a)	1/1	1/2	$\phi_{[\alpha} \psi_{\beta]}$; $\phi_{(\alpha} \psi_{\beta)}$
(h) III x $\bar{3}$ = (b)	1/-1	1/2	$\phi_\alpha \psi^\alpha$; $\phi_\alpha \psi^\beta - \frac{1}{3}$ Tr $\phi_\alpha \psi^\alpha$
(i) $\overline{\text{III}}$ x 3 = (b)	-1/1	1/2	$\phi^\alpha \psi_\alpha$; $\phi^\alpha \psi_\beta - \frac{1}{3}$ Tr $\phi^\alpha \psi_\alpha$
(j) $\overline{\text{III}}$ x $\bar{3}$ = (c)	-1/-1	1/2	$\phi^{[\alpha} \psi^{\beta]}$; $\phi^{(\alpha} \psi^{\beta)}$

(*) The physical meaning to be read in the right hand side is if the structure is colourful or colourless. Thus, for simplicity, we are going to write it with arabic numbers. The notation is $\psi_{(\alpha\beta)} = \frac{1}{2}(\psi_\alpha \psi_\beta + \psi_\beta \psi_\alpha)$ and $\psi_{[\alpha\beta]} = \frac{1}{2}(\psi_\alpha \psi_\beta - \psi_\beta \psi_\alpha)$.

TABLE II. Three Stones Product Representation(*)

	Yang/Yin Number	Spin	Tensor
Yang Composition (Set I)			
(a) $III \times III \times III = 1 \oplus 8 \oplus 8' \oplus 10$	3/0	0	(**)
(b) $III \times III \times \overline{III} = 3 \oplus 3 \oplus \overline{6} \oplus 15$	1/0	0	(***)
(c) $\overline{III} \times \overline{III} \times III = (\overline{b})$	-1/0	0	(****)
(d) $\overline{III} \times \overline{III} \times \overline{III} = (\overline{a})$	-3/0	0	(*****)
Yang Composition(+) (Set II)			
(e) $3 \times 3 \times 3 = (a)$	0/3	$\frac{1}{2}, \frac{3}{2}$	(S)
(f) $3 \times 3 \times \overline{3} = (b)$	0/1	$\frac{1}{2}, \frac{3}{2}$	(S)
(g) $3 \times \overline{3} \times \overline{3} = (\overline{b})$	0/-1	$\frac{1}{2}, \frac{3}{2}$	(S)
(h) $\overline{3} \times \overline{3} \times \overline{3} = (\overline{a})$	0/-3	$\frac{1}{2}, \frac{3}{2}$	(S)

(*) The notation is also with arabic numbers for RHS and with $T_{\alpha\beta\gamma} \equiv \phi_\alpha \phi_\beta \phi_\gamma$.

(**) The singlet tensor is: $\frac{1}{6}[T_{\alpha\beta\gamma} + T_{\gamma\alpha\beta} + T_{\beta\gamma\alpha} - T_{\beta\alpha\gamma} - T_{\alpha\gamma\beta} - T_{\gamma\beta\alpha}]$

The octet is: $\frac{1}{2}[T_{\alpha\beta\gamma} + T_{\alpha\gamma\beta} + T_{\beta\gamma\alpha} + T_{\gamma\beta\alpha} + T_{\gamma\alpha\beta} + T_{\beta\alpha\gamma} - T_{\beta\alpha\gamma} - T_{\beta\alpha\gamma} - T_{\alpha\gamma\beta} - T_{\gamma\alpha\beta} - T_{\gamma\beta\alpha} - T_{\alpha\beta\gamma}]$.

The decuplet is: $\frac{1}{2}[T_{\alpha\beta\gamma} + T_{\beta\gamma\alpha} + T_{\gamma\alpha\beta} + T_{\beta\alpha\gamma} + T_{\alpha\gamma\beta} + T_{\gamma\beta\alpha}]$.

(***) The triplet tensor is $\phi_{(\alpha\beta)}\phi^\gamma$. $\overline{6} \oplus 15$ gives $\phi_{[\alpha\beta]}\phi^\gamma$.

(****) It is as (***) but changing upper by lower indices and vice versa.

(*****) Similarly to the case above.

(+) For each tensor change the field ϕ by ψ.

(S) It is similar to the case above.

	Yang/Yin Number	Spin	Tensor
Yang Yin Crossing (Set III)(†)			
(i) III x III x 3 = (a)	2/1	1/2	(S)
(j) III x III x $\overline{3}$ = (b)	2/-1	1/2	(††)
(k) III x $\overline{\text{III}}$ x 3 = (b)	0/1	1/2	(S)
(ℓ) III x $\overline{\text{III}}$ x $\overline{3}$ = ($\overline{b}$)	0/-1	1/2	(S)
(m) $\overline{\text{III}}$ x $\overline{\text{III}}$ x 3 = ($\overline{b}$)	-2/1	1/2	(S)
(n) $\overline{\text{III}}$ x $\overline{\text{III}}$ x $\overline{3}$ = ($\overline{a}$)	-2/-1	1/2	(S)
(Set IV)(o)			
(o) III x 3 x 3 = (a)	1/2	0,1	(S)
(p) III x 3 x $\overline{3}$ = (b)	1/0	0,1	(oo)
(q) III x $\overline{3}$ x $\overline{3}$ = ($\overline{b}$)	1/-2	0,1	(S)
(r) $\overline{\text{III}}$ x 3 x 3 = (b)	-1/2	0,1	(S)
(s) $\overline{\text{III}}$ x 3 x $\overline{3}$ = ($\overline{b}$)	-1/0	0,1	(S)
(t) $\overline{\text{III}}$ x $\overline{3}$ x $\overline{3}$ = ($\overline{a}$)	-1/-2	0,1	(S)

(†) $T_{\alpha\beta\gamma} = \phi_\alpha \phi_\beta \psi_\gamma$

(††) The triplet tensor is $\phi_{(\alpha\beta)}\psi^\gamma$. Similarly, $\overline{6} \oplus 15$ gives $\phi_{[\alpha\beta]}\psi^\gamma$.

(o) $T_{\alpha\beta\gamma} = \psi_\alpha \psi_\beta \phi_\gamma$

(oo) The triplet tensor is $\psi_{(\alpha\beta)}\phi^\gamma$. Similarly, $\overline{6} \oplus 15$ gives $\psi_{[\alpha\beta]}\phi^\gamma$.

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