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NON-LINEAR PHOTONS IN THE UNIVERSE

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ABSTRACT

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I - INTRODUCTION

One of the most remarkable consequences of the theory of General Relativity was the prediction of the bending of light in a gravitational field. This proves that the photon has a passive gravitational energy and consequently, by means of the Equivalence Principle, it must have an active gravitational energy too — which should be responsible by the curvature of space-time generated by the photon itself. Such influence of gravitation in the behaviour of light has been shown by A. Eddington in a memorable expedition [1] . Since then, it has

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I - INTRODUCTION

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been the subject of a widely large number of experimental observations.

However, the set of equations which governs the coupled system of Electrodynamics and Gravitation may still be in an incomplete form. The main reason is due to the fact that we do not know the behaviour of Electric and Magnetic fields in a strong Gravitational field.

In order to achieve at a reasonable theoretical framework, one starts by assuming the so-called Minimal Coupling Principle. Due to this the equations of motion of Electrodynamics in a curved space can be obtained from Maxwell's equations in flat Minkowskii Universe whithout ambiguity. Additional terms which envolve the curvature of space-time are usually disregarded by a priori reasons, like the difficulty of compatibility of such terms with charge conservation for instance, or the high degree of arbitrariness contained in such coupling.

Irrespectly of these arguments, we start here a program in which an exhaustive and systematic analysis of these non-minimal coupling between vector W_μ and tensor $g_{\mu\nu}$ fields. The main reason which induced us to undertake such study is linked to the subject of non-linearities on Electrodynamics which is contained naturally in such kind of coupling. Traditionally, non-linearities on Electrodynamics are introduced either by an ad. hoc assumption (e.g. Born-Infeld) or by induction of quantum effects (Euler-Heisemberg; see also [2] Akhieser-Berehtski for a more detailed discussion). The general believe was that the non-linear theory could change drastically the properties of the field in the neighborhood of classical electron and so provides a succesful model to describe it.

Others models, however, have been proposed with different leitmotiv. It seems wortwhile to recall here a recent interesting suggestion which adds to the usual Maxwell's equations a term derived from a Lagrangian of the type $L = \lambda (W_\mu W^\mu)^2$ where λ is a constant. Such term, which further breaks gauge invariance, can give origin to inelastic photon-photon interaction. This term has been used tentatively to give an alternative explanation on galactic red-shift anomalies [3]. The theory we will explore breaks the gauge invariance too. In the first order of curvature term, we can have two possibilities: either $L_I = \sqrt{-g}\, R\, W_\mu W^\mu$ or $L_{II} = \sqrt{-g}\, R_{\mu\nu}\, W^\mu W^\nu$.

In the present paper we will limit our discussion to the case in which the Lagrangian L_I has to be added to Maxwell's Electrodynamics. The effect of adding such term to the equation of motion can be interpreted as giving to the photon a mass proportional to the scalar of curvature. As a consequence the theory will also break conformal invariance. The discussion on massive Electrodynamics has an extensive bibliograph. See reference [4] for an up-to-date review. However, as we will see in this paper, the mass $m_\gamma \sim \sqrt{R}$ has many very peculiar properties not contained in usual models.

II - THE MODEL

The equations of motion are obtained from the Lagrangian L which consists of three parts:

(1) $$L = L_A + L_B + L_C$$

where
$$L_A = \frac{1}{K}\sqrt{-g}\,(1+\lambda W_\mu W^\mu)\,R$$
$$L_B = -\frac{1}{2}\sqrt{-g}\quad F_{\mu\nu}F^{\mu\nu}$$
$$L_C = L_{matter}$$

in which $F_{\mu\nu} = W_{\mu\|\nu} - W_{\nu\|\mu}$; λ is a constant with the same dimensionality as Einstein's coupling constant K, that is $(\text{energy})^{-1}$ (length); R is the scalar of curvature thus defined $R = R_{\mu\nu\alpha\beta}\,g^{\mu\alpha}g^{\nu\beta}$; doube bar represents covariant derivative. From Lagrangian (1) by variation of $g_{\mu\nu}$ we obtain the equations of motion

$$(2)\quad (1+\lambda W^2)\,G_{\mu\nu} - \lambda\,\Box W^2 g_{\mu\nu} + \lambda\, W^2{}_{|\mu\|\nu} + \\ + \lambda R\, W_\mu W_\nu = -\,K\,E_{\mu\nu} - K\,T^*_{\mu\nu}$$

in which $T^*_{\mu\nu}$ represents the energy-momentum tensor of the matter; W^2 is the norm $W_\mu W^\mu$; and $E_{\mu\nu}$ is Maxwell's tensor:

$$(3)\qquad E_{\mu\nu} = F_{\mu\alpha}F^{\alpha}{}_{\nu} + \frac{1}{4}\,g_{\mu\nu}\,F_{\alpha\beta}F^{\alpha\beta}$$

By variation of W^μ on (3) we obtain

$$(4)\qquad F^{\mu\nu}{}_{\|\nu} = -\frac{\lambda}{K}\,R\,W^\mu + J^\mu$$

in which we have added an extra current J^μ. Taking the divergence of expression (4) yields the modified law of charge conservation

$$(5)\qquad J^\mu{}_{\|\mu} - (R\,W^\mu)_{\|\mu} = 0$$

At this point we can follow two distinct procedures: either we assume that charge is conserved and thus imposing the constraint $(RW^{\mu})_{||\mu} = 0$; or we allow for charge creation by the gravitational field. In this case the number of created particles depends on the value of the scalar of curvature through equation (5). Independently of this let us remark that creation of charge in our model can occurs only at those region of curved space-time in which the scalar of curvature is not null. This, of course, is not a sufficient condition but it is a necessary one.

The effect of a break on charge conservation on a cosmological scale has been analysed, some years ago, by Lyttleton and Bondi [5] and criticized by Hoyle [6]. The essential idea of Lyttleton-Bondi (LB) analysis rests on the observation that a slight difference in the magnitude of electric charges of the proton and the electron could give rise to a repulsive force, which in a cosmical scale could be an alternative explanation to the observed expansion of the Universe. The modification suggested by LB consists in adding a mass-term $\epsilon W_{\mu} W^{\mu}$ to Maxwell's Lagrangian, allowing for a non-null divergence of the potential vector W^{μ}. Then they construct a cosmological solution of an Universe filled with such massive photons. The result is a steady-state (de Sitter-type) cosmological configuration.

Hoyle [6] in a subsequent paper has shown that Lyttleton-Bondi model is equivalent to the introduction of a fluid with negative energy that could be constructed with a scalar field. As a consequence, the equation of motion which gives the behaviour of LB electrodynamics in an expanding steady--state homogeneous (isotropic) Universe is similar to the equation of Hoyle's C-field- which is responsible for matter creation. Thus, the effect of the proposed modification of Electrodynamics through Lyttleton-Bondi hypothesis is undistinguisable — in respect to cosmical effects — to Hoyle's model of continuous creation of matter.

Although there is a point of contact with Lyttleton--Bondi scheme of modified Electrodynamics, the model we advocate here is very distinct from their proposal. The crucial difference is contained on the introduction of non-linearities, through the dependence of the mass-term on the scalar of curvature. Actually, many new features appears in our model which has no equivalent in LB's. For instance, as we will show next, our model does not admit a cosmological steady-state configuration. Such solution, which is a typical property of Lyttleton-Bondi model is indeed the main point of contact of LB model and Hoyle's version of continuous creation of matter.

Let us come back now to our equation (2). Taking the trace of this equation

$$R = \kappa T^{*} - 3\lambda \Box W^{2} \tag{6}$$

where T^{*} is the trace of the energy-momentum tensor. Thus we obtain from equation (4)

$$F^{\mu\nu}{}_{\parallel\nu} = \frac{3\lambda^{2}}{\kappa}(\Box W^{2})\, W^{\mu} - \lambda T^{*} W^{\mu} + J^{\mu} \tag{7}$$

which exhibits explicitly the non-linearities of our model. It seems worthwhile to remark here that such type of non-linearity behaviour of our model can be introduced in an equivalent way without making appeal to non-minimal coupling with gravitation. Indeed, if we set a Lagrangian under the form

$$\mathcal{L}_N = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \epsilon \left(W^\mu W_{\mu \| \alpha} \right)^2$$

a straightforward calculation shows that the equation of motion obtained for such $\mathcal{L}_N$ is precisely equation (7) – (without the trace term, of course).

The wave equation for the potential vector W^μ is given by

$$(8) \quad \Box W^\mu + R^\mu{}_\alpha W^\alpha - \left(W^\nu{}_{\|\nu} \right)^{\|\mu} = \frac{3\lambda^2}{K} (\Box W^2) W^\mu$$

in the absence of currents and matter. The first two terms of this equation is nothing but de Rham's wave operator in curved space. The third term is proportional to the gradient of the variation of the scalar of curvature in the W^μ-direction. A case of particular interest occurs when we can neglect the second and the third terms and simplify equation (8) to the expression

$$(9) \quad \Box W^\mu - \frac{3\lambda^2}{K} (\Box W^2) W^\mu = 0$$

This equation has some very interesting properties by its own. Let us re-write it in a gaussian system of coordinates where $ds^2 = dt^2 - g_{ij}(x) dx^i dx^j$. We have for a generic vector E^μ

$$(10) \quad \Box E^\mu - \epsilon \left(\frac{\partial^2}{\partial t^2} E^2 \right) E^\mu = - \epsilon g^{ij} (E^2)_{|i\|j} E^\mu$$

in which ϵ is a constant.

The left-hand side has a striking analogy to the equation that governs the Electric field inside a non-linear dielectric, due for instance to the dependence of the dielectric constant on light intensity. It is then compeling to interprete equation (10) as giving origin to a sort of generalized relativistic Kerr effect.

III - NON-LINEAR EQUATION

The scalar equation associated to (9) has the form

$$\Box \phi + \frac{\mu^2}{2} \left(\Box \phi^2 \right) \phi = 0 \tag{11}$$

where μ is an arbitrary constant. This equation has an interest by its own. Let us examine some properties of it under two special circunstances: i) the background geometry is flat (Minkowskian); ii) the geometry is an expanding homogeneous and isotropic Friedmann Universe.

In case (i), let us look for a stationary solution, where ϕ depends only on one variable, say x . The equation (11) turns into:

$$\phi'' \left(1 + \mu^2 \phi^2 \right) + \mu^2 \phi \phi'^2 = 0 \tag{12}$$

A solution of this equation can be found under the form

$$m x + n = \text{arc sinh} (\mu \phi) + \phi \left(1 + \mu^2 \phi^2 \right)^{1/2} \tag{13}$$

where m and n are arbitrary constants. Let us now consider the case of Friedmann geometry. The infinitesimal length is given in a coordinate system $(t, \chi, \theta, \varphi)$ by:

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \sigma^2(\chi)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

The function $\sigma(\chi)$ may assume the values χ, $\sin\chi$ or $\sinh\chi$, in which cases the 3-geometry has constant curvature which is flat, positive or negative, respectivelly.

We set $\phi = \phi(t)$. Then equations (11) is

$$(14) \quad \frac{1}{a^3}\frac{d}{dt}\left(a^3\frac{d\phi}{dt}\right) + \frac{\mu^2}{a^3}\phi\frac{d}{dt}\left(a^3\phi\frac{d\phi}{dt}\right) = 0$$

A straightforward integration yields:

$$(15) \quad \operatorname{arc\,sinh}(\mu\phi) + \phi\left(1+\mu^2\phi^2\right)^{1/2} = B\int a^{-3}\,dt$$

in which B is an arbitrary constant. This can be directly integrated for different Friedmann models.

IV - THE ENERGY BALANCE

Let us return now to the original set of equations (2,4). In general, the divergence of the current does not vanishes. Thus, it gives a contribution to the balance of energy which we will now evaluate. Taking the divergence of equation (2) one obtains

$$(16)\quad W^2{}_{|\nu} G^{\mu\nu} - (\Box W^2)_{|\nu} g^{\mu\nu} + (W^2)_{|\varepsilon\|\nu\|\lambda} g^{\varepsilon\mu} g^{\nu\lambda} + $$
$$+ (R W^\mu W^\nu)_{\|\nu} = -\frac{K}{\lambda} E^{\mu\nu}{}_{\|\nu} - \frac{K}{\lambda} \overset{*}{T}{}^{\mu\nu}{}_{\|\nu}$$

Now, we have

$$E^{\mu\nu}{}_{\|\nu} = F_{\mu\alpha} F^{\alpha\nu}{}_{\|\nu} + F_{\mu\alpha\|\beta} F^{\alpha\beta} + \frac{1}{2} F^{\alpha\beta} F_{\alpha\beta\|\mu}$$

or, using the anti-symmetry of $F_{\mu\nu}$:

$$(17)\qquad E^{\mu\nu}{}_{\|\nu} = F_{\mu\alpha} F^{\alpha\nu}{}_{\|\nu}$$

From equation (4) we obtain, like in Maxwell's Electrodynamics

$$(18)\qquad E^{\mu\nu}{}_{\|\nu} = F^{\mu}{}_{\alpha} \overset{*}{J}{}^{\alpha}$$

in which the total current $\overset{*}{J}{}^{\alpha}$ is defined by the expression

$$(19)\qquad \overset{*}{J}{}^{\alpha} = J^{\alpha} - \frac{\lambda}{K} R W^{\alpha}$$

Now, we have

$$(\Box \phi)_{|\mu} = \Box (\phi_{|\mu}) + R_{\mu}{}^{\varepsilon} \phi_{|\varepsilon}$$

From (16) and using this relation we find

$$(20)\quad -\frac{1}{2} (W^2)_{|\nu} g^{\mu\nu} R + \frac{K}{\lambda} J^{\alpha}{}_{\|\alpha} W^{\mu} + R W^{\nu} W^{\mu}{}_{\|\nu} =$$
$$= -\frac{K}{\lambda} F^{\mu\alpha} \overset{*}{J}_{\alpha} - \frac{K}{\lambda} \overset{*}{T}{}^{\mu\nu}{}_{\|\nu}$$

or finally, after some simplifications

(21) $$\overset{*}{T}{}^{\mu\nu}{}_{\parallel\nu} = -F^{\mu\alpha} J_{\alpha} - J^{\alpha}{}_{\parallel\alpha} W^{\mu}$$

The first term of the right-hand side gives the rate of work of the field; the second term is the contribution to the energy due to the created particles.

V - THE COSMICAL SOLUTION

In this section we will analyse a solution of our set of equations of non-minimal coupling of electrodynamics and gravitation in the search for a cosmical solution. We will try here to answer the following question: assuming the existence of an Universe filled with such non-linear photons, what are the global properties of such Cosmos ? As we will show there is a a solution of our set (2,4) of equations which represents an homogeneous and isotropic Universe. However, contrary to usual Friedmannian cosmologies in which an explicit function for the radius of the Universe with time is not in general available, our solution has a simple explicit form as we will see.

As there is no previleged direction in the space, in which the electric and the magnetic vectors could point to we conclude that they must be null. Thus, from equation (4) the scalar of curvature must vanishes:

(22) $$R = 0$$

and as a consequence the charge is conserved. Equation (22) may be written equivalently

(23) $$\Box\, W^2 = 0$$

Let us define a function $\Omega \equiv 1 + \lambda W^2$. Then, the set (2,4) of equations can be written into the form

(24) $$R_{\mu\nu} = -\frac{\Omega_{|\mu||\nu}}{\Omega}$$

(25) $$\Box\, \Omega = 0$$

We look for a solution of this set of equation by setting a geometry:

(26) $$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \sigma^2(\chi)(d\theta^2 + \sin^2\theta\, d\varphi^2)\right]$$

After some simple calculations we obtain the equations for $a(t)$ and $\Omega(t)$. The value of the curvature are:

$$R^0{}_0 = 3\,\frac{\ddot{a}}{a}$$

(27) $$R^1{}_1 = -\frac{\ddot{a}}{a} - 2\,\frac{\dot{a}^2}{a^2} + \frac{2}{a^2}\,\frac{\sigma''}{\sigma}$$

$$R^2{}_2 = R^3{}_3 = -\frac{\ddot{a}}{a} - 2\,\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\left(\frac{\sigma''}{\sigma} + \frac{\sigma'^2 - 1}{\sigma^2}\right)$$

and the co-variant derivative of Ω are :

$$\Omega^{|0}{}_{||0} = \ddot{\Omega}$$

(28) $$\Omega^{|1}{}_{||1} = \Omega^{|2}{}_{||2} = \Omega^{|3}{}_{||3} = -\frac{\dot{a}}{a}\,\dot{\Omega}$$

From (28) we obtain the result that the 3-curvature ${}^{(3)}R$ must be a constant. Let us define $\epsilon \equiv -\frac{1}{6}{}^{(3)}R$. The ϵ may assume the values $0, +1, -1$. Correspondingly the function $\sigma(\chi)$ is χ , sin χ or sinh χ . In each of these cases a solution can be found and written:

$$a(t) = (-\epsilon t^2 + bt + c)^{1/2} \tag{29}$$

$$\Omega = \frac{\Omega_0}{a}(-2\epsilon t + b) \tag{30}$$

Let us make some comments on these solution. We remark first of all that, as we have said – a simple explicit form for the function a(t) is available in the Cosmos filled with such non-linear photons.

The constants b, c and Ω_0 are not completelly arbitrary. The constraint they have to satisfy is linked to the definying equation of Ω . As in the isotropic world there is no previleged direction the vector W^μ must be of the form $W^\mu = (\dot{\phi}, 0, 0, 0)$. We have set a derivative on ϕ just to recall that W^μ must be a gradient.

Thus, we have

$$1 + \lambda\dot{\phi}^2 = \Omega_0(-2\epsilon t + b)(-\epsilon t^2 + bt + c)^{-1/2} \tag{31}$$

Let us examine this relation for the three possible values of ϵ separately.

In case $\epsilon = 0$, then $\lambda \dot{\phi}^2 = \Omega_0 \frac{b}{a} - 1$
If λ is negative, then Ω_0 must be negative too once b is a positive constant.

In the closed Universe, $\lambda \dot{\phi}^2 = \frac{\Omega_0}{a}(-2t+b)-1$. In case of a negative λ then Ω_0 must be positive and b negative. Finally for the open model if λ is negative b must be positive and Ω_0 negative. Now, let us turn for the function a(t). The possibility of real solution is dominated by the sign of $\Delta \equiv b^2 + 4\epsilon C$. In case of closed model, a positive value of Δ implies the existence of two real roots t_1 and t_2. The Universe starts to exist at $t = t_1$ and ends at $t = t_2$. The value $t_2 - t_1$ measure the total life of such Universe. The maximum point of expansion is for the value $t = \frac{b}{2}$. A notable consequence of the above set of equation is the impossibility of a steady-state regime. Indeed, for the Euclidean section the equation of motion are

(32) $$-3 \frac{\ddot{a}}{a} = \frac{\ddot{\Omega}}{\Omega}$$

(33) $$\frac{\ddot{a}}{a} + 2 \frac{\dot{a}^2}{a^2} = - \frac{\dot{a}}{a} \frac{\dot{\Omega}}{\Omega}$$

(34) $$\dot{\Omega}\, a^3 = \text{constant}$$

It is easy to recognize that this set do not admit as solution of a(t) the de-Sitter function e^{Ht}, whith an arbitrary constant H. This could well be guessed by equation (22) which is nothing, in Lyttleton-Bondi formulation, but the annihilation of the photon mass. Thus, although there is a point of contact between our present model and L-B's suggestion, their main result — besides others properties — does not occurs in our theory.

VI - THE ANISOTROPIC UNIVERSE

Although there is no possibility to have a non-null electric and/or magnetic field as a source of an isotropic world, this is not the case in an anisotropic Cosmos. Indeed, cosmological solutions of Maxwell's equations with a previleged direction has been analysed by many authors.

In the present situation we will show the possibility of having non-linear photons as the main source of anisotropic Universe.

Remarkable enough, our solution will have a null electric and magnetic field but a non-null potential vector pointing in a previleged direction. We start by setting the general equations (2,4) under such conditions, for a Bianchi type-I cosmological model. The fundamental length is given by:

$$ds^2 = dt^2 - a^2(t)\,dx^2 - b^2(t)\,dy^2 - c^2(t)\,dz^2 \tag{35}$$

Then a straightforward calculation gives, from equations (24) and (25) the set:

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} = -\frac{\ddot{\Omega}}{\Omega} \tag{36}$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{c}}{c} = \frac{\dot{a}}{a}\frac{\dot{\Omega}}{\Omega} \tag{37}$$

(38) $$\frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{b}}{b}\frac{\dot{c}}{c} = \frac{\dot{b}}{b}\frac{\dot{\Omega}}{\Omega}$$

(39) $$\frac{\ddot{c}}{c} + \frac{\dot{a}}{a}\frac{\dot{c}}{c} + \frac{\dot{b}}{b}\frac{\dot{c}}{c} = \frac{\dot{c}}{c}\frac{\dot{\Omega}}{\Omega}$$

(40) $$(a b c \dot{\Omega})^{\cdot} = 0$$

in which a dot, as usual, means time-derivative. We limit our discussion here to the case in which $\dot{b} = \dot{c} = 0$. Then there remains only two equations

(41) $$\frac{\ddot{a}}{a} = \frac{\dot{a}}{a^2}\frac{m}{\Omega}$$

(42) $$\dot{\Omega} = \frac{m}{a}$$

where m is a constant. Then, we obtain

(43) $$a\ddot{a} = \text{constant}$$

(44) $$\Omega = m\int \frac{dt}{a}$$

APPENDIX I

It seems wortwhile to call attention to the fact that the present model is not equivalent to a local re-scaling of all existing masses in the Universe, leaving the photon mass to be null. The ultimate reason for this is the behaviour of the theory under a conformal transformation.

The situation is very different in, for instance, scalar-tensor theories. Indeed, many authors have shown that scalar-tensor theories of Jordan-type (with a cosmical variation of the gravitational coupling constant) can be transformed conformally to theories with continuous creation of matter (either steady-state or not). This is a consequence of the existence of a unique definying function for both theories and on the behaviour of these models under conformal map. Harrison has shown that a conformal transformation $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \varphi^{S} g_{\mu\nu}$ coupled to the change of the scalar field $\varphi \rightarrow \tilde{\varphi} = \varphi^{\mu}$ induces the equality:

$$\delta \int \sqrt{-\tilde{g}} \left\{ \tilde{R} \tilde{\varphi}^{C} + \tilde{\omega} \tilde{\varphi}_{|\alpha} \tilde{\varphi}_{|\beta} \tilde{g}^{\alpha\beta} \tilde{\varphi}^{C-2} + k \tilde{L}_{m} \tilde{\varphi}^{D} \right\} d^4x \;\overset{\circ}{=}$$

$$\overset{\circ}{=} \delta \int \sqrt{-g} \left\{ R \varphi^{A} + \omega \varphi_{|\alpha} \varphi_{|\beta} g^{\alpha\beta} \varphi^{A-2} + k L_{m} \varphi^{B} \right\} d^4x$$

where the symbol $\overset{\circ}{=}$ means equality of the Euler-Lagrangian equations (that is, up to a divergence term). A direct calculation can relate the values of $\tilde{\omega}$, $\tilde{C}$ and $\tilde{D}$ in terms of ω, μ, A and S. Thus, all scalar-tensor theories are conformally equivalent. This however is not the situation in vector-tensor theories The point is that if we try to generate a mapping which brings our model (with a space-time dependent photon mass) to a theory in which all others masses in the Universe are altered (retaining Maxwell's Electrodynamics) then we inevitably introduce non-linearities on the equation of motion of the vector field. The reason is simple to analyse in the example of a conformal map. In order to eliminate the non-minimal coupling term we are obliged to set the conformal function to be proportional to some power of W^2. This will give origin to the non-linear terms, as can be easily seen. Thus, our model cannot be reduced to a re-scaling of masses in Maxwell's Electrodynamics. Actually this seems to be a generic situation for non-minimal coupling with gravitation for non-null spin fields.

APPENDIX II

The cosmological solution presented in section V is stable against a small perturbation generated by the introduction of a small quantity of matter. Actually, this property does not depend on our specific model but is a consequence of having no density of matter in the expanding background.

As a consequence of the energy balance equation (21) and due to the absence of electric and magnetic fields, the energy momentum tensor of the matter must be conserved. Let us consider a fluid (dust) with an energy-momentum tensor given by $T_{\mu\nu} = (\delta\rho)\, V_\mu V_\nu$; where $\delta\rho$ is a small density. We choose the co-moving frame in order to set the fluid velocity V^μ to have the value $V^\mu = \delta^\mu_0$. Conservation of $T^\mu{}_\nu$, projected in the V^μ-direction gives

$$(\delta\rho)^{\cdot} + (\delta\rho)\theta = 0$$

In the Euclidean section case, using the results obtained above, the expansion Θ equals to $\frac{3b}{2}(bt+c)^{-1}$. Thus a direct integration gives

$$\delta\rho = (\delta\rho)_0 \, (bt+c)^{-3/2}$$

Thus, as time goes on the total perturbation decreases showing the stability of our model under a small injection of matter in our non-linear photon Cosmos.

Actually, one can show a result more stronger than this, e.g. that our model Universe connot share the bending of space-time with a finite density of matter.

This can be seen by a direct inspection on equation (22) and (25), which imply that R = 0 and $\Box\Omega = 0$. These two equation specify the functions a(t) and $\Omega(t)$, giving no possibility of inserting another function $\rho(t)$ in our equations. Thus, the Cosmos generated by our non-linear photons seems to be a very aristocratic one.

APPENDIX III

We will give here, for completeness, the equation of motion obtained from the Lagrangian $L = \sqrt{-g}\,(R + \frac{\epsilon}{K} W^{\mu} W^{\nu} R_{\mu\nu}) + \mathcal{L}_{Max}$
From the variational principle we obtain

(A-1) $$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{\epsilon}{2}\Big\{ (W^{\alpha} W^{\beta})_{|\alpha||\beta}\, g_{\mu\nu} - \Box(W_{\mu}W_{\nu}) + (W^{\beta} W_{(\nu})_{||\mu)||\beta} + 2 R_{\alpha(\mu} W_{\nu)} W^{\alpha} - R_{\alpha\beta} W^{\alpha} W^{\beta} g_{\mu\nu} = - k \, T^{*}_{\mu\nu}$$

and

(A-2) $$F^{\mu\nu}{}_{\|\nu} = -\frac{\epsilon}{K} R^{\mu}{}_{\nu} W^{\nu}$$

It seems worthwhile to remark here that even in the case of a homogeneous and isotropic Universe, the equation of motion obtained from Lagrangian $L_{I} = \sqrt{-g}\, R W_{\mu} W^{\mu}$ are distinct from those given by $L_{II} = \sqrt{-g}\, R_{\mu\nu} W^{\mu} W^{\nu}$. Indeed, the set (A-1,2) of equations does not admit a Friedmann-like Universe. This can be seen by the following arguments. Due to the isotropy of the Universe, the electric and magnetic vectors are null. Thus, from (A-2) $R_{\mu\nu} W^{\nu}$ must vanishes. However, the unique non-null component of the potential vector W^{α} is for $\alpha = 0$. Thus, we obtain the equation $R^{\mu}{}_{0} = 0$

Due to the symmetry conditions of such Universe, $R^{i}{}_{0}$ are identicelly null and it remains the equation

(A-3) $$R^{0}{}_{0} = 0$$

Now, it is straightforward to verify that (A-3) is not compatible with equations (22,24).

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