CBPF-NF-046/03

On the Dynamics of Dissipative Gravitational Collapse

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Abstract

The dynamical equations for dissipative collapse in, both, the streaming out and the diffusion approximation, are presented and coupled to a causal transport equation in the context of Israel–Stewart theory. It is obtained that the resulting evolution (at any time scale) will critically depend on a quantity defined in terms of thermodynamic variables. Prospective applications of this result to some astrophysical scenarios are discussed.

Key words: gravitation, relativity, hydrodynamics, stellar dynamics, radiation mechanisms: general, diffusion.

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1 Introduction

Some years ago, Misner and Sharp (Misner & Sharp 1965) and Misner (Misner 1965) provided a full account of the dynamical equations governing the adiabatic, and the dissipative relativistic collapse in the streaming out approximation.

The relevance of dissipative processes in the study of gravitational collapse cannot be over emphasized. Indeed, dissipation due to the emission of massless particles (photons and/or neutrinos) is a characteristic process in the evolution of massive stars. In fact, it seems that the only plausible mechanism to carry away the bulk of the binding energy of the collapsing star, leading to a neutron star or black hole, is neutrino emission (Kazanas & Schramm 1979).

In the diffusion approximation, it is assumed that the energy flux of radiation, as that of thermal conduction, is proportional to the gradient of temperature. This assumption is in general very sensible, since the mean free path of particles responsible for the propagation of energy in stellar interiors is in general very small as compared with the typical length of the object. Thus, for a main sequence star as the sun, the mean free path of photons at the centre, is of the order of 2 cm. Also, the mean free path of trapped neutrinos in compact cores of densities about 10^{12} g. cm^{-3} becomes smaller than the size of the stellar core (Arnett 1977, Kazanas 1978).

Furthermore, the observational data collected from supernova 1987A indicates that the regime of radiation transport prevailing during the emission process, is closer to the diffusion approximation than to the streaming out limit (Lattimer 1988).

However in many other circumstances, the mean free path of particles transporting energy may be large enough as to justify the free streaming approximation. Therefore it is advisable to include simultaneously both limiting cases of radiative transport, diffusion and streaming out, allowing to describe a wide range of situations.

In a recent work (Herrera & Santos 2003) we have studied the effects of dissipation, in both limiting cases of radiative transport, within the context of quasi–static approximation. This assumption is very sensible because the hydrostatic time scale is very small for many phases of the life of a star. It is of the order of 27 minutes for the sun, 4.5 seconds for a white dwarf and 10^{-4} seconds for a neutron star of one solar mass and 10km radius (Schwarzschild 1958; Kippenhahn & Weigert 1990; Hansen & Kawaler 1994). However, during their evolution, self–gravitating objects may pass through phases of intense dynamical activity, with time scales of the order of magnitude of (or even smaller than) the hydrostatic time scale, and for which the quasi–static approximation is clearly not reliable, e.g.,the collapse of very massive stars (Iben 1963), and the quick collapse phase preceding neutron star formation, see for example (Myra & Burrows 1990) and references therein. In these cases it is mandatory to take into account terms which describe departure from equilibrium, i.e. a full dynamic description has to be used.

Thus the extension of Misner dynamical equations as to include dissipation in the form of a radial heat flow (besides pure radiation) is our first task in this paper. This is presented in Section 3. Then in the following Section, the resulting dynamical equation is coupled to the transport equation obtained in the context of the Müller–Israel–Stewart theory (Müller 1967, Israel 1976, Israel & Stewart 1976, 1979).

After doing that we show that the effective inertial mass density of a fluid element reduces by a factor which depends on dissipative variables. This result was already known (see Herrera 2002 and references therein), but being valid only, just after leaving the equilibrium, on a time scale of the order of relaxation time. The novelty here, and the main result of this paper is, on the one hand, that such reduction of the effective inertial mass density is shown to be valid at an *arbitrary time scale*, and on the other, that the"gravitational force" term in the dynamical equation is also reduced by the same factor, as expected from the equivalence principle. Prospective applications of this result to astrophysical scenarios are discussed at the end.

In the next section the field equations, the conventions, and other useful formulae are introduced.

2 The energy–momentum tensor and the field equations

In this section we provide a full description of the matter distribution, the line element, both, inside and outside of the fluid boundary and the field equations this line element must satisfy. Since we are going to follow closely the Misner approach (Misner 1965) we shall use comoving coordinates (for a description of gravitational collapse in non–comoving coordinates, see (Herrera et al 2002) and references therein).

2.1 The interior spacetime

We consider a spherically symmetric distribution of collapsing fluid, which, for sake of completeness, we assume to be locally anisotropic, undergoing dissipation in the form of heat flow and free streaming radiation, bounded by a spherical surface Σ. For such system the energy–momentum tensor is given by

$$T^{\alpha\beta}_{-} = (\mu + P_\perp)V^\alpha V^\beta + P_\perp g^{\alpha\beta} + (P_r - P_\perp)\chi^\alpha\chi^\beta + q^\alpha V^\beta + V^\alpha q^{\text{eta}} + \epsilon l^\alpha l^\beta, \qquad (1)$$

where, μ is the energy density, P_r the radial pressure, $P_\perp$ is the tangential pressure, ϵ is the radiation density, V^α is the four velocity of the fluid, q^α is the heat flux, χ^α is a unit four vector along the radial direction and l^α is a null four vector. These quantities have to satisfy

$$V^\alpha V_\alpha = -1, \;\; V^\alpha q_\alpha = 0, \;\; \chi^\alpha\chi_\alpha = 1, \;\; \chi^\alpha V_\alpha = 0, \;\; l^\alpha l_\alpha = 0. \qquad (2)$$

We assume the interior metric to Σ to be comoving, shear free for simplicity, and spherically symmetric, accordingly it may be written as

$$ds^2 = -A^2(t,r)dt^2 + B^2(t,r)(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2), \qquad (3)$$

and hence

$$V^\alpha = A^{-1}\delta^\alpha_0, \;\; q^\alpha = q\delta^\alpha_1, \;\; l^\alpha = A^{-1}\delta^\alpha_0 + B^{-1}\delta^\alpha_1, \;\; \chi^\alpha = B^{-1}\delta^\alpha_1, \qquad (4)$$

where q is a function of t and r and we have numbered the coordinates $x^0 = t$, $x^1 = r$, $x^2 = \theta$ and $x^3 = \phi$. Now the Einstein's field equations become with the help of (1-4)

$$8\pi T^-_{00} = 8\pi(\mu+\epsilon)A^2 = -\left(\frac{A}{B}\right)^2\left[2\frac{B''}{B} - \left(\frac{B'}{B}\right)^2 + \frac{4}{r}\frac{B'}{B}\right] + 3\left(\frac{\dot{B}}{B}\right)^2, \qquad (5)$$

$$8\pi T^-_{01} = -8\pi(qB+\epsilon)AB = -2\left(\frac{\dot{B}'}{B} - \frac{B'}{B}\frac{\dot{B}}{B} - \frac{A'}{A}\frac{\dot{B}}{B}\right), \qquad (6)$$

$$8\pi T^-_{11} = 8\pi(P_r+\epsilon)B^2 = \left(\frac{B'}{B}\right)^2 + \frac{2}{r}\frac{B'}{B} + 2\frac{A'}{A}\frac{B'}{B} + \frac{2}{r}\frac{A'}{A} - \left(\frac{B}{A}\right)^2\left[2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 - 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right], \qquad (7)$$

$$8\pi T^-_{22} = \frac{8\pi T^-_{33}}{\sin^2\theta} = 8\pi r^2 P_\perp B^2 = r^2\left[\frac{B''}{B} - \left(\frac{B'}{B}\right)^2 + \frac{1}{r}\frac{B'}{B} + \frac{A''}{A} + \frac{1}{r}\frac{A'}{A}\right] - r^2\left(\frac{B}{A}\right)^2\left[2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 - 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right], \qquad (8)$$

where the dot and prime stand for differentiation with respect to t and r. The rate of expansion $\Theta = V^\alpha{}_{;\alpha}$ of the fluid sphere is given, from (3) and (4), by

$$\Theta = 3\frac{\dot{B}}{AB}, \qquad (9)$$

and from (6) we have

$$8\pi(qB+\epsilon)B = \frac{2}{3}\Theta'. \quad (10)$$

Since $q > 0$ and $\epsilon > 0$ then from (10) we have $\Theta' > 0$ meaning that, if the system is collapsing $\Theta < 0$, q and/or ϵ decrease the rate of collapse towards the outer layers of matter. If $q = 0$ and $\epsilon = 0$ from (10) $\Theta' = 0$, which means that the collapse is homogeneous.

The mass function $m(t, r)$ of Cahill and McVittie (Cahill & MacVittie 1970) is obtained from the Riemann tensor component $R_{23}{}^{23}$ and is for metric (3)

$$m(t,r) = \frac{(rB)^3}{2} R_{23}{}^{23} = \frac{r^3}{2}\frac{B\dot{B}^2}{A^2} - \frac{r^3}{2}\frac{B'^2}{B} - r^2 B'. \quad (11)$$

2.2 The exterior spacetime

The exterior spacetime to Σ of the collapsing body is described by the outgoing Vaidya spacetime which models a radiating star and has metric

$$ds_+^2 = -\left[1 - \frac{2m(v)}{\rho}\right] dv^2 - 2dvd\rho + \rho^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (12)$$

where m, the total mass inside Σ, is a function of the retarded time v. The surface Σ described by the comoving coordinate system (3) is $r = r_\Sigma =$ constant, while in the non comoving coordinate system (12) is $\rho = \rho_\Sigma(v)$. Matching the interior spacetime (3) with source (1) to the exterior spacetime (12) by using Darmois junction conditions we obtain

$$(P_r)_\Sigma = (qB)_\Sigma, \quad (13)$$

$$(qB+\epsilon)_\Sigma = \frac{1}{4\pi}\left(\frac{L}{\rho^2}\right)_\Sigma, \quad (14)$$

$$(rB)_\Sigma = \rho_\Sigma, \quad (15)$$

$$\left(\frac{r^3}{2}\frac{B\dot{B}^2}{A^2} - \frac{r^3}{2}\frac{B'^2}{B} - r^2 B'\right)_\Sigma = m(v), \quad (16)$$

$$A_\Sigma dt = \left(1 - \frac{2m}{\rho} + 2\frac{d\rho}{dv}\right)_\Sigma^{1/2} dv, \quad (17)$$

where L is defined as the total luminosity of the collapsing sphere as measured on its surface and is given by

$$L = L_\infty \left(1 - \frac{2m}{\rho} + 2\frac{d\rho}{dv}\right)^{-1}, \quad (18)$$

and where

$$L_\infty = \frac{dm}{dv} \quad (19)$$

is the total luminosity measured by an observer at rest at infinity. The result (13) represents the continuity of the radial flux of momentum across Σ which only the heat flow q appears. However, for the total radiation leaving Σ (14) the radiation ϵ contributes as well as q. Although it might seem to be obvious, it is perhaps important to stress that the radiation ϵ has the same null property associated to the exterior null radiation that it produces, while the heat flux q, producing the exterior null radiation too, is not a null flux. Relation (15) is the equality of the proper radii as measured from the perimeter of the spherical surface Σ in both frames (3) and (12). The expression for the total mass (16) is the corresponding mass function (Cahill & Mac Vittie 1970) given by (8). The relationship of proper times measured on Σ with both frames (3) and (17) is given by (17).

3 Dynamical equations

For studying the dynamical properties of the field equations and following Misner and Sharp, let us introduce the proper time derivative D_t given by

$$D_t = \frac{1}{A}\frac{\partial}{\partial t}. \tag{20}$$

Then using (20) we can describe the velocity U of the collapsing fluid as

$$U = rD_tB < 0 \;\; (\textit{in the case of collapse}), \tag{21}$$

then (11) can be rewritten as

$$\frac{(rB)'}{B} = \left[1 + U^2 - \frac{2m(t,r)}{rB}\right]^{1/2} = E. \tag{22}$$

The right hand side of (22) can be interpreted as being the energy density E of a collapsing fluid element. Next, by taking the proper time derivative of (11) we obtain

$$\begin{aligned} D_tm = r^3\frac{B\dot{B}\ddot{B}}{A^3} + \frac{r^3}{2}\left(\frac{\dot{B}}{A}\right)^3 - r^3\frac{B\dot{A}\dot{B}^2}{A^4} \\ + \frac{r^3}{2}\frac{\dot{B}B'^2}{AB^2} - r^3\frac{B'\dot{B}'}{AB} - r^2\frac{\dot{B}'}{A}. \end{aligned} \tag{23}$$

Considering (6) and (7) we can rewrite (23) as

$$D_tm = -4\pi\left[(P_r+\epsilon)r^3\frac{B^2\dot{B}}{A} + (qB+\epsilon)r^2B(rB)'\right], \tag{24}$$

and with (21) and (22) it becomes

$$D_t m = 4\pi \left[-(P_r + \epsilon)U - (qB + \epsilon)E \right] (rB)^2, \tag{25}$$

which gives the rate of variation of the total energy inside a surface of radius rB. In the right hand side of (25) $(P_r + \epsilon)|U|$ (in the case of collapse $U < 0$) increases the energy inside rB through the rate of work being done by P_r and the induction field produced by ϵ and already observed in (Lindquist, Schwartz & Misner 1965). Clearly here the heat flux q does not appear since it does not produce an induction field. The second term $-(qB + \epsilon)E$ is the matter energy leaving the spherical surface.

Another proper derivative that helps us to study the dynamics of the collapsing system is the proper radial derivative D_R, where

$$R = rB, \tag{26}$$

constructed from the radius of a spherical surface, as measured from its perimeter inside Σ, being

$$D_R = \frac{1}{R'}\frac{\partial}{\partial r}. \tag{27}$$

Then by taking the proper radial derivative (27) of (11) we obtain

$$D_R m = \frac{B}{(rB)'}\left[-r^3\frac{B'B''}{B^2} + r^2\frac{B''}{B} + \frac{r^3}{2}\left(\frac{B'}{B}\right)^3 - \frac{3r^2}{2}\left(\frac{B'}{B}\right)^2 \right.$$
$$\left. -2r\frac{B'}{B} - r^3\frac{A'\dot{B}^2}{A^3} + \frac{r^3}{2}\frac{B'\dot{B}^2}{A^2 B} + r^3\frac{\dot{B}\dot{B}'}{A^2} + \frac{3r^2}{2}\left(\frac{\dot{B}}{A}\right)^2 \right]. \tag{28}$$

Considering (5) and (6) then (28) becomes

$$D_R m = 4\pi \left[\mu + \epsilon + (qB + \epsilon)\frac{rB\dot{B}}{(rB)'A} \right] (rB)^2, \tag{29}$$

and with (21) and (22) we finally have

$$D_R m = 4\pi \left[\mu + \epsilon + (qB + \epsilon)\frac{U}{E} \right] (rB)^2. \tag{30}$$

This expression gives the total energy entrapped between two neighboring spherical surfaces with respect to proper radius inside the fluid distribution. The first term on the right hand side of (30) $\mu + \epsilon$ is due to the energy density plus the induction field and no heat flux appears. The second term $(qB + \epsilon)U/E$ is negative (in the case of collapse) and measures the out flux of heat and radiation.

Finally, we can obtain the acceleration D_tU of a collapsing particle inside Σ. In order to do that we start from (7) and (11) which allows us to write

$$D_tU = -\left[m + 4\pi(P_r+\epsilon)(rB)^3\right](rB)^{-2} + \frac{A'}{A}\frac{(rB)'}{B^2}. \tag{31}$$

Calculating the r component of the Bianchi identities, $T^{1\beta}_{-\ ;\beta} = 0$, from (1) we obtain

$$\begin{aligned} P_r' + \epsilon' + (\mu + P_r + 2\epsilon)\frac{A'}{A} + 2(\epsilon + P_r - P_\perp)\frac{(rB)'}{rB} \\ +(5qB + 4\epsilon)\frac{\dot{B}}{A} + \dot{q}\frac{B^2}{A} + \dot{\epsilon}\frac{B}{A} = 0. \end{aligned} \tag{32}$$

Substituting the expression A'/A from (32) into (31) and considering (11), (20), (26) and (27) we obtain

$$\begin{aligned} (\mu + P_r + 2\epsilon)D_tU = -(\mu + P_r + 2\epsilon)\left[m + 4\pi(P_r+\epsilon)R^3\right]\frac{1}{R^2} \\ -E^2\left[D_R(P_r+\epsilon) + 2\frac{\epsilon + P_r - P_\perp}{R}\right] \\ -E\left[(5qB + 4\epsilon)\frac{U}{R} + BD_tq + D_t\epsilon\right]. \end{aligned} \tag{33}$$

Equation (33) has the “Newtonian” form

$$Force = Mass\ density \times Acceleration \tag{34}$$

The first term in the right hand side of (33) represents the gravitational force. It shows that the gravitational force acting on a particle has a Newtonian part with m and a purely relativistic gravitational contribution due to P_r and ϵ. The second term in square brackets represent the hydrodynamical forces. It consists of the usual pressure gradient term (including the contribution of the radiation to the pressure) $D_R(P_r+\epsilon) < 0$ counteracting collapse, and the anisotropic force term $\epsilon + P_r - P_\perp$ which can be positive or negative thus, respectively, acelerating more or counteracting the rate of collapse. In these two terms the appearance of ϵ is due to the contribution of radiation to the total energy density and radial pressure. The last term in square brackets contains the specific contribution of dissipation to the dynamics of the system. The first term within this bracket is positive ($U < 0$) showing that the out flux of $q > 0$ and $\epsilon > 0$ diminish the total energy inside the collapsing sphere thereby reducing the rate of collapse. It is interesting to observe the different effects that q and ϵ have on the dynamical behaviour of the collapsing fluid. The heat flux q helps only to slow down the rate of collapse by diminishing the energy inside the fluid sphere by producing an exterior outflowing radiation. On the other hand, the radiation density ϵ behaves not only in a similar way as q by diminishing the energy of the collapsing sphere through the exterior outflow of radiation, but contributes too as an induction field to the gravitational energy, first observed in (Lindquist, Schwartz & Misner 1965), and contributes to the radial pressure P_r. The effects of $D_t\epsilon$ have been discussed in detail in (Misner 1965). Thus it remains to analyse the effects of D_tq, this will be done in the next section after introducing the transport equation.

Before coming to the next section, we observe that from (33) the limit of hydrostatic equilibrium when $U = 0$, $q = 0$ and $\epsilon = 0$ can be achieved, producing

$$D_R P_R + \frac{2(P_R - P_\perp)}{R} = -\frac{\mu + P_r}{R(R-2m)}\left(m + 4\pi P_r R^3\right), \tag{35}$$

which is just the generalization of the TOV equation for anisotropic fluids (Bowers & Liang 1974), obtained in comoving coordinates in (Chan, Herrera & Santos 1993) while studying dynamical instability for radiating anisotropic collapse.

4 Transport equation and its consequences

As we mentioned before we shall use a transport equation derived from the Müller-Israel-Stewart second order phenomenological theory for dissipative fluids (Müller 1967, Israel 1976, Israel & Stewart 1976, 1979).

Indeed, it is well known that the Maxwell-Fourier law for the radiation flux leads to a parabolic equation (diffusion equation) which predicts propagation of perturbation with infinite speed (see Joseph & Preziosi 1989, Jou, Casas-Vázquez J.& Lebon 1988, Maartens 1996, Herrera & Pavón, 2002 and references therein). This simple fact is at the origin of the pathologies (Hiscock & Lindblom 1983) found in the approaches of Eckart (Eckart 1940) and Landau (Landau & Lifshitz 1959) for relativistic dissipative processes.
To overcome such difficulties, different relativistic theories with non-vanishing relaxation times have been proposed in the past (Müller 1967, Israel 1976, Israel & Stewart 1976, 1979, Pavón, Jou & Casas-Vázquez 1982, Carter 1976). The important point is that all these theories provide a heat transport equation which is not of Maxwell-Fourier type but of Cattaneo type (Cattaneo 1948), leading thereby to a hyperbolic equation for the propagation of thermal perturbation. Thus the corresponding transport equation for the heat flux reads (Maartens 1996)

$$\tau h^{\alpha\beta}V^\gamma q_{\beta;\gamma} + q^\alpha = -K h^{\alpha\beta}(T_{,\beta} + T a_\beta) - \frac{1}{2}\kappa T^2 \left(\frac{\tau V^\beta}{\kappa T^2}\right)_{;\beta} q^\alpha, \tag{36}$$

where $h^{\mu\nu}$ is the projector onto the three space orthogonal to V^μ, κ denotes the thermal conductivity, and T and τ denote temperature and relaxation time respectively. Observe that, due to the symmetry of the problem, equation (36) only has one independent component, which may be writtten as:

$$\tau(qB)\dot{}B + qAB^2 = -K(TA)' - \frac{\kappa T^2 qB^2}{2}\left(\frac{\tau}{\kappa T^2}\right)^{\cdot} - \frac{3\tau \dot{B}Bq}{2}. \tag{37}$$

Now, using (20) and (31)

$$BD_t q = -\frac{\kappa T}{\tau E}D_t U - \frac{\kappa T'}{\tau B} - \frac{qB}{\tau}(1 + \frac{\tau U}{R}) -$$
$$-\frac{\kappa T}{\tau E}\left[m + 4\pi(P_r + \epsilon)R^3\right]R^{-2} - \frac{\kappa T^2 qB}{2AE}\left(\frac{\tau}{\kappa T^2}\right)^{\cdot} - \frac{3UBq}{2R}. \tag{38}$$

We can couple the transport equation in the form above (38) to the dynamical equation (33), in order to bring out the effects of dissipation (in the diffusion approximation) on the dynamics of the collapsing sphere. With that purpose, let us replace (38) into (33) (putting $\epsilon = 0$), then we obtain after some rearrangements

$$(\mu + P_r)(1-\alpha)D_t U = F_{grav}(1-\alpha) + F_{hyd} +$$
$$+\frac{E\kappa T'}{\tau B} + \frac{EqB}{\tau} - \frac{4qBEU}{R} + \frac{\kappa E T^2 qB}{2A\tau}\left(\frac{\tau}{\kappa T^2}\right)^{\cdot} + \frac{3UBq}{2R}. \tag{39}$$

Where F_{grav} and F_{hyd} are defined by

$$F_{grav} = -(\mu + P_r)\left[m + 4\pi P_r R^3\right]\frac{1}{R^2}, \tag{40}$$

and

$$F_{hyd} = -E^2\left[D_R P_r + 2\frac{P_r - P_\perp}{R}\right], \tag{41}$$

where α is given by

$$\alpha = \frac{\kappa T}{\tau(\mu + P_r)}. \tag{42}$$

We can now analyze the overall effects of dissipation (in the diffussion approximation) on the evolution of the collapsing sphere.

First of all observe that as α tends to 1, the effective inertial mass density of the fluid element tends to zero. This effect was known (see Herrera 2002 and references therein), but only to be valid just after the system abandons the equilibrium, on a time scale of the order of (or smaller than) the relaxation time. Here we see that it is present all along the evolution. Furthermore we see that F_{grav} is also multiplied by the factor $(1-\alpha)$. Indicating that the effective gravitational attraction on any fluid element decreases by the same factor as the effective inertial mass (density). Which of course is to be expected, from the equivalence principle. It is also worth mentioning that F_{hyd} is in principle independent (at least explicitly) on this factor.

Next observe that the third and the fourth terms, as well as the fifth and the last terms, on the right hand side of (39), are of oposite sign and of the same order of magnitude (at least in the case of not too strong gravitational field). Finally, the sign and the order of magnitude of the sixth term is clearly, model dependent. Furthermore this term as well as the last one on the right hand of (39) are absent in the "truncated" version of the theory (see Triginer & Pavón 1995).

With these comments above in mind, let us imagine the following situation: A collapsing sphere evolves in such a way that the value of α keeps increasing and approaches the critical value of 1. As this process takes place, the ensuing decreasing of the gravitational force term would eventually lead to a change of the sign of the right hand side of (39). Since that would happen for small values of the effective inertial mass density, that would imply a strong bouncing of the sphere, even for a small absolute value of the right hand side of (39).

For this picture to be physcally meaningful one should first answer to the following questions:

- How close may α approach the critical value?
- In what physical scenarios one could expect values of α close to the critical value?

Since these questions are related to each other, we answer to them, simultaneously.

First of all it should be mentioned that from the analysis of stability and causality in dissipative relativistic fluids (Hiscock & Lindblom 1983), it follows that causality and hyperbolicity (which imply stability) require for dissipative viscous free systems

$$\tau > \frac{\kappa T}{\mu + p} + \frac{\kappa}{n}\frac{c_s^2}{c_p}, \tag{43}$$

$$\tau > \frac{\kappa T}{1 - c_s^2}\left[\frac{1}{\mu + p} + \frac{1}{nT}\left(\frac{1}{c_v} - \frac{c_s^2}{c_p}\right) - \frac{2\alpha_p}{nc_v\kappa_T(\mu + p)}\right] \tag{44}$$

and

$$\tau > \frac{\kappa}{nc_s^2 c_v}\left[\frac{2\alpha_p T}{\kappa_T(\mu + p)} - 1\right], \tag{45}$$

where n and c_s denote the particle density and the sound speed, c_p and c_v are the specific heat at constant pressure and volume, κ_T is the thermal expansion coefficient and α_p the isothermal compressibility. These expressions are found from equations (146-148) in (Hiscock & Lindblom 1983), taking the limit $\beta_o, \beta_2 \to \infty$ and $\alpha_i = 0$ (this method was applied in Maartens 1996 to the case in which only bulk viscous perturbations were present). It should be kept in mind that the conditions above, are obtained within a linear perturbative scheme.

Obviously, condition (43) is violated at the critical point (in fact it is violated, slightly below it). However as we shall see below, it is not difficult to find physical conditions for which the numerical values of variables entering in the definition of α lead to $\alpha = 1$. Therefore, the relevant question is: Can a physical system actually reach the critical point? If the answer to this question is negative, then it should be explained how a given system avoids the critical point. Since, as mentioned before, numerical values of κ, T, τ, μ, and P_r, leading to $\alpha \approx 1$ may correspond to a non very exotic scenario. On the other hand, a positive answer seems to be prohibited by causality and stability conditions. However, we shall conjecture here that this might not be the case. In fact, the vanishing of the effective inertial mass density at the critical point, indicates that linear approximation is not valid at that point. So it seems that the behaviour of the system close to the critical point cannot be studied with a linear perturbative scheme, in which case it might be possible for a given system to attain the critical point. Furthermore, in the general case (including viscosity) it may happen that causality breaks down beyond the critical point (Herrera & Martínez 1996). Thus, it appears that there exist situations where a given physical system may attain the critical point and even go beyond it.

Indeed, condition $\alpha \approx 1$ can be accomplished in non very exotic systems. One of them is an interacting mixture of matter and neutrinos, which is a well-known scenario during the formation of a neutron star in a supernova explosion. In this case the heat conductivity coefficient is given by (Weinberg 1971,Shapiro 1989)

$$\kappa = \frac{4}{3} b T^3 \tau, \tag{46}$$

where τ is the mean collision time and $b = 7N_\nu a/8$, with N_ν the number in neutrino flavors and a the radiation constant. Assuming that two viscosity coefficients vanishes, and $p \ll \mu$ then

$$\alpha = \frac{\kappa T}{\tau_\kappa \left(\mu + p\right)} \simeq \frac{\kappa T}{\tau \mu}. \tag{47}$$

Using usual units, the critical point is overtaken if

$$T > \left(\frac{6\mu c^3}{7N_\nu a}\right)^{1/4} \sim 4.29 \times 10^8 \mu^{1/4}, \tag{48}$$

where we have adopted $\tau \sim \tau_\kappa$, $N_\nu = 3$, T is in Kelvin and μ is given in g cm^{-3}. The values of temperature, for which $\alpha \approx 1$, are similar to the expected temperature that can be reached during hot collapse in a supernova explosion (Shapiro & Teukolsky 1983 [§ 18.6]).

5 Conclusions

Following the scheme developped by Misner and Sharp, we have established the set of dynamical equations governing the evolution of collapsing dissipative spheres, taking into account, both, the free–streaming and the difussion approximation. We have further coupled the dynamical equation with a heat transport equation obtained from the Müller-Israel-Stewart theory. The resulting equation brings out the relevance of a critical point ($\alpha = 1$) for which, both the effective inertial mass density and the gravitational frorce term vanish. We have shown that in principle that critical point may be attained under acceptable physical conditions (e.g. a supernova scenario) and have speculated about the possibility that in that case, a collapsing sphere bounces. Of course the eventual application of this model to a supernova, would require much more details about the astrophysical settings.

Before concluding we would like to make the following remarks:

1. It should be noticed that the appearance of the factor $1 - \alpha$ in the inertial mass density and the gravitational force term, is produced by the first term on the left of equation (36). But this is the term which introduces causality in the transport equation and therefore, is to be expected in any causal theory of dissipation. Accordingly our main result is also expected to hold for a general family of theories which includes the Müller-Israel-Stewart theory. However, whereas the mere appearance of the factor $1 - \alpha$ in the dynamical equation is a very general result, the possibility of reaching the critical point and the physical consequences derived from that, will depend on the specific theory of dissipation to be adopted.

2. Observe the formal similarity between the critical point and the equation of state for an inflationary scenario ($\mu = -P_r$) without dissipation. This kind of equation of state has been recently proposed to describe the interior of a cold compact object without event horizons, and which would represent an alternative to black holes (Mazur & Mottola 2001). One could speculate with the possibility that such interior could be described instead, by a dissipative fluid with α tending to 1.

3. In the same line of arguments, collapsing objects endowed with strong magnetic moments have been recently proposed also as alternatives to black holes (Robertson & Leiter 2003). In these objects the magnetic dipole field drives them to radiate, leading to steady collapse at an Eddington limit rate. As mentioned before, the condition of such steady collapse, $D_tU \approx 0$, may be reached as α tends to 1, without invoking the presence of strong magnetic fields. Alternatively the simultaneous

action of both mechanisms would enhance the possibilities of formation of such objects.

4. It should be clear that the analysis presented here depends strictly on the validity of the diffusion approximation, which in turn depends on the assumption of local thermodynamical equilibrium (LTE). Therefore, only small deviations from LTE can be considered in the context of this work. Thus when we state that the effective inertial mass density decreases by the factor $(1-\alpha)$, at all time scales, this means at time scales within which the system is not very far from LTE.

5. For the sake of completeness we have considered an anisotropic fluid (instead of an isotropic one, $P_r = P_\perp$), leaving the origin of such anisotropy completely unspecified. As it is apparent, anisotropy does not affect the most important result obtained here (e.g. the existence of the critical point). However, should anisotropy be related to viscosity, then for consistency the anisotropic pressure tensor should be subjected to the Israel-Stewart causal evolution equation for shear viscosity.

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