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INELASTIC NEUTRINO-NUCLEUS SCATTERING IN HIGHLY EVOLVED STARS

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A complete expression of the neutrino-nucleus scattering cross section is presented. The relevance of the inelastic channels for the neutrino opacity in supernova problem is discussed.

Since the pioneering work of Colgate and White[1], the role played by the neutrino in supernova events has been investigated extensively[2-5]. At present, it seems that, at the early stage of the presupernova collapse, the neutrino just carry energy from the inner core to outside of the star, but, at the later stage, when the density becomes high enough ($\gtrsim 10^{10}$ g cm^{-3}), the dynamics of the supernova depends very sensitively on the details of the neutrino transport mechanism in the stellar matter. In this respect, Bruenn et al.[6] proposed, in a qualitative basis, criteria for matter ejection via momentum and energy transfer of neutrinos, and via hydrodynamic bounce. The hydrodynamic bounce may also depend critically on neutrino confinement.

Using the current theory of weak interaction, many authors calculated the neutrino opacity in stellar matter, taking into account all possible processes. The main contribution to neutrino opacity comes from the neutrino-electron, neutrino-neu tron and neutrino-nucleus scattering[7]. With respect to neutrino--nucleus scattering, Freedman[8] showed the importance of the coherent effects in the elastic channel, whose cross section is proportional do A^2, just like the electron-nucleus Mott scattering cross section is proportional to Z^2.

However, the Freedman's calculations did not considered the effects of the nuclear structure, although he pointed out the possible importance of them. As the neutrino energies, in the range of density and temperature we are interested, comes rise to some tens of MeV, it is desirable to include these effects. In this letter, we present an expression of neutrino-nucleus scattering including the inelastic channels, i.e., the nuclear excitation effects.

We start with the effective Lagrangean density for the neutrino-nucleus scattering,

$$\mathcal{L}_{eff} = \frac{G}{\sqrt{2}}\, j_\mu J^\mu \quad , \tag{1}$$

where j_μ and J^μ refer neutrino and nucleonic currents, respectively. In Eq. (1), we assumed $E_\nu << m_B$, m_B denoting the mass of the intermediate boson. The neutrino current, in obvious notation, is given by as usual

$$j_\mu = \bar{\nu}\, \gamma^\mu (1-\gamma_5)\nu \tag{2}$$

and the nucleonic current, obtained from the Salam-Weinberg theory[9] is

$$J^\mu = \bar{n}\, \frac{\tau_3}{2}\, \gamma^\mu (1-\gamma_5) n - 2\, \mathrm{sen}^2\theta_W\, \bar{n}\, \frac{1+\tau_3}{2}\, \gamma^\mu\, n \tag{3}$$

For convenience, we decompose the nucleonic current into three parts:

$$J = a_0 \, J_{SV} + a_1 \, J_{VV} - J_{VA} \quad ,$$

where the first term is the isoscalar-vector (SV) current, the second, the isovector-vector (VV) current and the last, the isovector-axial (VA) current, with $a_0 = -\mathrm{sen}^2\theta_W$ and $a_1 = 1-2\mathrm{sen}^2\theta_W$.

Denoting the neutrino momentum by $k = (\omega, \mathbf{k})$ and nuclear state by $|N\rangle$, the transition rate $W_{i\to f}$ from the initial state $|N_i k_i\rangle$ to the final state $|N_f k_f\rangle$, in natural units, is given by

$$W_{i\to f} = 2\pi \left| \langle N_f k_f | L_{eff} | N_i k_i \rangle \right|^2 \delta(E_f + \omega_f - E_i - \omega_i) \quad , \tag{5}$$

where $L_{eff} = \int \mathcal{L}_{eff} \, d^3 r$.

After a straightforward trace calculation, we get

$$W_{i\to f} = \frac{G^2}{(2\pi)^2} \frac{1}{\omega_i \omega_f} \delta(E_f + \omega_f - E_i - \omega_i) \times$$

$$\Big[k_i \cdot \langle J\rangle \; k_f \cdot \langle J\rangle^* - k_i \cdot k_f \; \langle J\rangle \cdot \langle J\rangle^*$$

$$+ k_i \cdot \langle J\rangle^* \; k_f \cdot \langle J\rangle \Big] \quad , \tag{6}$$

where $a \cdot b = a_\mu b^\mu = a^0 b^0 - \mathbf{a} \cdot \mathbf{b}$, and $\langle J^\mu \rangle = \langle N_f | J^\mu | N_i \rangle$.

In the nonrelativistic approximation for nucleons, we get the final expression for the double differential cross section

$$\frac{d^2\sigma}{d\omega_f d\Omega} = \frac{G^2}{(2\pi)^2} \omega_f^2 \Big[\Big\{ a_0^2 \Big| \int e^{-i\mathbf{q}\cdot\mathbf{r}} \Big|^2 +$$

$$+ \frac{1}{4} a_1^2 \Big| \int \tau_3 \, e^{-i\mathbf{q}\cdot\mathbf{r}} \Big|^2 \Big\} (1+\cos\theta) +$$

$$+ \frac{1}{4} C_A^{\;2} \left| \int \tau_3 \, \boldsymbol{\sigma} \, e^{-i\mathbf{q}.\mathbf{r}} \right|^2 (1-\frac{1}{3}\cos\theta) \Bigg] \quad , \qquad (7)$$

where $C_A \simeq 1.2$ is the axial vector renormalization constant, taken from the nuclear beta decay theory. In Eq. (7), we adopted the similar notation of nuclear beta decay matrix element for an operator Ω as

$$\int \Omega = \int d^3r_1 \ldots d^3r_A \; \psi_f^*(\mathbf{r}_1,\ldots,\mathbf{r}_A) \sum_{n=1}^{A} \Omega_n \psi_i(\mathbf{r}_1,\ldots,\mathbf{r}_A) \quad , \quad (8)$$

where $\psi_{i(f)}$ is the nuclear wave function for the initial (final) state, $\mathbf{q} = \mathbf{k}_f - \mathbf{k}_i$ the neutrino momentum transfer, and the summation is taken over all A nucleons.

We can expand the nuclear matrix elements appearing in Eq. (7) as

$$\left| \int e^{-i\mathbf{q}.\mathbf{r}} \right|^2 \simeq \left| \int \mathbb{1} \right|^2 + \left| \mathbf{q}. \int \mathbf{r} \right|^2 + 0\left[(qR)^4\right]$$

$$\left| \int \tau_3 \, e^{-i\mathbf{q}.\mathbf{r}} \right|^2 \simeq \left| \int \tau_3 \right|^2 + \left| \mathbf{q}. \int \tau_3 \mathbf{r} \right|^2 + 0\left[(qR)^4\right]$$

$$\left| \int \tau_3 \, \boldsymbol{\sigma} \, e^{-i\mathbf{q}.\mathbf{r}} \right|^2 \simeq \left| \int \tau_3 \, \boldsymbol{\sigma} \right|^2 + \left| \mathbf{q}. \int \mathbf{r} \tau_3 \, \boldsymbol{\sigma} \right|^2 + 0\left[(qR)^4\right] .$$

This is justified because, for $E_\nu \sim 20$ MeV, $\langle 1-\cos\theta \rangle = 0.66$[10] and $R \sim 5$fm, we get $(qR)^2 \sim 10^{-1}$.

The terms $|\int \mathbb{1}|^2$ and $|\int \tau_3|^2$ contribute only to the elastic channels, since the operators $\mathbb{1}$ and τ_3 commute with the total Hamiltonian. On the other hand, contributions to inelastic scattering come from all of the rest.

The first term $|\int \mathbb{1}|^2$ is equal to $A^2 \, \delta(E_i - E_f)$, so that the cross section due to this term coincides exactly with the Freedman's formula. Tubbs and Schramm[10] extended the

Freedman's formula by including the other elastic term $|\int\tau_3|^2$, which is equal to $(N-Z)^2\ \delta(E_i-E_f)$.

The ratio $|\int\tau_3|^2 / |\int \mathbb{1}|^2 \sim 0.5\%$, if the abundance peak is assumed to be localized on $^{56}_{26}Fe$. For neutron-rich nuclei, which are more probable for collapsing core of supernova, this ratio increases even up to 12% at the neutron drip line for A = = 56. Therefore, the VV current contribution is of the certain importance at the later stage of collapsing core.

The main contributions to the inelastic part of the cross section come from the dipole term of SV current $|\mathbf{q}.\int\mathbf{r}|^2$, and from the monopole term of the VA current $|\int\tau_3\boldsymbol{\sigma}|^2$, which are proportional to $(qR)^2\ A^2$ and $(N-Z)^2$, respectively. Direct comparison of these terms with respect to A^2 gives the contribution of $\sim$ 10% for the former and $\sim$12% at maximum for the latter. Furthermore, the latter term is enhanced by the factor $(\frac{1}{2}\ C_A/a_0)^2 \simeq 3$.

However, the total inelastic cross section is reduced by a factor due to the smaller phase space available. For the SV term, this reduction is much more effective due to the presence of $q^2 \simeq \omega^2(1-\cos\theta)$. Consequently, we conclude the principal contribution to inelastic process comes from the VA current. In order to estimate the VA cross section, it is necessary to known the energy dependence of the strength function $S(E) \equiv \rho(E)\ \overline{|\int\tau_3\boldsymbol{\sigma}|^2}$, where $\rho(E)$ is the nuclear level density. A possible contribution to this strength function is the giant magnetic monopole resonance. If the resonance energy is small ($\sim$5MeV) compared to the neutrino incident energy, the reduction of phase space may not be critical.

We presented in this letter the complete formula for the neutrino-nucleus scattering with the purpose of application to the neutrino transport problem in the highly evolved stellar core. In addition to the coherent elastic scattering terms discussed by Freedman and by Tubbs and Schramm, it is pointed out a possible importance of inelastic scattering due to the giant magnetic monopole resonance. We should note that, if this effect is appreciable, the angular distribution of scattered neutrinos will be modified by the same amount. Furthermore, even in less favorable cases, the energy loss of neutrinos by inelastic neutrino-nucleus scattering is much greater than that in the elastic scattering due to nucleus recoiling ($\sim$ 0.01%). This suggests that the inelastic channels should be taken into account in a more detailed treatment of questions like neutrino thermalization and neutrino confinement.

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