ISSN 0029-3865

CBPF-NF-044/84

# STOCHASTIC BEHAVIOUR OF DE SITTER UNIVERSES

by

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ABSTRACT

A stochastization procedure applied to mini-universes (friedmons) leads to a nonzero average minimum radius and thus to the exclusion of classical singularities.

Key-words: Stochastization; Mini-cosmos.

Although one of the most basic concepts concerning the Universe, nowadays, be that of its unity – associated to the image of totalization we commonly use to picture it – recently there have been speculations about more complex structures, in which totalities like (e.g.) de Sitter cosmos were able to exist as distinct entities. Some authors have proposed to think of such mini-cosmos making up gatherings of bubbles, each evolving inside almost unconnected regions; others, as nuclei of (elementary) particles. The main known effort in this direction is due to Markov[1], who analysed the case of "friedmons" : mini-Friedman-like universes.

Of course, one such idea, in order to be seriously considered, should provide a mechanism of interaction among these mini-cosmos, lest their isolate existence, closed around themselves, would render senseless any collective affirmation about their behaviour. It seems that the most direct way to do that would be to split such collections of mini-cosmos into individual systems, each one perceiving the remaining systems as a perturbative effect of random character – as in a stochastic process. Here we develop a simple example of this procedure; other stochastic approaches to cosmological questions are due to Ginzburg et al.[2], Novello[3], and Gruszczak et al.[4].

De Sitter universe is characterized by a Riemannian metric line element (written in the Gaussian co-moving system of coordinates) of the form

$$ds^2 = dt^2 - R^2(t)d\Omega^2 \quad , \tag{1}$$

where

$$d\Omega^2 = d\chi^2 + f(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (2)$$

and

$$f(\chi) = \sin\chi, \sinh\chi \quad \text{or } \chi \quad . \qquad (3)$$

Einstein's equations, concerning the only free function, the radius R(t), are, in the vacuum,

$$3\dot{R}^2 + \frac{3\varepsilon}{R^2} = -\Lambda \qquad (4a)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{\varepsilon}{R^2} = -\Lambda \qquad (4b)$$

where $\varepsilon = 0, \pm 1$ is the curvature index, and $\Lambda$ a cosmological constant.

If we take $\varepsilon = -1$ ("open" case), these equations reduce to

$$\dot{R}^2 + \frac{\Lambda}{3} R^2 = 1 \quad , \qquad (5)$$

which is nothing but the "conservation of energy" condition for an harmonic oscillator.

Setting $R = q$, $\dot{R} = p$ (for the associated unit-mass momentum) and $\omega^2 = \frac{\Lambda}{3}$, we have for the associated Hamiltonian $H(q,p)$ the usual expression,

$$H = \frac{p^2}{2(m)} + \frac{(m)\omega^2}{2} q^2 =$$

$$= \frac{p^2}{2} + \frac{\omega^2}{2} q^2 \quad . \qquad (6)$$

The corresponding equations of motion are

$$\dot{q} = p/(m) = p \quad , \qquad \dot{p} = -\omega^2 (m) q = -\omega^2 q \quad , \qquad (7)$$

with general classical solution

$$q_{c\ell}(t) = q_0 \cos\omega t + p_0 \sin\omega t \qquad (8)$$

$$p_{c\ell}(t) = p_0 \cos\omega t - q_0 \sin\omega t \qquad (9)$$

where $(q_0, p_0)$ are initial data at $t = 0$, and the frequency $\omega$ is related to the cosmological constant $\Lambda$ by

$$\omega = \sqrt{\frac{\Lambda}{3}} \, . \qquad (10)$$

The Schrödinger equation for the wavefunction $\psi(x,t)$ associated to this classical harmonic oscillator structure, that is,

$$ih \frac{\partial}{\partial t} \psi = - \frac{\hbar^2}{2(m)} \nabla^2 \psi + (m) \frac{\omega^2}{2} x^2 \psi \quad , \qquad (11)$$

admits solutions of the form

$$\psi = \sqrt{\rho} \, e^{i/\hbar \, S} \quad , \qquad (12)$$

such that the quantum probability density $\rho(x,t)$ and phase $S(x,t)$ are

$$\rho = \frac{1}{\sqrt{2\pi}\,\sigma} \, e^{-\frac{1}{2}\left(\frac{x - x_{c\ell}}{\sigma}\right)^2} \qquad (13)$$

$$S = x p_{c\ell} - \frac{1}{2} p_{c\ell} q_{c\ell} + \frac{\hbar \omega t}{2} \qquad (14)$$

along with (Gaussian) mean value and variance given by

$$\langle x\rangle = x_{c\ell} \quad , \tag{15a}$$

$$\sigma^2 = (\langle x^2\rangle - \langle x\rangle^2) = \frac{\hbar}{2\omega(m)} \quad . \tag{15b}$$

In the so-called hydrodynamical or Madelung-fluid description of quantum mechanics[5], density $\rho$, phase S and Hamiltonian H of the classical correspondent of a given quantum system are related to a fluid velocity $\vec{v}$, defined by

$$\vec{v} = \frac{\nabla S}{m} \quad , \tag{16}$$

and which satisfies a law of conservation of probability,

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0 \quad , \tag{17}$$

in such a way that the dynamics is expressed by

$$\frac{\partial S}{\partial t} + H_M = 0 \quad , \tag{18}$$

where $H_M$ is built as

$$H_M = H_{classical} + H_{diffusion} = \frac{p^2}{2m} + U - \frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho} \quad . \tag{19}$$

Using the phase given by eq. (14), we obtain

$$v = \left|\frac{\nabla S}{(m)}\right| = \frac{p_{c\ell}}{(m)} \quad , \tag{20}$$

and using the density given by eq. (13), we can also obtain the osmotic velocity $\delta\vec{v}$ defined by

$$\delta\vec{v} = \nu \frac{\nabla\rho}{\rho} \quad ; \tag{21}$$

with diffusion coefficient $\nu$ equal to the usual value,

$$\nu = \hbar/2(m) \quad , \tag{22}$$

this amounts to

$$\delta v = -\omega(x - <x>) \quad . \tag{23}$$

Then Nelson's forward and backward drift velocities $v_{(+)}$ and $v_{(-)}$ [6] turn out to be

$$v_{(+)} = v + \delta v = p_{c\ell}/(m) - \omega(x-<x>) \tag{24a}$$

$$v_{(-)} = v - \delta v = p_{c\ell}/(m) + \omega(x-<x>) \tag{24b}$$

so that the pertinent Langevin equation can be written as

$$dq(t,W) = v_{(+)}(q(t),t)dt + dW(t) = [p_{c\ell}/(m)-\omega(x-<x>)]dt+dW(t) \quad , \tag{25}$$

in which $dW(t)$ is a Wiener process[7] such that

$$E[dW^i] = 0 \tag{26a}$$

$$E[dW^i dW^j] = 2\nu\delta^{ij}dt = \hbar\delta^{ij}dt \tag{26b}$$

where E[ ] stands for an expectation value procedure. Hence, for de Sitter case we obtain

$$dR(t,W) = [\dot{R}_{c\ell}(t)-\omega(R(t)-R_{c\ell}(t))]dt+dW(t) . \tag{27}$$

The manifestly Gaussian nature of the problem implies that[8]

$$E[R(t,W)] = R_{c\ell}(t) \tag{28a}$$

$$E[R^2(t,W)] = R^2_{c\ell}(t) + \sigma^2 = R^2_{c\ell}(t) + \frac{\sqrt{3}}{2}\frac{\hbar}{\sqrt{\Lambda}} \tag{28b}$$

according to eq. (15b). We can see from these expressions that the net effect of the environment is the preclusion of the collapse of the model: the classical (i.e., non-stochastic) singularity disappears, since the minimum of the average of the square radius $R^2(t)$ (which is the relevant quantity to be put into the line element $dS^2$ of the geometry) results proportional to $\Lambda^{-1/2}$. Thus, if $\Lambda$ is small, then the radius is great. In fact, a simple evaluation (in units $m = \hbar = c = 1$) gives $\hbar/\sqrt{\Lambda} \sim 10^{-9}$cm, so that the corresponding minimum radius lenght is of the order of $10^{-2}$cm – by many orders of magnitude greater than Planck's lenght. This results seems at first glance very strange, since there is a widely spread belief that the classical singularity could be avoided by quantum effects, whose occurrence would become important near Planck's lenght – and here, instead, stochastization led to a stopping of the collapse at almost macroscopic dimensions (in this connection, see also the work of Trautman[9]). We note, however, that our evaluation employed the present, very small, value of the cosmological constant; if this is allowed to change, then it is possible that in the past those "pure" quantum effects have played their admitted role. We remark also that it is possible to fix arbitrarily the value of the cosmological constant for each bubble or mini-cosmos, thereby resulting different sorts of minimum radii.

The authors wish to thank Dr. Hans Heintzmann for valuable discussions.

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