CBPF-NF-044/01

A Model for the Pressure Fluctuations inside Fuselages

Luiz C.L. Botelho

Permanent address:Departamento de Física
Universidade Federal Rural do Rio de Janeiro
23851-180 Itaguaí, Rio de Janeiro, RJ, Brasil

Abstract

We estimate the pressure fluctuation inside fuselages.

One of the most important point on the designers of fuselages for missiles, torpedous, etc... with very sensitive electronics and radioactive liquid explosives inside is to estimate the turbulent pressure component transmission to the fuselage interior ([1]).

In this brief report, we propose a mathematical-pedagogical model to analyze this problem.

Let us consider a infinite beam backed on the lower side by a space of depth d which is filled with a fluid of density ρ_2 and sound speed v_2. On the upper side of the beam which is modelling the torpedous' fuselage there is a supersonic boundary-layer turbulent pressure P. The fluid on the upper side of the beam which is on the turbulence is supposed to have a free stream velocity U_∞, density ρ_1 and sound speed v_1.

The effective equation governing the "outside" pressure in our model is given by $(d \le z < \infty)$

$$\left(\frac{\partial}{dt} + U_\infty \frac{\partial}{\partial x}\right)^2 p_1(x,z,t) - v_1^2 \left(\frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 z}\right) p_1(x,z,t) = 0 \qquad (1.a)$$

with the boundary condition (2^{nd} Newton's law)

$$-\rho_1 \left(\frac{\partial}{dt} + U_\infty \frac{\partial}{\partial x}\right)^2 W(x,t) = \frac{\partial p_1(x,z,t)}{\partial z}\Big|_{z=d} \qquad (1.b)$$

Here the beam's deflection $W(x,t)$ is given by the beam small deflection equation

$$B\ \frac{\partial^4 W(x,t)}{\partial^4 x} + m\ \frac{\partial^2 W(x,t)}{\partial^2 t} = P(x,t) + (p_1 - p_2)(x,d,t) \tag{2}$$

with B denoting the bending rigidity, m the mass per unit lenght of the beam and $p(x,z,t)$ the (supersonic) boundary-layer pressure fluctuation of the exterior medium.

The searched induced pressure $p_2(x,z,t)$ in the fluid interior medium $(0 \leq z \leq d)$ is governed by

$$\frac{\partial^2 p_2}{\partial^2 t}(x,z,t) - (v_2)^2 \left(\frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 z} \right) p_2(x,z,t) = 0 \tag{3.a}$$

$$\frac{\partial p_2(x,d,t)}{\partial z} = -\rho_2\ \frac{\partial^2}{\partial^2 t}\ W(x,t) \tag{3.b}$$

The solution of eq. (1-a) and eq. (1-b) is straightforward obtained in the Fourier domain

$$\tilde{p}_1(k,z,w) = -\rho_1 \left\{ \frac{v_1(w+U_\infty k)^2}{\sqrt{v_1^2 k^2 - (w+U_\infty k)^2}} \right\} \tilde{W}(k,w)$$
$$exp(-\frac{1}{v_1} - \sqrt{v_1^2 k^2 - (w+U_\infty k)^2})(z-d) \tag{4}$$

where the deflection beam $\tilde{W}(k,w)$ is explicitly given by

$$\tilde{W}(k,w) = (\tilde{P}(k,w) - \tilde{p}_2(k,w,d)$$
$$\left\{ Bk^4 - mw^2 + \frac{\rho_1(w+U_\infty k)^2 v_1}{\sqrt{v_1^2 k^2 - (w+U_\infty \cdot k)^2}} \right\}^{-1} \tag{5}$$

At this point, we solve our problem of determining the pressure $\tilde{p}_2(k,w,z)$ in the interior domain eq. (3-a), eq. (3-b) if one knows the pressure $\tilde{p}_2(k,w,d)$ on the "fuselage". Let us, thus, consider the Taylor's serie in the z-variable $(0 \leq z \leq d)$ around $z = d$, namely

$$\tilde{p}_2(k,w,z) = \tilde{p}_2(k,w,d) + \frac{\partial p_2(k,w,z)}{\partial z}\Big|_{z=d}(z-d) + \cdots + \frac{1}{k!}\frac{\partial^k \tilde{p}_2(k,w,z)}{\partial^k z}\ (z-d)^k + \cdots \tag{6}$$

From the boundary condition eq. (3-b), we have the explicitly expression for the second-derivative on the depth z

$$\frac{\partial p_2(k,w,z)}{\partial z}\Big|_{z=d} = +\rho_2 w^2\ \tilde{W}(k,w) =$$
$$+\rho_2 w^2[\tilde{P}(k,w) - \tilde{p}_2(k,w,d)] \times \left\{ Bk^4 - mw^2 + \frac{\rho_1(w+U_\infty k)^2 v_1}{\sqrt{v_1 k^2 - (w+U_\infty k)^2}} \right\}^{-1} \tag{7}$$

The second z-derivative of the interior pressure (and the higher ones!) are easily obtained recursively from the wave equation (3-a) ($k \geq 0, k \in \mathbb{Z}^{+}$)

$$\frac{\partial^{2+k}}{\partial^{2+k} z} \tilde{p}_2(k,z,t)\Big|_{z=d} = \left(\frac{1}{(v_2)^2} \times (-w^2 + (v_2)^2 k^2)\right) \left(\frac{\partial^{2+k-1}}{\partial^{2+k-1}} \tilde{p}_2(k,z,t)\right)\Big|_{z=d)} \tag{8}$$

Let us finally comment that the general turbulent pressure is assumed to be expressible in a Stieltjes integral

$$P(x,t) = \int \int_{-\infty}^{+\infty} e^{ikx} e^{i(w-ku)t} dF(k,u) dG(u) \tag{9}$$

where $G(u)$ is a random function ([2]).

Acknowledgements

We are thankful to CBPF/MCT for warm hospitality. We herewith state our acknowledgements to Alexander Von Humboldt Stiftung-Deutschland for financial support during 1991–1992 where my project research on waves physics has begun with Av.H Fellowship, besides all this research was carried out under the philosophy-"SLV's-Vehicles for the third world".

REFERENCES

[1] D.J. Mead, "Free wave propagation in periodically supported infinite beams", J. Sound Vib, 11, 181 (1970).
- R. Vaicaitis and Y.K. Lin, "Response of finite periodic beam to turbulence boundary-layer pressure excitation", AIAAJ 10, 1020 (1972).

[2] L. Maestrello, J.H. Monteith, J.C. Manning and D.L. Smith, "Measured response of a complex structure to supersonic turbulent boundary layers, AIAA 14th Aerospace Science Meeting, Washington, DC (1976).