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The Effect of Contaminants on the Rio de Janeiro's Beaches Surface Waves – A Mathematical Exactly Soluble Model

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Abstract

We model the effect of carioca's línguas negras on the coastal waves (beaches) by means of a fluid with an effective viscosity $\nu_{sh} \equiv \nu_{\text{shear}}$

One of the most interesting problems in Fluid Waves Physics with a large spectrum for applications is that one related to the propagation of waves on surface fluids ([1]).

Although its intrinsic mathematical difficulty to solve the associated motion equations, it is very fortunate for hydrodynamicist to have few models exactly soluble ([1]). Another point worth remark is that among these fews exactly soluble models, one never considers the existence of internal friction on the fluid motion of the form of usual mechanical rigid-body linear damping proportional to velocity; a experimental fact relevant for irrotational fluid motions at very low frequencies, like water in reservoirs, bays, oceans, etc...

Our aim in this note is to re-analyze the surface waves on fluids in the presence of the above cited mechanical damping ([2]).

A word for applications: It is well known that contaminents of biological origin, like the famous "línguas negras" (plumes) on the Carioca's beaches, can be heavily spread in the fluid medium, and, thus being an ecological candidate for our study ([3]).

Let us start our analysis by considering the usual (irrotational and incompressible) two-dimensional Navier-Stokes for very low fluid velocities (the so called Bernoulli-equation!) in the presence of the gravity and a (small) viscosity parameter ν_{sh}

$$\text{grad}\left(\frac{\partial\sigma(x,z;t)}{\partial t} + \frac{1}{\rho}\ \rho(x,z;t) + \nu_{sh}\ \sigma(x,z;t) + gz\right) = 0 \tag{1}$$

Here $\sigma(x,z;t)$ is the hydrodynamical potential. The incompressibility condition on the fluid is taken as a realistic assumption, since the carioca's línguas negras do not have any sensible thermodynamical behaviour on the fluid

$$\frac{\partial^2\sigma(x,g;t)}{\partial^2 x} + \frac{\partial^2\sigma(x,z;t)}{\partial^2 z} = 0 \tag{2}$$

Finally, the elevation fluid surface $\eta(x,t)$ at a time $t > 0$ (for small fluid oscillations) is given by the wave equation at $z = 0$

$$0 = -\rho\frac{\partial\sigma(x,z;t)}{\partial t}\Big|_{z=0} - pg\ \eta(x,t) \tag{3}$$

As a last very important hypothesis, we consider our fluid surface ilimited $(-\infty < x < \infty)$ and the fluid volume has a finite depth $h(-h \leq z \leq 0)$.

In order to solve the Wave's fluid systems eq. (1)-eq. (3) we consider the "ansatz"

$$\sigma(x,z;t) = exp(-\nu_{sh}\cdot t)\overline{\sigma}(x,z;t) \tag{4}$$

It is straightforward to see that the Fourier transformed fluid velocity potential satisfies the following ordinary differential equation

$$\hat{\Phi}(k;z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx\ e^{ikx} \overline{\sigma}(x,z;t) \tag{5.a}$$

$$\hat{\Phi}(k,z;t) = \overline{A}(k,t) cosh[k(z+h)] \tag{5.b}$$

$$\frac{d^2\overline{A}(k;t)}{d^2t} - \nu_{sh}\frac{d\overline{A}(k;t)}{dt} + w^2(h)\overline{A}(k,t) = 0 \tag{5.c}$$

with the depth-dependent dispersion relation

$$w^2(h) = g\ tgh[k(z+h)] \tag{6}$$

At this point we point out that the modulation wave factor (see eq. (4)) $A(k,t) = exp(-\nu_{sh}t)\overline{A}(k,t)$ is given explicitly by the following formulae

$$A(k;t) = e^{-\left(\frac{\nu_{sh}}{2}\right)t}\left[\alpha(k)e^{i\Omega(h,k_q\nu_{sh})t} + \beta(k)e^{-i\Omega(h,k,\nu_{sh})t}\right] cosh(k(z+h)) \tag{7}$$

where

$$\Omega(h,k,\nu_{sh}) = \left(\sqrt{4w^2(h) - (\nu_{sh})^2}\right)\Big/2 \tag{8}$$

The coefficients $\alpha(k)$ and $\beta(k)$ are given by means of the initial conditions

$$\sigma(x,0,0) = 0 \qquad \leftrightarrow \qquad \alpha(k) + \beta(k) = 0 \tag{9}$$

$$\frac{\partial}{\partial t}\sigma(x,z,t)\Big|_{z=0;t=0} = gf(x) \leftrightarrow \alpha(k) = \frac{g}{2i\Omega(h,k,\nu_{sh})}\frac{\tilde{f}(k)}{cos(kh)} \tag{10}$$

here

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{ikx} f(x)dx \tag{11}$$

By grouping together all the results above obtained, we get our final formulae for the hydrodynamical potential given as a Fourier Integral

$$\sigma(x,z;t) = \frac{ge^{-\left(\frac{\nu_{sh}}{2}\right)t}}{\sqrt{2\pi}}\int_0^{+\infty} \frac{2\tilde{f}(k)}{\sqrt{4\Omega^2(h,k,\nu_{sh}) - \nu_{sh}^2}}$$
$$\frac{cosh[k(z+h)]}{cos(kh)}\{sen(kx + \Omega(h,k,\nu_{sh})t) - sen(kx - \Omega(h,k,\nu_{sh})t)\} \tag{12}$$

As an important application of eq. (12), let us take the initial profile, a periodic pulse $f(x) = \sum_{n=-\infty}^{+\infty} f_n\ exp\{\frac{2\pi n}{L}x\}$. We have, thus, the following leading profile for $t > 0$ and

ν_{sh} small

$$n(x,t) \cong -\frac{1}{\sqrt{2\pi}} \left\{ \sum_{n=-\infty}^{+\infty} \frac{e^{-\left(\frac{\nu_{sh}}{2}\right)t}}{\sqrt{4\Omega^2(\nu_{sh}, h, \frac{2\pi n}{L}) - \left(\frac{\nu_{sh}}{2}\right)^2}} \Omega(\nu_{sh}, h, \frac{2\pi n}{L}) \right.$$
$$\left. \left(cos\left[(\frac{2\pi n}{L})x + \Omega(\nu_{sh}, h, \frac{2\pi n}{L})t \right] - cos\left[(\frac{2\pi n}{L})x - \Omega(\nu_{sh}, h, \frac{2\pi n}{L})t \right] \right) \right\} \quad (13)$$

As a consequence of the above formulae, one can make the following sad conclusion for the surf in Rio de Janeiro: the heavy presence of contaminants of the kind $\nu_{sh} \cdot t$ (línguas negras) attenuates strongly the beaches' waves.

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REFERENCES

[1] Ian N. Sneddon, "Fourier Transforms", McGraw-Hill Book Company, 1951.

[2] L. Landau et Lifchitz, "Mécanique des Fluides", Editions Mir, Moscow, 1971.

[3] A.R. Osborne et al., "Topics in Ocean Physics", International School of Physics Enrico Fermi, 1982.